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Abstract– This paper presents experimental results on the characterization of dynamics and synchronization of networks of nonlinear oscillators with dynamic links. The results are obtained using a new experimental setup. Accurate evaluation of synchronization with dynamic coupling is reported, with reference to a network of Chua oscillators, each settled onto a periodic orbit. The observed synchronization levels, as function of the dynamic link parameters, give a picture of the synchronization area in parameter space which is in agreement with previous theoretical predictions.

1. Introduction

Many natural and engineered systems, such as biological neural networks, metabolic systems, electrical power grids, the World Wide Web, etc. can be classified as complex networks, composed by a large number of highly interconnected elementary dynamical units (nodes), whose collective behavior differs substantially from their individual "stand-alone" dynamics [1].

A particular type of collective behavior is synchronization, where all nodes of the network converge towards some common dynamic solution [2]. The problem of studying synchronization in networks of non linear oscillators has attracted considerable research effort. Much of this effort has been dedicated to the investigation of the relationship between the synchronization dynamics and the nature and the strength of the coupling, as well as the topology of the network. Fewer results are available as experimental verification of synchronization. Basic results are reported in [3] for a simple two nodes system, whereas in [4] the case of four nodes with diffusive (resistive links) have been explored, for various coupling topologies. A CNN dedicated architecture implementation is presented in [5].

A new setup, designed to extend and generalize previous experimental work, has been realized as described in [6] by some of the authors, where the specific problem of synchronization in the presence of non identical nodes has been addressed.

The present paper is aimed at using this novel experimental test bed to study the case of dynamic link (*R-C*) in a network of four Chua's. Effects such as fingering in the synchronization region and doubled thresholds, theoretically predicted in [7] but never confirmed experimentally, are observed here for the first time.

2. The Experimental Setup

The experimental setup, shown in Fig. 1, consists of a set of *N* Chua circuits acting as network dynamical nodes, with adjustable parameter values. The nodes are coupled via configurable resistive and dynamical links, in prescribed topologies. A unique characteristic of the setup is the possibility of reconfiguring, by means of a digital control, the properties of the single nodes and of the interconnecting network.

A modular USB multi-channel acquisition system is used to measure and monitor the variables of interest in the experimental setup, allowing real time evaluation of the synchronization level. The whole network is controlled from a PC in LabVIEW, so realizing the complete automation of the experimental campaign.

A snapshot of the experimental setup is shown in Fig. 1, where the modular structure and main subparts are evidenced; a more detailed description, exceeding the scope of the present paper, can be found in [6]. In what follows we will briefly describe its main features.

2.1. Chua's circuits layer implementation

The realization of the network nodes as Chua's circuits follows the classical implementation by Kennedy [8], with the further improvement proposed by Aguirre et al. in [9] (inductor-less realization). A schematic representation of an ideal Chua's circuit is shown in Fig. 2, together with the characteristic of the "Chua's diode".

Passive component selection to 0.1% tolerance has been considered for the realization, in order to build almost identical circuits. The values of the nominal circuit parameters are reported in Tab. 1. For such combination of nominal parameters, and assuming *R* as basic bifurcation parameter for the (uncoupled) circuit, the value of *R*=1985Ω guarantees a limit cycle attractor, as desired.
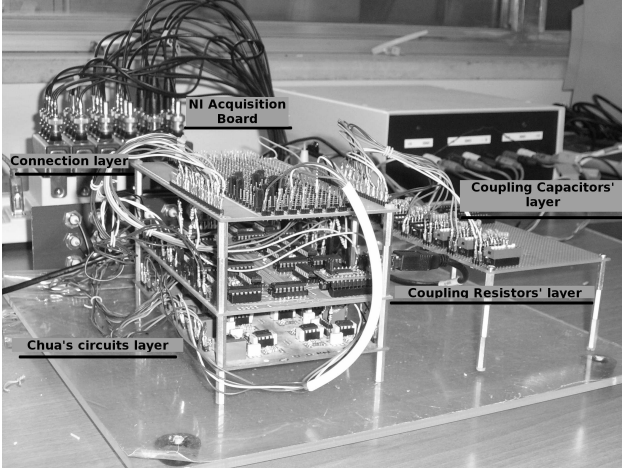
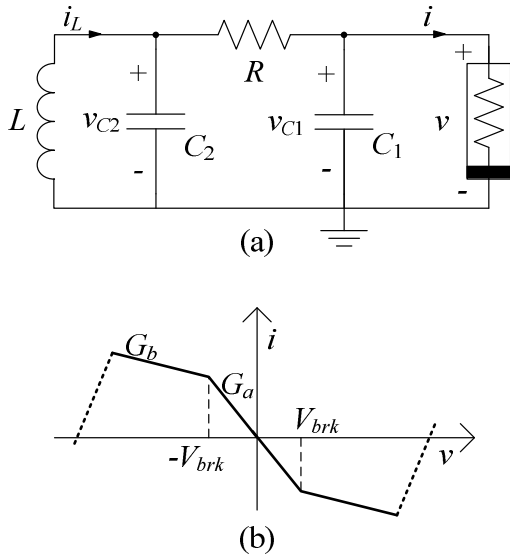Figure 1. The complete experimental setup.

