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Statistical Analysis of Chaotic Neuron in the Mutually-Connected Chaotic Search Method

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Abstract—To find near-optimal solutions of combinatorial optimization problems, a method which uses mutually-connected chaotic neural network (CNN) has already been proposed. However, it is not so easy to generate feasible solutions of the problems from the CNN, because an output of a chaotic neuron takes an analog value. Each neuron generates a complicated spike time-series. In this paper, to decide good solutions of the combinatorial optimization problems from the CNN, we analyzed complexity of the spike time-series from each chaotic neuron by using a statistical measure, such as coefficient of variation (CV) and local variation of interspike intervals (LV), which are frequently used in the field of neuroscience.

1. Introduction

The quadratic assignment problem (QAP) is one of the most famous combination optimization problems [1]. In the QAP, N facilities and N places where facilities are located. Let d_{ij} be the distance between the location i and the location j and f_{mn} be the flow between the facility m and the facility n. The goal of QAP is to allocate facilities to locations so that the sum of costs defined by the "flow between facilities and the product of distance between places" is minimized. It is a constraint that each facility can be placed in only one place and only one facility can be placed in one place. QAP can be formulated as follows:

$$\text{minimize} \quad \sum_{i=1}^{N}\sum_{m=1}^{N}\sum_{j=1}^{N}\sum_{n=1}^{N} x_{im}x_{jn} \tag{1}$$

$$\text{subject to} \quad \sum_{m=1}^{N} x_{im} = 1 \qquad (i = 1, 2, ..., N) \tag{2}$$

$$\sum_{m=1}^{N} x_{im} = 1 \qquad (m = 1, 2, ..., N) \tag{3}$$

$$x_{im} \in \{0, 1\} \qquad (i, m = 1, 2, ..., N) \tag{4}$$

where x_{im} is a decision variable. If x_{im} is 1 when the facility i is allocated to a location m, and otherwise 0. Eq. (1) represents minimization of the total sum of the allocation costs. Eqs. (2)-(4) are constraint conditions. Eq. (2) represents that each facility is arranged in one location. Eq. (3) represents that each one location is assigned to only one facility. Eq. (4) specifies that the decision variable x_{im} takes 0 or 1.

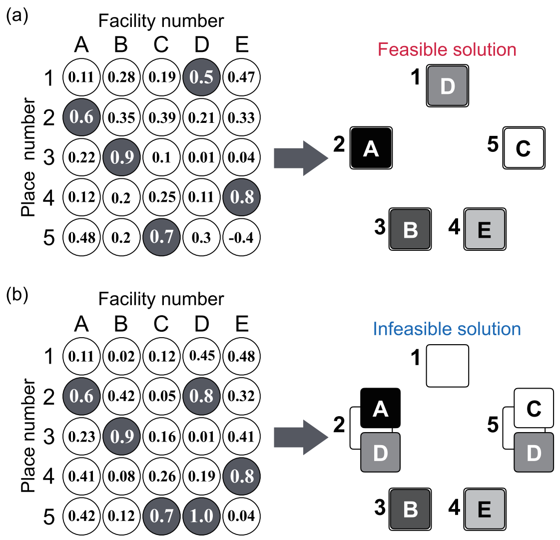

Figure 1: Relationship between a state of a chaotic neural network and a solution of the quadratic assignment problem. Filled circles by gray represent firing neurons (The neuron fires when an output value is be higher than or equal to 0.5). For example, in (a), facility D is assigned to location 1, because the (1,D)th neuron fires. In (a), a single neuron fires in each row and each column. Thus, a feasible solution of the QAP can be generated from the CNN. However, in (b), multiple neurons fire in the 2nd and 5th row, and the 4th column. The solution generated from the CNN cannot be satisfy the constraints of the QAP.

