2017 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2017, Cancun, Mexico, December 4-7, 2017

Assimilating nonlinear dynamics with FORCE-learning : A perspective from chaotic synchronization

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Abstract—We propose an approach for assimilating dynamical systems using FORCE-learning which is a version of reservoir computing (RC) framework. In this approach, a direct coupling between FORCE-learning system and the target system is employed, which enables us to treat the problem of the system identification in terms of synchronization phenomena. An example of chaotic systems is used as a demonstration and we investigate how our approach is useful for modeling general nonlinear dynamics.

1. Introduction

Recently, the paradigm known by the name of reservoir computing (RC) has attracted much attention as a new way for simple training of large recurrent neural networks (RNNs), which has been introduced independently as echo state networks[1] and liquid state machines[2]. Furthermore, Sussillo and Abbott proposed a version of RC, called FORCE-learning[3] and how chaotic activity in a RNN is useful for temporal pattern generations has been explored[3, 4]. In this study, we propose an approach for modeling nonlinear dynamical systems using the combinations of *synchronization phenomena* and the FORCE-learning.

2. Model

We introduce a type of coupled dynamical systems defined as

$$\begin{cases} \dot{x} = f(x), \\ \tau\dot{y} = -y + gW_{\rm rec}\tanh(y) + W_{\rm fb}z + K(x-z), \\ z = W_{\rm out}y. \end{cases} \quad (1)$$

Here, the function $f(\cdot)$ in the first equation represents a chaotic dynamical system as target where $x \in \mathbb{R}^D$ are its state variables. The second and third equations describe the dynamics of a recurrent neural network (called a *reservoir*) where $y \in \mathbb{R}^N$ (N denotes the number of neurons) and $z \in \mathbb{R}^D$ are state variables of the reservoir and the output neurons, respectively. Let $W_{\rm rec}$ be a $N \times N$ matrix that identifies the connectivity among neurons in the reservoir and g be its intensity that represents the degree of nonlinearity.

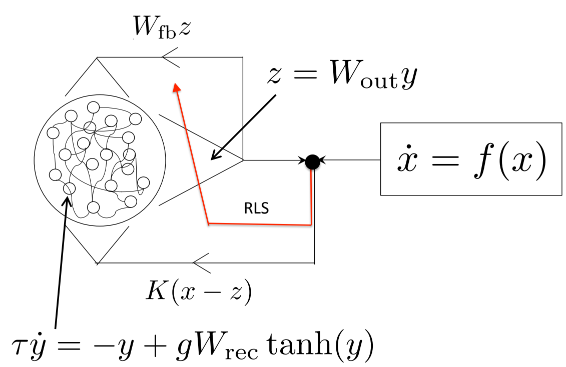

Figure 1: Schematic plot of the proposed learning framework.

Also, let $W_{\rm fb}$ be a $N \times D$ matrix that identifies the feedback from the output to the reservoir neurons. In this paper, we employ a sparse Gaussian random matrix with the connectivity probability p, the zero mean and the standard deviation $1/\sqrt{pN}$ as $W_{\rm rec}$. We also use a dense uniformly distributed random matrix with the zero mean and the standard deviation $a_{\rm fb}/\sqrt{3}$ as $W_{\rm fb}$. The state z is determined by the linear projections of y as defined in the third of Eqs. (1) whose weight matrix $W_{\rm out}$ (a $D \times N$ matrix) is updated in an on-line learning manner so as that the error between x and z converges to zero. Here, the recursive least squares (RLS) is employed for this update.

In this model, we also provide the *direct* coupling between x and z as shown in the last term of the second equation("$+K(x-z)$"). This direct coupling tern plays a role of imposing the reservoir to synchronize with the chaotic dynamics generated from $f(\cdot)$. Schematic plot for this system is shown in Fig. 1.