(a)

(b)

Figure 2. Chua's circuit (a) and characteristic of non-linear resistor (b).

A single PCB with 8 Chua's dynamical nodes has been realized as the elementary unit of such a modular system (Chua's circuits layer in Fig. 1).

TABLE I. CHUA'S CIRCUIT ELEMENTS NOMINAL VALUES

Name	Description		
	parameter type	*value*	*unit*
C_1	Capacitance	10	nF
C_2	Capacitance	100	nF
L	Inductance	18	mH
G_a	Conductance	-0.756	mS
G_b	Conductance	-0.409	mS
V_{brk}	Voltage	1.089	V

2.2. Coupling impedances layer implementation

Multi-valued discrete impedances (R,C) sets have been realized with digitally controlled switches, enabling their insertion in a series-parallel scheme. This allows to accurately evaluate the dynamics and synchronization of the coupled *N*-nodes network as a function of the link impedances values.

Each link resistor is realized as a series of 8 step values; the desired value of the coupling resistance R_c is obtained by setting the switches, which are finally controlled by a USB interface cascaded to a logic port and a decoder. In Fig. 3 a block schematic of the resistors set is shown. For the present experiments the module has been designed to scan the range 0÷12750 Ω in 2^8 =256 uniform steps of 50 Ω. Moreover, an "offset" precision resistor can be placed in series, in order to widen the range of values assumed by the coupling resistors, keeping the maximum accuracy of 50 Ω steps.

Dynamic link coupling impedances are implemented by capacitor sets C_c connected in parallel to resistors R_c. The implementation of a switched PCB, is in principle quite similar to the one described above, although series connections have to be replaced by parallel ones. For the experiment reported in this paper a simpler version, with DIP switches has been used. Each link capacitor value is obtained as a parallel of 8 step values. We considered the range 0÷1.035 nF in steps of 0.035 nF. Once the proper capacitance value has been chosen, each capacitor is placed in parallel to the corresponding link resistor in order to implement a dynamic link between the nodes of the network.

2.3. Interconnection layer implementation

A key property of the set-up is its general and reconfigurable topology. It is realized by means of the
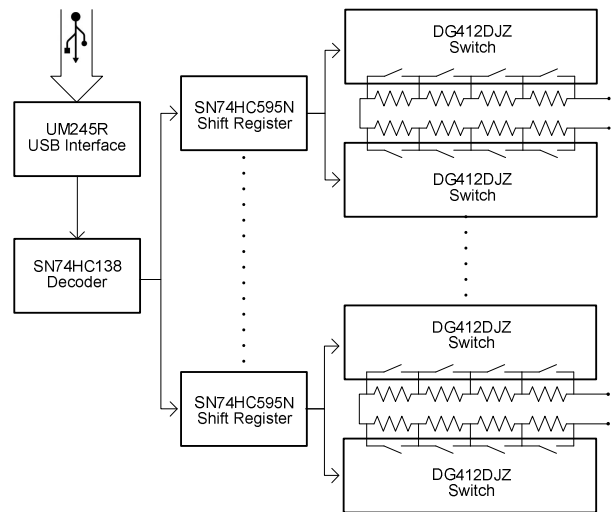

Figure 3. USB controlled resistor for the coupling resistors' layer.

connection layer, visible in Fig. 1. At the present status of the project, arbitrary topologies are defined (before starting an experimental campaign) with the configuration of jumpers on a connection matrix PCB. Further developments will allow such definition again with a USB controlled set of switches.

For the sake of brevity, in the present work we consider only a basic network configuration with 4 nodes, fully connected (Fig. 4). Further and more extended results will be published elsewhere.

2.4. Automatic data acquisition and experiment control

The data acquisition board allows to record the time evolution of the Chua's circuit state variables for each node of the network, namely the voltage v_{c1} across capacitor C_1, the voltage v_{c2} across capacitor C_2 and the i_L inductor current. Because of the relatively large number of variables (channels) to be concurrently acquired, a modular National Instruments USB acquisition system (NI CompactDAQ with 8 NI9215 modules) has been used, allowing a maximum of 32 channels per single USB unit.

The experiment control and data acquisition is governed by specific software developed within the LABVIEW environment. Data batches are saved either automatically or at the operator request.

3. Experimental Results

Here we report results of synchronization experiments carried out on a 4-nodes network, with dynamic undirected couplings, having equal weights on each link. The coupling has been implemented on the first state variable v_{c1}.

