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Sparse Optimization of Physical Distribution Systems based on Maximum Hands-off Control

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Abstract—In this article, we propose a novel design method for physical distribution on a network based on maximum hands-off control. In this formulation, we can take account of the tradeoff between minimizing the total cost and reducing the CO_2 emissions. The design is described as an optimal control for a discrete-time dynamical system with equality/inequality constraints. This is equivalent to a convex optimization problem, which can be efficiently solved by numerical optimization softwares.

1. Introduction

Physical distribution system is a system for efficient movement of finished products from the end of the production line to the consumers. Usually, there exist multiple warehouses and transportations, and hence the distribution system forms a network. It is important for an efficient distribution to optimally manage the product flow over the network to minimize the time and the cost. On the other hand, it is also important to consider the emission of CO_2 in a networked physical distribution system. We then face a tradeoff between the efficiency and the CO_2 emission.

To consider this tradeoff, we propose to use the technique of *maximum hands-off control* [2, 3, 1]. In this formulation, we describe the cost as a quadratic function of the amount of transformations (i.e. control) and the CO_2 emission as the ℓ^1 norm of the control. The ℓ^1 norm leads to a sparse control, with which the CO_2 emission can be dramatically reduced by stopping transportation vehicles. The optimization can be described as a finite-dimensional convex optimization, which can be efficiently solved by a numerical optimization toolbox, such as CVX in MATLAB [1].

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[1]`http://cvxr.com/cvx`

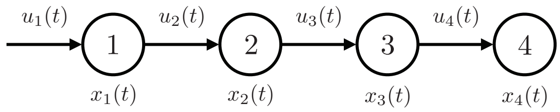

Figure 1: Physical distribution network

2. Sparse Optimization

Here we consider a network as shown in Figure 1. In this figure, node i is the i-th warehouse, $t \in \{0, 1, 2, \ldots\}$ denotes the discrete time, $x_i(t)$ is the stock amount of warehouse $i \in \{1, 2, \ldots, n\}$, and $u_i(t)$ is the amount of transportation to i. Let $\boldsymbol{x}(t) \triangleq [x_1(t), \ldots, x_n(t)]^\top$ and $\boldsymbol{u}(t) \triangleq [u_1(t), \ldots, u_n(t)]^\top$. Then the dynamical model of the distribution system in Figure 1 is obtained by

$$\boldsymbol{x}(t+1) = \boldsymbol{x}(t) + B\boldsymbol{u}(t), \qquad (1)$$

where B is the incidence matrix. For this dynamical system, we minimize the ℓ^1/ℓ^2 cost

$$J(\boldsymbol{u}) = \sum_{t=1}^{T} \boldsymbol{u}(t)^\top R\boldsymbol{u} + \lambda \|\boldsymbol{u}(t)\|_1.$$

With linear constraints on $\boldsymbol{x}(t)$ and $\boldsymbol{u}(t)$, the design problem is described by a convex optimization problem, which can be solved very efficiently [3, 1].

References

[1] T. Ikeda, M. Nagahara, and S. Ono, “Discrete-valued control of linear time-invariant systems by sum-of-absolute-values optimization,” *IEEE Trans. Autom. Control*, vol. 62, no. 6, pp. 2750–2763, 2017.

[2] M. Nagahara, D. E. Quevedo, and D. Nešić, “Maximum hands-off control: a paradigm of control effort minimization,” *IEEE Trans. Autom. Control*, vol. 61, no. 3, pp. 735–747, 2016.

[3] M. Nagahara, J. Østergaard, D. E. Quevedo, “Discrete-time hands-off control by sparse optimization,” *EURASIP Journal on Advances in Signal Processing*, 2016:76, Dec. 2016.