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Localized Modes in a One–dimensional Resonant Circuit Array Consisting of Overlapped Square Coils

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Abstract—Localized modes in a resonant circuit array consisting of square coils and capacitances are numerically investigated for utilizing the localized modes in wireless power transfer. When a receiving coil approaches to the surface of the resonant circuit array, a localized mode appears around the position of the receiving coil. The angular frequency and the localization strength strongly depend on the position of the receiving coil if the overlap of the square coils in resonant circuit array is small. However, we have revealed that there exists an optimal overlap at which the property of the localized modes only slightly changes with respect to the receiving coil's position.

1. Introduction

Wireless power transfer based on electromagnetic resonance has recently attracted academic attention [1, 2]. In these days, many of devices such as smartphones, laptops, tablet PCs etc. are required to be wireless. Some of electric devices have already become wireless for power charge; for example, electric toothbrush is wirelessly charged by using magnetic induction to prevent electrical short by water. Although the wireless power transfer is very useful for charging electric devices, the efficiency of power transfer strongly depends on the relative positions of transmitting and receiving coils. In order to improve the efficiency with respect to the position of the receiving coil, an array configuration of transmitting coils is proposed [3]. However, the efficiency is still varied with respect to the receiving coil's position [4].

Recently, it has been reported that a localized mode arises when a receiving coil approaches to the surface of a transmitting coil array which consists of uniform planar coils and capacitors [5]. The localized mode is an eigenmode of linearly coupled resonators in which an impurity, for example the resonant frequency is different from the others, is induced [6]. For the transmitting coil array, each resonator consisting of a planar coil and a capacitance is coupled to adjacent resonators via mutual magnetic flux. Therefore, the transmitting coil array can be assumed to be a resonant circuit array. An impurity is also induced by mutual magnetic induction between the receiving coil and the transmitting coils beneath the receiving coil. Thus, the impurity will disappear if the receiving coil becomes far away from the surface. Therefore, the localized mode will be useful for wireless power transfer using a transmitting coil array.

In this paper, we focus on localized mode in a one–dimensional resonant circuit array in which a square planar coils are straightly arranged with an overlap. In Sec. 2, localized mode in a resonant circuit array is introduced. Then a circuit equation is derived in Sec. 3. In the following section, the frequency dependency and the localization strength of localized mode are investigated with respect to the position of receiving coil and the amount of overlap of transmitting coils. In the final section, we will make a brief summary.

2. Localized mode in resonant circuit array

Localized mode will appear in a resonant circuit array if an impurity exists. Let us consider a simple example of resonant circuit array shown in Fig. 1. Each resonant circuit interacts only with adjacent circuits. The circuit equation is described as:

$$v_n = -LC\left(\frac{d^2 v_n}{dt^2} + k\frac{d^2 v_{n-1}}{dt^2} + k\frac{d^2 v_{n+1}}{dt^2}\right), \qquad (1)$$

where k is coupling coefficient between adjacent coils. Dispersion relation between the angular frequency and the wave number of the voltage distribution,

$$\omega = \frac{\omega_0}{\sqrt{1 + 2k\cos\kappa}}, \quad \left(k \leq \frac{1}{2}\right), \qquad (2)$$

can be obtained by assuming $v_n(t) = e^{-\mathbf{j}(\kappa n \pm \omega t)}$, where $\omega_0 = 1/\sqrt{LC}$ is the resonant frequency of each resonator and $\mathbf{j}$ is the imaginary unit. As shown in Fig. 2, the dispersion relation has the upper bound and the lower bound for the angular frequency which are described as:

$$\omega_{\mathrm{U}} = \frac{\omega_0}{\sqrt{1 - 2k}}, \quad \omega_{\mathrm{L}} = \frac{\omega_0}{\sqrt{1 + 2k}}, \qquad (3)$$