Many approximation algorithms have been proposed to find the good near-optimal solutions. As one of the algorithms, an algorithm using chaotic neural network (CNN) has been proposed [2, 4]. In this algorithm, in order to solve the N size of QAP, we arrange chaotic neurons in an $N \times N$ grid. The states of CNNs arranged in the $N \times N$ grid represent solutions of the QAP. Then, if the (i, m) chaotic neuron fires, the facility i is assigned to the location m (Fig. 1(a)). However, it is difficult to always obtain a feasible solution of the QAP, because multiple neurons fire at the ith row (or the mth column) and no neuron fires at the ith row (or the mth column) (Fig. 1(b)). To generate a feasible solution from the state of the CNN, a method by using the values of the internal state has been proposed [4]. The method greedily selects one neuron from each row and each column so that a sum of the values of the internal state becomes large. Then, to obtain a better solution, a local search method which exchanges selected neurons to increase the sum of the value of the internal state has been proposed [6, 7].

The chaotic neurons in the CNN generate very complicated spike time-series to continue the search beyond local optimality. We have analyzed the spike time-series generated from each chaotic neuron by using statistical methods which are frequently used in the field of neuroscience when the chaos search method searches for an optimal solution of a motif extraction problem [8, 9]. As a result, we clarified that a spike time-series generated from a neuron corresponding to optimal solution show characteristic behavior. That is, by using the analysis results, we can find optimal solutions of the motif extraction problem. Then, in this paper, to construct an optimal solution of the QAP from the state of the CNN, we analyzed the spike time-series generated of the chaotic neuron by using statistical measures such as the coefficient of variation (C_V) and the local variation L_V.

2. Mutually-Connected Chaotic Neural Network

For solving the N-size QAP, N^2 chaotic neurons [2] are required, and these are arranged on an $N \times N$ grid [3, 4, 5]. An internal state of the (i, m)th chaotic neuron for the QAP is defined as follows:

$$\begin{aligned} y_{im}(t+1) &= k_r y_{im}(t) + \sum_{j=1}^{N}\sum_{n=1}^{N} w_{im;jn} f(y_{jn}(t)) \\ &\quad -\alpha f(y_{im}(t)) + \theta_{im} \end{aligned} \tag{5}$$

where k_r is a decay parameter of a refractory effect and α is a strength parameter of a refractory effect. The chaotic neurons are coupled each other with a synaptic connection weight. $w_{im;jn}$ is the synaptic weight between the (i, m)th neuron and the (j, n)th neuron. The synaptic weight between the (i, m)th neuron and the (j, n)th neuron, and the threshold of the (i, m)th neuron are defined as follows:

$$\begin{aligned} w_{im;jn} &= -2\Big\{A(1-\delta_{mn})\delta_{ij} + B\delta_{mn}(1-\delta_{ij}) \\ &\qquad + \frac{d_{ij} f_{mn}}{q}\Big\} \end{aligned} \tag{6}$$

$$\theta_{im} = A + B \tag{7}$$

where δ_{ij} is Kronecker's delta and q is a normalization parameter. θ_{im} is a bias of the (i, m)th chaotic neuron, and $f(\cdot)$ is an output function of the chaotic neuron. As an output function, a sigmoidal function is used: $f(y) = 1/(1 + \exp(-y/\epsilon))$, where ϵ is a gradient parameter of the sigmoidal function.

In a single iteration, all neurons are asynchronously updated. After updating the internal state of all neurons, a solution of the QAP is decided from the state of CNN by using a solution method described in Section 3.

3. Solution Decision Method

In the CNN, if the (i, m)th chaotic neuron fires, the ith facility is assigned at the mth location. However, it is difficult to obtain feasible solutions of the QAP from outputs of the neurons, because multiple neurons fire at the ith row (or the mth column) and no neuron fires at the ith row (or the mth column). Namely, it does not satisfy constraints of the QAP.

To generate a good solution of the QAP from the CNN, first an feasible solution is constructed by using a solution decision method [3, 4]. This method is greedy algorithm to maximize a sum of the internal state of firing neurons. The procedure of the method is described as follows:

1. The (i, m)th chaotic neuron, that gives the maximum value of the internal state among all of the internal states is selected. Then, we set $x_{im} = 1$, namely the element at the ith row and the jth column in the solution matrix X is set to 1.