3. Results

As a demonstrative example, we employ the Lorenz system

$$\begin{cases} \dot{x}_1 = -\sigma x_1 + \sigma x_2 \\ \dot{x}_2 = -x_1 x_3 + r x_1 - x_2 \\ \dot{x}_3 = x_1 x_2 - b x_3, \end{cases} \tag{2}$$

with the parameters $\sigma = 10, r = 28$ and $b = 8/3$, and consider to copy its generic dynamics into the reservoir using the approach introduced in the previous section.

Figures 2 show results *after* the training for four different values of the degree g of nonlinearity. Here, we use $N = 500$ neurons as elements of the reservoir, $p = 0.5$ as the connection probability, $k = 1$ as the coupling strength between the reservoir and the Lorenz system, and $a_{fb} = 0.3$ as the feedback strength, respectively. Figure 2(a) shows the time series of an output neuron $z_1(t)$ *after* training. For all four panels, the coupling term between the reservoir and the Lorenz systems is maintained until an intermediate stage and is cut off at a time moment indicated by an arrow. Figure 2(b) shows the corresponding 3d plots of $z(t)$ after the coupling is cut off. For $g = 0.5$, i.e., in the case of weak nonlinearity, the reservoir dynamics traces the Lorenz chaos before the coupling is cut off. However, once the coupling is cut off, the reservoir dynamics settles down to a limit cycle and cannot assimilate the Lorenz chaos any more. For larger values of g (= 0.9 and 1.2), although both of time series $z_1(t)$ become similar to that of the Lorenz system, the corresponding 3d plot is far different from the Lorenz system for $g = 0.9$. Then, for $g = 1.8$, the chaotic nature of the reservoir dynamics is too strong, it fails to assimilate the Lorenz system.

Next, we investigate the dependence of performance on the parameter k, the coupling strength between the reservoir and the Lorenz systems during the training. Results are shown in Fig. 3. When k is small ($k = 0.001$), alternations of cycles between the up and down sides are observed even after the coupling is cut off, but there is also some “bias” toward the up side which is different from the appearance of the original Lorenz system. On the other hand, for large value of k (= 10), the reservoir dynamics settles down to a limit cycle, i.e., assimilation is failed. These results indicates us that the existence of proper direct coupling is useful for good performance of assimilation.

Finally, we compare the statistics of the Lorenz system and that of the reservoir system. Figures 4 show the probability distribution functions concerning with the period τ of alternations between two butterfly cycles for the original Lorenz system (Fig. 4 (a)) and for the output from a reservoir (Fig. 4 (b)), respectively. Here we use a reservoir network with $N = 1000$, $p = 0.1$, $g = 1.2$, $a_{fb} = 0.3$, and $k = 1$, respectively. Although the qualitative structure, i.e., the exponential decay of the probability distribution can be reproduced in the reservoir system, there is also quantitative difference from between two systems.

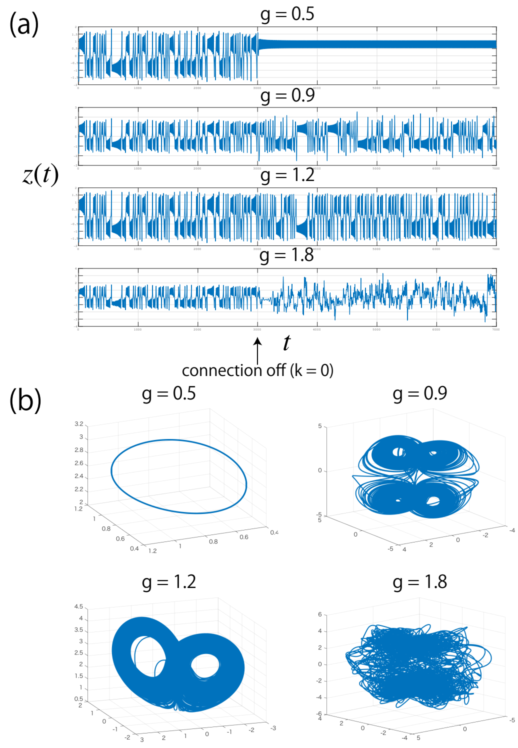

Figure 2: (a) Time series of the output neurons generated from the reservoir for $g = 0.5, 0.9, 1.2$ and 1.8. (b) Corresponding 3d plots.

