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Relation between dynamics of network systems and those of theirs sub-networks [(1)]

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Summary. Biological networks such as gene regulatory networks, neural networks, and metabolic networks are generally complex even from the network topology point of view [17, 18]. However, the understanding of the dynamics of such network systems is crucial to identify mechanisms behind many kinds of biological processes and diseases, and to decode the mysteries of life. Statistical studies on the topology of real world networks revealed some very intriguing features [17] including power-law degree distributions [2, 25, 36], local community structures [4, 11, 13] and network motifs [14, 6]. There is a large body of work devoted to identifying communities or motifs in biological networks [17, 35, 22, 18, 23, 6, 14]. Interestingly, only very few works focused on using modular idea to study dynamics of network systems: the dynamics of a complex network can be understood by studying its subnetwork systems. In order for this idea to work, the dynamics of the subnetworks need to be preserved or partially preserved in the original network. A simple example where this is true is when a subnetwork does not receive input from the rest of the network. However, the situation becomes quite subtle when the subnetwork and its complementary subnetwork have mutual interactions.

In this work, we address the relations between dynamics of the subnetworks and that of the whole system in a Boolean network mathematical modeling framework.

Mathematical models have proven to be indispensable tools for network systems. Among various mathematical modeling frameworks, coupled differential equations and Boolean networks are popular for modeling regulatory networks[7, 26, 16, 20, 21, 3, 10, 33, 1, 12, 29, 30, 31, 15, 5, 28]. Network systems are often represented by directed graphs, wherein components are represented by nodes and interactions by arrows. An n-node Boolean network system is a discrete dynamical system with the form $X(t+1) = F(X(t))$, where $X = (x_1, \cdots, x_n)$, x_i represents the state variable of the i^{th} node, $F = (f_1, \cdots, f_n)$ and f_i is the governing function of the i^{th} node with its value being either 0 or 1. Boolean networks have been widely used to model biological regulatory networks [7, 26, 16, 20, 21, 3, 10, 33, 1, 12]. They can be set up in situations where information on the detailed kinetic interactions is not available and can provide valuable insights [19, 12, 24, 27, 8, 9, 32].

In this work, we particularly consider networks formed by two subnetworks connected at a *cutting node*, which we will define next. A node is called a *cutting node* of a connected network if the removal of the node leads to two or more disjoint subnetworks. Furthermore, we introduce the notion of a network being agreeable. Let G be the network of the whole system formed by G_1 and G_2 connected at a cutting node c. Let $x_c(t, *)$ be the value of the cutting node in the system * (here * can be G_1, G, or G_2) at time t. We say that G is agreeable if $x_c(t, G) = z_0$ whenever $x_c(t, G_1) = x_c(t, G_2) = z_0$. Additionally, we give an example of an updating scheme for the cutting node that guarantees that a network system is agreeable. We show that if a network is agreeable and its subnetworks have only cycles, then the whole system has only cycles. We then prove that if X_0 is a fixed point of G, then X_0 restricted to the phase-space of one of the subnetwork systems must be a fixed point of that system.

We also discuss the relations between the product of the transition diagrams (a representation of trajectories) of the subnetwork systems and that of the whole system. In a Boolean network system, the transition diagram of the system represents the dynamics of the system. When the dynamics of the subnetworks are all independent, then the dynamics of the whole network is just the product of the subnetworks. However, when they are not independent, the relation is not all that transparent. We discuss the construction of the transition diagram of a network $\mathcal{G}$ from the transition diagram of the product system of its subnetworks and present an algorithm to construct the transition diagram of $\mathcal{G}$. This algorithm can be very useful when the subnetworks are large and their transition diagrams are ready to use. On the other hand, the algorithm provide a rather clear view on the relations between the dynamics of the whole network and that of its subnetworks.

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