As for the evaluation of synchronization levels, we have calculated the index I_s which describes the rms distance of trajectories $x_i^j(t)$ from the "average" trajectory $\overline{x}^j(t)$. After defining the difference for the j-th state of the i-th node as $\Delta x_i^j = x_i^j - \overline{x}^j$, the index can be defined (in percent) as:
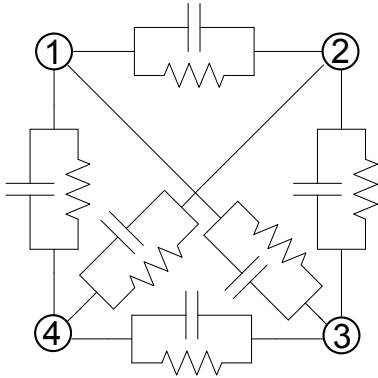
$$I_s = 100 \sqrt{\frac{1}{N_S N_C} \sum_{i=1}^{N_C} \sum_{k=1}^{N_S} \left(\frac{\Delta x_i^j(k)}{\left| x_{max}^j - x_{min}^j \right|} \right)^2} \quad (1)$$

where N_C is the number of node circuits, N_S the number of (measured) samples and $\left| x_{max}^j - x_{min}^j \right|$ is a normalizing factor for each set of measurements. In our case we have $N_C = 4$, $N_S = 2500$.

Each node has been settled onto a periodic orbit. The four nodes have been coupled with 6 parallel RC couples, so forming a fully connected all-to-all network topology, as shown in Fig. 4. The coupling resistance R_c has been varied in the range $0 \div 74$ kΩ with a step of 100 Ω; for each fixed value of the resistance, the coupling capacitance in parallel has been varied between 0 and 520 pF with steps of about 19 pF. As a consequence, a total of more than 18000 coupling configurations has been analyzed. For each configuration the wave forms of the three state variables v_{c1}, v_{c2}, and i_L have been recorded and the synchronization index I_s has been calculated.

As an example, in Fig. 5 we have shown the synchronization index, calculated on the first state variable v_{c1}, as a function of the coupling resistance when the coupling capacitance is C_c=143 pF and C_c=390 pF.

We notice that: a) the transition from the synchronized to the unsynchronized is sharp; b) the higher the coupling capacitance, the higher is the threshold represented by the coupling resistance value; c) for high values of the coupling capacitance there is a new transition from the unsynchronized state to the synchronized state; d) a threshold can be found in the capacitance value, above which no transition occurs.

All results from the 18000 configurations are summarized in Fig. 5, where synchronization (white) and de-synchronization (gray) regions can be distinguished.
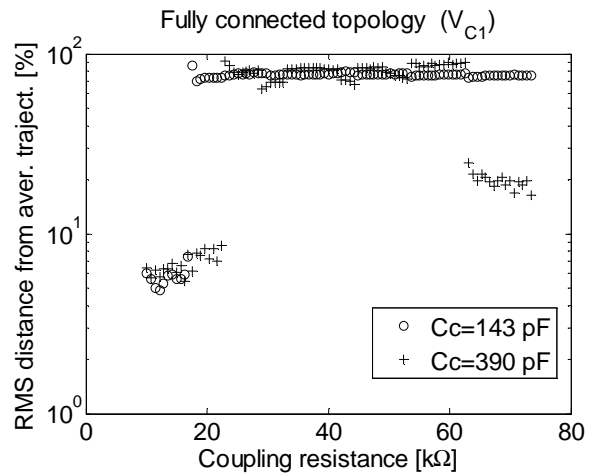

Figure 4. 4 nodes all-to-all network topology with dynamic links .

Figure 5. synchronization index as a funcrion of link resistance R_c for a dynamic link. C_c=143 pF and C_c=390 pF.
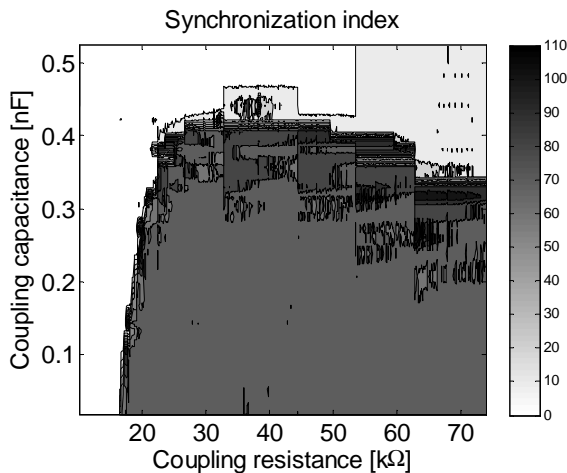
Figure 6. synchronization index as a funcrion of coupling resistance R_c and capacitance C_c.

The experimental results confirm the theoretical findings in [7] where the case of networks of Chua's with dynamic coupling was presented, using Harmonic Balance methods together with an MFS approach. We wish to emphasize that, while having been validated numerically in 8, this is the first time these theoretical results find an experimental confirmation.

Acknowledgement

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