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Behavioral Modeling of Switched Descriptor System via Backward Euler Method

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Abstract—The behavioral modeling of switched descriptor system with impulse modes is presented, in which the backward Euler method is used for taking an arbitrary input. This method is so simple that it can be used for modeling of various nonlinear phenomena. In the numerical examples, we demonstrate that the proposed method generates an appropriate stead-state model of class-E amplifier, which is a resonant power amplifier.

1. Introduction

Nonlinear phenomena are expressed by a switched linear dynamical system which is often written by a state equation. However, all the phenomena cannot be necessarily expressed by the state equation, then, the system is expressed by a set of differential algebraic equations, which is called descriptor system [1]. The typical situation is modeling of power electronics circuits. Passive linear circuits are switched for power converters [2]. The dynamical systems of equivalent circuit models correspond to the switched dynamical systems.

The behavioral model presents an efficient simulation model, which is used for making a design guidance [5] or generating a cost function for the optimization problem. The behavioral modeling of class-E amplifiers is proposed [8], in which the Weierstarss canonical form is used for obtaining the behavioral model. However, the input waveform must be known a prior and tedious calculations are necessary for obtaining the behavioral model.

In this paper, the behavioral modeling of switched descriptor system is presented, in which the backward Euler method is used for allowing an arbitrary input waveform. Although the backward Euler method is not accurate, it is advantageous to avoid a numerical instability. The descriptor system usually has impulse modes, which cause the instability for the numerical integration. If an accurate numerical integration formula is used, the simulation may be broken down. Therefore, the backward Euler method is used as a compromise.

In the numerical examples, we demonstrate that the proposed method can capture the steady-sate response of class-E amplifier, in which the class-E amplifier behaves both DC/AC and AC/AC converters.

2. Descriptor System

The descriptor system is expressed by a set of linear algebraic differential equations,

$$E\frac{dx(t)}{dt} = Ax(t) + Bu(t), \quad (1)$$

$$y(t) = Cx(t), \quad (2)$$

where $x(t) \in R^n$ is the state, $u(t)$ is the input, and $y(t)$ is the output. $A \in R^{n\times n}$, $B \in R^{n\times p}$, and $C \in R^{m\times n}$ are the coefficient matrices. The solution of (1) is obtained by the Weierstrass canonical form:

$$WET = \begin{bmatrix} I_d & 0 \\ 0 & N \end{bmatrix}, \quad WAT = \begin{bmatrix} \Lambda & 0 \\ 0 & I_{n-d} \end{bmatrix}. \quad (3)$$

In (3), W and T are the transformation matrices, and Λ is the Jordan form composed of the finite eigenvalues associated with the matrix pencil $\lambda E - A$. The degree of the characteristics polynomial is assumed to be d, then, Λ is $d \times d$ matrix. N is a nilpotent, the eigenvalues of which are corresponding to infinity. I_l is $l \times l$ identity matrix. The Weierstrass canonical form is numerically calculated via QZ transform to the matrices E and A, and the solutions of Sylvester equations [7].

Using (3), we can rewrite (1) into

$$\dot{x}_s(t) = \Lambda x_s(t) + B_s u(t), \quad (4)$$

$$N\dot{x}_f(t) = x_f(t) + B_f u(t), \quad (5)$$

where

$$T^{-1}x(t) = \begin{bmatrix} x_s(t) \\ x_f(t) \end{bmatrix}, \quad WB = \begin{bmatrix} B_s \\ B_f \end{bmatrix}. \quad (6)$$

The general solutions of (4) and (5) are respectively obtained by

$$x_s(t) = e^{\Lambda(t-t_0)}x_s(t_{0-0}) + f_s(t), \quad (7)$$

$$x_f(t) = -B_f u(t) - \sum_{i=1}^{\mu-1}\left(N^i\delta^{(i-1)}(t-t_0)x_f(t_{0-0}) + N^i B_f u^{(i)}(t)\right), \quad (8)$$

where $e^{\Lambda t}$ is the matrix exponential of Λt and μ, which is called index, satisfies $N^{\mu} = 0$. The function $f_s(t)$ depends on the input $u(t)$. The time-domain response of (1) includes impulses unless $\ker N = x_f(t_{0-0})$.