2. To satisfy the constraints of the QAP (Eqs. (2) and (3)), for other neurons in the ith row and the mth column, we set $x_{ik} = 0$ $(k \neq m)$ and $x_{lm} = 0$ $(l \neq i)$.

3. The neurons which have already been selected in Steps 1 and 2 are excluded from the candidate at step 1. Steps 1 and 2 are repeated N times to make a feasible solution.

Next, the initial solution is improved to increase the values of internal state $y(t)$ by a simple local search method [6, 7]. In the local search method, two neurons are deleted from selected neurons and new two neurons are selected to satisfy the constraints of the QAP.

4. Statistical Analysis of Chaotic Neuron

To obtain a good solution by using an approximate solution, it is important how to escape or avoid a local optimal solution. In the CNN, a refractory effect of the chaotic neuron effectively controls to escape form the local optimal solution. By the refractory effect, each neuron generates a complicated spike train.

To find a good solution by the CNN, the parameters in the CNN must be set to appropriate values. First, we investigated the performance of the CNN by changing the values of parameters α and k_r in the refractory effect that works an important role in solution search. In the simulation, we used tai20a which is QAPLIB benchmark problem. We use θ ($\equiv \theta_{im}$ for all i and m) finely tune the performance. The value of parameter α in the CNN method (Eq.(5)) is set to between from 0.05 to 2.00 by step size 0.05. The value of parameter k_r is set to between from 0.05 to 0.95 by step size 0.05. Other parameters are as follows: $A, B = 0.40$, $\theta = 3.0$, $\epsilon = 0.002$, $q = 90,000$. Figure 2 shows the performances of the CNN. From Fig. 2, it can be seen that a good solution can be obtained by setting the parameter to an appropriate value.

Second, statistical analysis of spike time-series generated from the neurons is carried out to clarify whether an optimal solution can be constructed using the analysis results. To reveal such characteristic property, In the analysis, we analyzed complexity of the spike time-series from each chaotic neuron by using statistical measures, such as coefficient of variation and local variation of interspike intervals, which are frequently used in the field of neuroscience [11]. If the spike time-series of the chaotic neuron corresponding to a optimal solution may exhibit characteristic response, we can construct an optimal solution by using this characteristic response.

The coefficient of variation (C_V) of interspike intervals is a measure of randomness of interval variation. The C_V is defined as

$$C_V = \frac{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(T_i - \bar{T})^2}}{\bar{T}} \tag{8}$$

where T_i is the ith interspike interval (ISI), n is the number of ISIs, and $\bar{T} = \frac{1}{n}\sum_{i=1}^{n} T_i$ is the mean ISI. For spike intervals that are independently exponentially distributed, C_V is 1 in the limit of a large number of intervals. For a regular spike time-series in which T_iT is constant, $C_V = 0$.

The local variation (L_V) of interspike intervals is a measure of the spiking characteristics of an individual neuron [11]. The L_V is defined as

$$L_V = \frac{1}{n-1}\sum_{i=1}^{n-1}\frac{3(T_i - T_{i-1})^2}{(T_i + T_{i+1})^2}. \tag{9}$$

For spike intervals that are independently exponentially distributed, $L_V = 1$. For a regular spike time-series in which T_i is constant, $L_V = 0$.

Parameters of the chaotic neuron model were set to values for which good solutions were obtained ($\alpha = 1.28$ and $k_r = 0.94$), and analysis of spike time-series generated from each neuron model was carried out. Fig. 3 shows the results of each statistic. The dots in Figure 3 represent a neuron corresponding to the optimal solution. For example, the dot in the 1st row and 10th column means that the factory 10 is assigned to location 1 in the optimal solution. An optimal solution can be obtained if the neurons corresponding to these dots can be specified by using the result of the statistical analysis of each neuron.