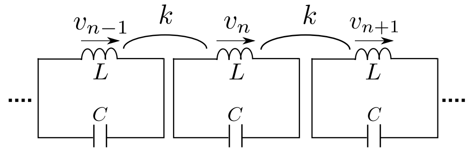


Figure 1: A resonant circuit array

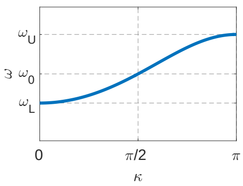


Figure 2: Dispersion relation

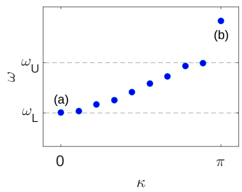
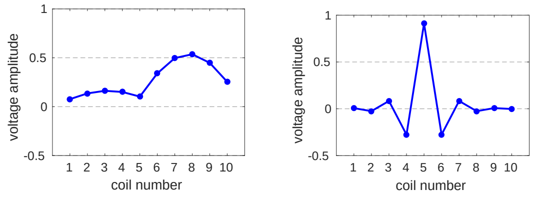
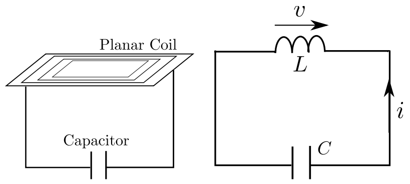


Figure 3: Dispersion relation in ten resonant circuits. $k = 0.1$



(a) mode #1 (b) mode #10

Figure 4: Amplitude distribution. $k = 0.1$. An impurity exists 5th coil.



(a) A planar coil and a capacitor (b) Equivalent circuit

Figure 5: One unit of resonant circuit

respectively. An impurity will cause an isolated mode outside the band. For resonant circuit array, approaching shorted coil induces a light mass impurity to the resonant circuit array [5]. Then an isolated mode is caused above the upper bound of the band.

The number of resonant circuits are assumed to be 10 in this paper. Fig. 3 shows the dispersion relation of the small resonant circuit in which the coupling coefficient is $k = 0.1$ and an impurity is added at 5th coil. The amplitude distributions of the voltage for the lowest frequency mode and the highest frequency mode are shown in Figs. 10(a) and 10(b), respectively. Obviously, the amplitude of the voltage is localized around the impurity for the highest frequency mode whose frequency is above the upper bound ω_U, whereas no voltage localization is observed for the lowest frequency mode whose frequency is not below the lower bound ω_L. Therefore, the localized mode is caused above the upper bound of the band for the resonant circuit array.

The localization of the localized mode depends on the position and the distance of the receiving coil. To discuss quantitative change of the amplitude distribution of the localized mode, we define the localization strength S in this paper. the definitions of the localization strength are,

$$S_n = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(N-\mu)^2|v_n| \Big/ \sum_{n=1}^{N}|v_n|}, \qquad (4)$$

$$S_x = S_n r_{tx}, \qquad (5)$$

where μ is the center of localization $\mu = \frac{1}{N}\sum_{n=1}^{N} n|v_n| \Big/ \sum_{n=1}^{N}|v_n|$, and r_{tx} denotes the distance between neighboring coils. The localization strength S_n is related to variance of the amplitude distribution on the coil index n. On the other hand, S_x gives the variance in real space x. Note that if S is smaller, the localized mode is strongly localized.

3. Circuit equation

In this paper, we consider the resonant circuit array as the transmitting circuit in which each resonator consists of a square coil and a capacitor as shown in Fig. 5. For receiving circuit, the shorted planar square coil is assumed for simplicity. By considering all–to–all coupling among planar square coils in the transmitting and the receiving circuit, the circuit equations,

$$\begin{cases} v_n = \sum\limits_{m=1}^{N} M_{mn}\dfrac{di_m}{dt} + M_{nR}\dfrac{di_R}{dt} \\ \dfrac{di_n}{dt} = -C\dfrac{d^2v_n}{dt^2} \\ 0 = L_R\dfrac{di_R}{dt} + \sum\limits_{m=1}^{N} M_{mR}\dfrac{di_m}{dt}, \end{cases} \qquad (6)$$

is derived, where L_R is the self–inductance of the receiving coil, M_{mn} and M_{nR} represent mutual inductance between mth and nth coils and between nth and the receiving coil, respectively. By using the third equation of Eqs. (6), the following equation is finally obtained,

$$v_n = -LC\sum_{m=1}^{N}(k_{mn} - k_{mR}k_{nR})\frac{d^2v_m}{dt^2}, \qquad (7)$$

where $k_{mn} = M_{mn}/L$, $k_{mR}k_{nR} = M_{mR}M_{nR}/LL_R$. Eq. (7) can be rewritten in matrix form,

$$\boldsymbol{v} = -\boldsymbol{K}\frac{d^2\boldsymbol{v}}{dt^2}, \qquad (8)$$

$$\boldsymbol{K}(m,n) = LC(k_{mn} - k_{mR}k_{nR}). \qquad (9)$$

If the matrix $\boldsymbol{K}$ is regular, we can obtain the following equation.