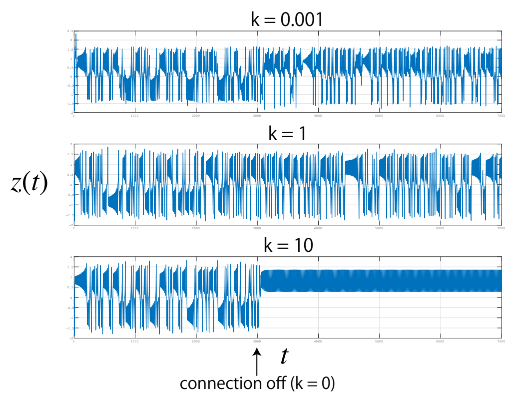

Figure 3: Time series of the output neurons generated from the reservoir for $k = 0.001, 1$ and 10.

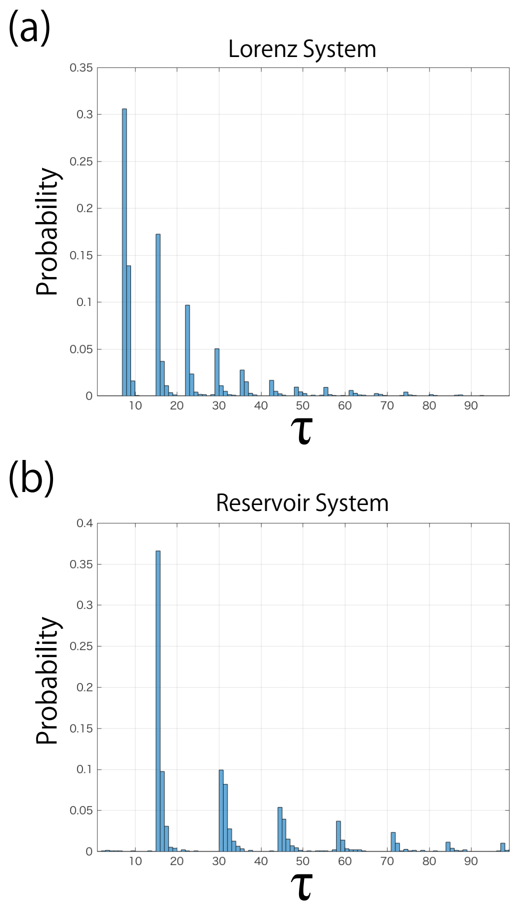

Figure 4: Probability distribution functions of alternation period between two butterfly cycles for (a) the original Lorenz systems and (b) the reservoir system.

4. Summary and Discussion

In summary, we proposed an approach for assimilating chaotic dynamical systems using FORCE-learning combined with the direct coupling term between the target and the reservoir systems. Depending on the choice of parameters associated with the reservoir system, intrinsic dynamics of the target system is successfully reproduced by the reservoir in a qualitative manner, which has potential applications to data assimilation problems. We need, however, further investigations and modifications of our proposed approach to improve quantitative performance. In the presentation, we will discuss how the learning of dynamical systems by RC can be interpreted as chaotic synchronization problems.

Acknowledgments

This work was supported by MEXT KAKENHI Grant Numbers 25120011, and 16H01617.

References

[1] H. Jäger, and H. Haas, Science **308**, 78 (2004).

[2] W. Maas et al., Neural Comp. **14**, 2351(2002).

[3] D. Sussillo and L.F. Abbott, Neuron **63**, 544(2009).

[4] H. Suetani, ISCS 2014: Interdisciplinary Symposium on Complex Systems, 47. Springer International Publishing (2015).