When the descriptor system expresses a linear passive circuit, the passivity of the network is guaranteed. Then, the index μ is at most 2. Hence, the solutions (1) are rewritten by

$$x(t) = \alpha(t)x(t_{0-0}) + \beta(t), \quad (9)$$

where

$$\begin{aligned} \alpha(t) &= T\begin{bmatrix} e^{\Lambda(t-t_0)} & 0 \\ 0 & -N\delta(t-t_0) \end{bmatrix} T^{-1}, \\ \beta(t) &= T\begin{bmatrix} f_s(t) \\ f_f(t) \end{bmatrix}, \\ f_f(t) &= -B_f u(t) - NB_f u^{(1)}(t). \end{aligned}$$

The expression (9) presents a behavioral transient model. However, the function $f_s(t)$ must be closely written and the input is restricted to a typical function such as $sin(\omega t)$.

3. Backward Euler Method

Explicit numerical integration methods such as forward Euler and Runge-Kutta methods cannot be applied to (1), because the matrix E is singular and (1) cannot be converted into a state equation. Therefore, we apply the backward Euler method as an implicit numerical integration method. Then, the equation (1) is expressed as

$$E\frac{x(n+1) - x(n)}{h} = Ax(n+1) + Bu(n+\frac{1}{2}), \quad (10)$$

where $x(n+1)$ is the solution at $t = t_{n+1}$ and h is a time step size. Then, the solution $x(t+1)$ is expressed by

$$x(n+1) = (E - hA)^{-1}\left(Ex(n) + hBu(n+\frac{1}{2})\right). \quad (11)$$

The backward Euler method is used for regularizing the matrix E. In this case, we can use more accurate numerical integration formula such as Gear method, BDF, and so on. However, these accurate methods are not necessarily suitable. As the time-domain response of descriptor system includes impulses as (8), there may be some singular points in a time interval. If an accurate numerical integration method is used, it would try to follow an impulse accurately, which breaks the numerical stability and the simulation is down. To avoid the breakdown, we use the backward Euler method, even though the accuracy is not better.

The relationship between $x(n+1)$ and $x(0)$ is obtained by

$$\begin{aligned} x(n+1) &= ax(n+1) + bu(n+\frac{1}{n}) \\ &= a^{n+1}x(0) + a^n bu(\frac{3}{2}) + \ldots + bu(n+\frac{1}{2}) \\ &= \alpha x(0) + \beta, \end{aligned} \quad (12)$$

where $a = (E - hA)^{-1}E$ and $b = (E - hA)^{-1}hB$.

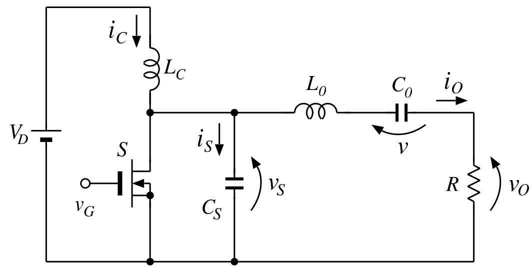


Figure 1: Basic configuration of class-E DC/AC and AC/AC converters.

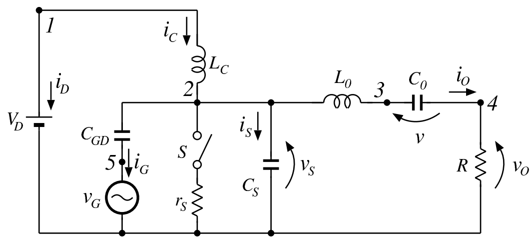


Figure 2: Simplified circuits of class-E amplifier. The gate-to-drain capacitance is considered with an ideal switch, in which each number is identified as a node.