From Fig. 3 (a), it can be seen that neurons corresponding to the optimal solution do not frequently fire. Similarly, it can be seen that the values of C_V and L_V do not behave distinctive behaviors even though they are neurons corresponding to the optimal solution (Figs. 3 (b) and (c)). From these results, it was found that it is difficult to identify neurons corresponding to the optimal solution from the spike sequence of each neuron.

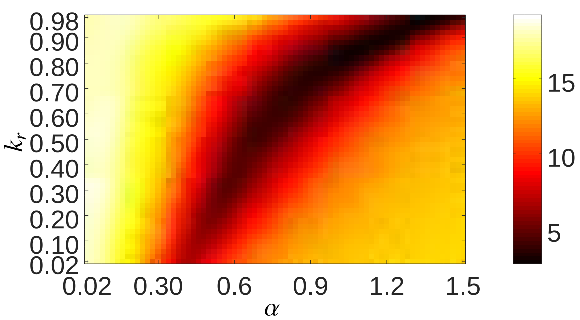

Figure 2: Percentages of average gaps between obtained solutions in 50 trials.

5. Conclusions

To find the near optimal solution of the QAP, the method by using mutually-connected chaotic neuron network (CNN) is proposed. In this method, the state of the neural network represents the solution of QAP. Its performance depends on how to determine the solution from the state of CNN. In this paper, we analyzed the behavior of the chaotic neuron constituting the mutually coupled chaotic neural network and examined whether it is possible to identify neurons corresponding to the optimal solution by using the analysis results. Specifically, we analyzed the spike sequence generated by each neuron using statistical measures such as the coefficient of variation (C_V) and the local variation L_V used in the field of neuroscience. As a result of analysis, it was found that it is difficult to identify neurons corresponding to the optimal solution from the spike sequence of each neuron. In the future, we will analyze the similarity of spike time series among multiple neurons by using spike time metric.

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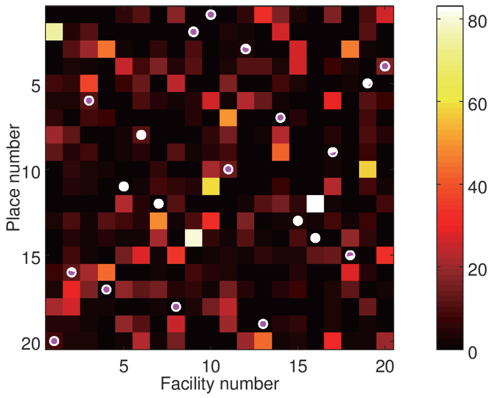
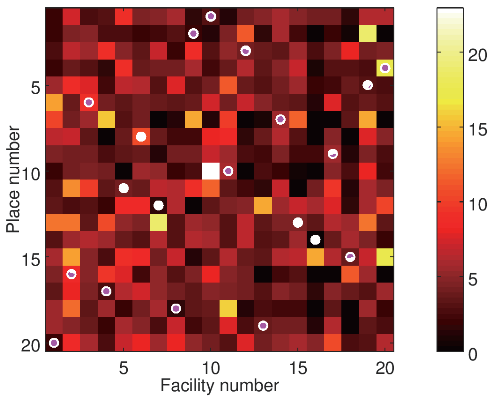
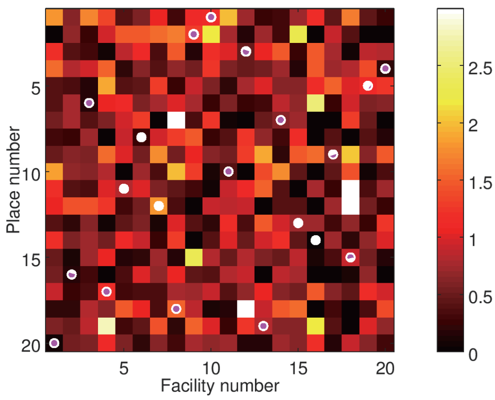


(a) Firing rate



(b) C_V



(c) L_V

Figure 3: The values of (a) the firing rate, (b) C_V , and (c) L_V of all neurons for tai20a. The dots represent neurons corresponding to the optimal solution.

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