4. Numerical Results

We demonstrate that the proposed method captures the steady-state response of class-E amplifier [6]. The basic configuration of class E-amplifier is shown in Fig. 1. When the MOSFET of Fig. 1 is replaced with an ideal switch and the gate-to-drain capacitance, the equivalent circuit model is shown in Fig. 2. It is known that this circuit generates an impulse mode [8]. The class-E amplifier has the two features, in which the circuit constructs a DC/AC converter when a pulse waveform is given at v_G and a AC/AC converter when a sinusoidal waveform is given. Moreover, the circuit has the two state of on and off with respect to the ideal switch; the two descriptor systems are switched. For the on state, $x(t) = x_1(t)$ in (1), and for the off state, $x(t) = x_2(t)$.

As the boundary condition at $t = t_{1-0}$, the following relationship is obtained,

$$x_2(t_{1-0}) = x_1(t_1) = \alpha_1 x_1(t_1) + \beta_1. \quad (13)$$

Moreover, the steady-state condition is obtained by

$$x_1(t_{0-0}) = x_2(t_2) = \alpha_2 x_1(t_1) + \beta_2. \quad (14)$$

Using (13) and (14), we can express the initial condition, which generates the steady-state response, as follows:

$$x_1(t_{0-0}) = (I - \alpha_2(t_2)\alpha_1(t_1))^{-1}(\alpha_2(t_2)\beta_1(t_1) + \beta_2(t_2)). \quad (15)$$

It should be noted that α_1, α_2, β_1, and β_2 are calculated by using (12).

As parameters of class-E amplifier as shown in Fig. 2, the following values were given, $V_D = 5V$, $C_S = 6nF$, $C_0 = 3nF$, $L_C = 7.96mH$, $L_0 = 7.96\mu H$, $R = 5\Omega$, $r_S = 0.16\Omega$, and $C_{GD} = 0.178nF$. The pulse wave with 50% duty ratio and $V_G = 5V$ amplitude was given to the gate. Then, the two modified nodal analysis equations are written as

$$C\frac{x_1(t)}{dt} + G_{on}x_1(t) = B_{on}u(t) \tag{16}$$

$$C\frac{x_2(t)}{dt} + G_{off}x_2(t) = B_{off}u(t) \tag{17}$$

where

$$G_{on} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1/r_S & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/R & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$G_{off} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/R & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_a & -C_0 & 0 & -C_{GD} & 0 & 0 & 0 & 0 \\ 0 & -C_0 & C_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -C_{GD} & 0 & 0 & C_{GD} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & L_C & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & L_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

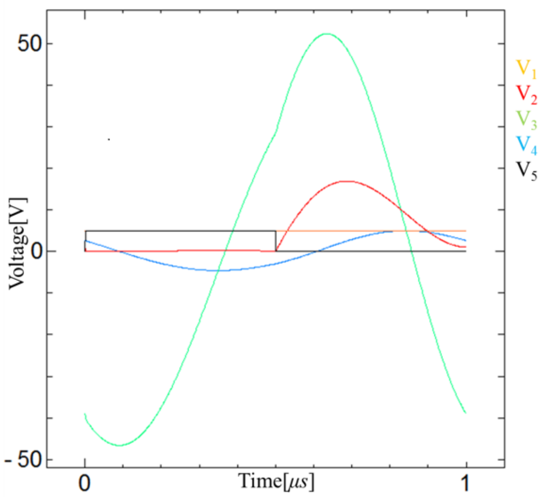
$$B_{on} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$

$$B_{off} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

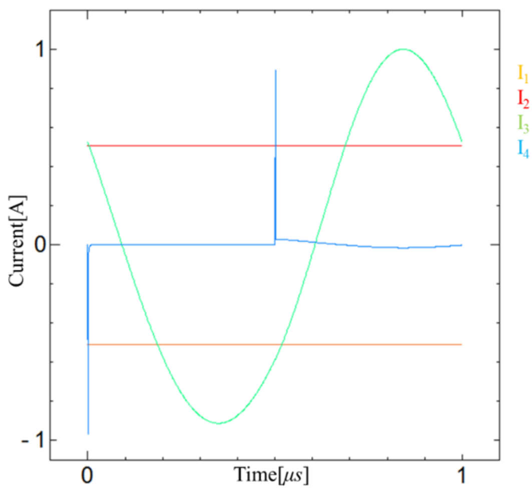
and $C_a = C_{GD} + C_S + C_0$ and $u(t) = [V_D V_G]^T$. Then, these equations are rewritten as (1) and the behavioral modeling was applied to the descriptor systems.

Figure 3(a) and 3(b) shows the state-state responses at the nodes and branches, respectively. In Fig. 3(a), V_1, V_2, and V_4 imply the driven input, the switch, and output voltages, respectively. At 1 μs, the switch voltage is almost zero; hence, the zero voltage switching is achieved. On the other hand, the blanch currents have spikes at 0.5 and 1 μ. It appears that the zero current switching is not achieved. However, it is the influence of impulse modes and artifical spikes are added to the current waveform.

Figure 4(a) and 4(b) shows the state-state responses at the nodes and branches, respectively, in which a sinusoidal waveform $u(t) = 3.2 + 6\sin\omega t$ was given. We can see that appropriate waveforms are obtained as a AC/AC converter



(a)

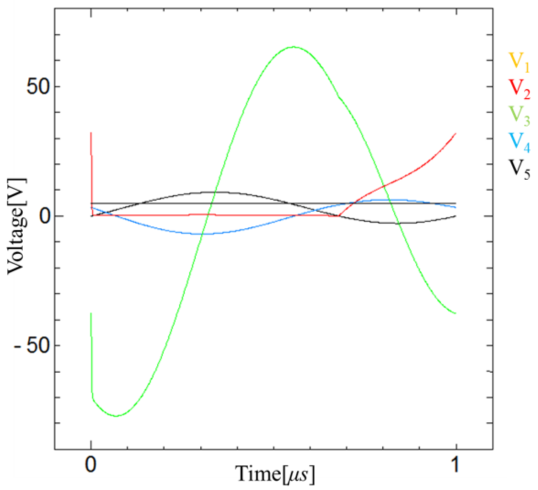


(b)

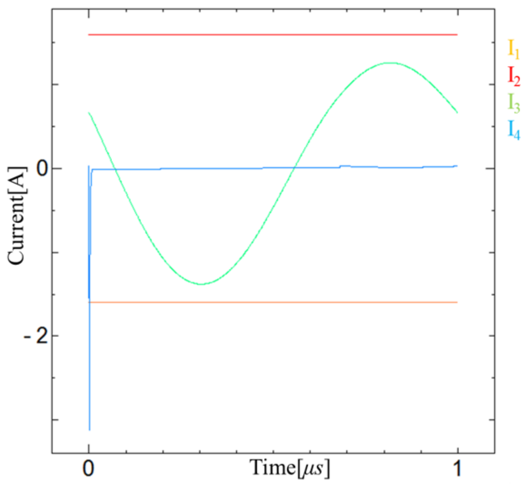
Figure 3: Steady state response of the circuit shown in Fig. 1, in which a pulse waveform is given at the gate.

5. Conclusions

The behavioral model of switched descriptor system is presented, in which the backward Euler method is used for taking an arbitrary input. This method is applied to class-E amplifier, which is a resonant power amplifier. It can be also applied to various power electronics circuits. Although a nonlinear element is treated as an ideal switch, the consideration of nonlinear characteristic other than switch is necessary for modeling of a system accurately. Therefore, we will try to include the nonlinear effects flexibly for the modeling.



(a)



(b)

Figure 4: Steady state response of the circuit shown in Fig. 1, in which a sinusoidal waveform is given at the gate.

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