$$\frac{d^2\boldsymbol{v}}{dt^2} = -\boldsymbol{K}^{-1}\boldsymbol{v} \qquad (10)$$

The equation is the same form as an equation of motion of coupled spring–mass system. In Eq. (10), $\boldsymbol{K}^{-1}(i,j),(i \neq$

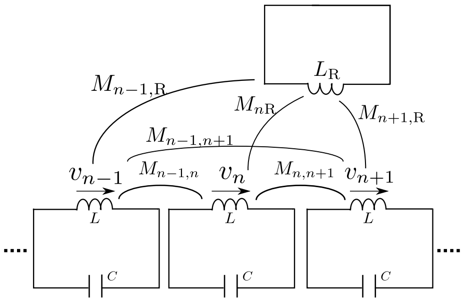


Figure 6: A resonant circuit array and a receiving coil

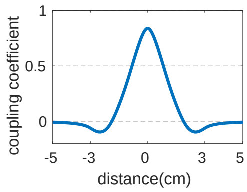


Figure 7: Coupling between coils A_{sq}

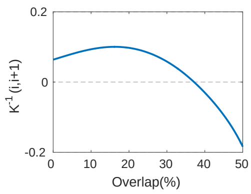


Figure 8: $K^{-1}(i, i+1)$ in only resonant circuit array

j) represents the spring coefficient between ith and jth masses. The square root of the absolute value of eigenvalues of $\boldsymbol{K}^{-1}$ correspond to the angular frequencies of the eigenmodes. The eigenvectors represent the amplitude distributions of the voltage in the transmitting circuit. In this paper, eigenvalues and the eigenvectors are numerically calculated. The highest frequency mode is identified as the localized mode. In the following section, $LC = 1$ is assumed.

4. Overlap dependency of localized modes

4.1. Planar square coils array

A transmitting coil is assumed to be square with side length b. The square coil has a square hole with dimensions $a \times a$. In this paper, two different coils are considered. The smaller coil is labeled A_{sq} and the larger B_{sq}. Both coils are made of the same wire of radius rw. The pitch of winding is w and the number of turns is N. The self–inductances of these coils are computed by using Neumann's formula. The mutual inductance among A_{sq} are shown in Fig. 7. Table. 1 lists the values of the parameters mentioned above.

In this paper, we assume that a transmitting coil array consists of ten coils of A_{sq}. For receiving circuit, two cases are considered: RX-A_{sq} and RX-B_{sq} have the coil A_{sq} and B_{sq}, respectively. For numerical simulations, the position of the receiving circuit is assumed to be on the 5th coil (position α) or between 5th and 6th coils (position β). The overlaps of transmitting coils are varied from 0% to 50%.

Table 1: The size of coils

coil	a	b	rw	w	N	L
A_{sq}	1cm	3cm	0.5mm	1mm	11	2.87μH
B_{sq}	1cm	6cm	0.5mm	1mm	26	24.0μH

4.2. Effective coupling coefficient of transmitting resonant circuit array

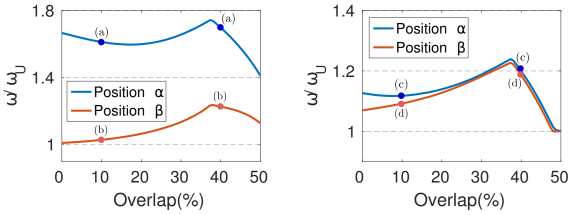
As shown in Fig. 7, the mutual inductance between two A_{sq} coils has zeros when the distance of the coils is about ±2cm. This implies that the effective coupling coefficient $K^{-1}(i, i+1)$ will be zero at a certain overlap. Fig. 8 shows $K^{-1}(i, i+1)$ with respect to the overlap of transmitting coils. Obviously, $K^{-1}(i, i+1)$ becomes zero at 37% overlap. Generally, localized mode strongly localizes if the coupling coefficient is smaller [7]. Thus, it is expected that the 37% overlap will be the best for concentrating the voltage or the current distribution in the transmitting resonant circuit array.

4.3. Frequency and localization strength of localized modes

Figures 9 show the angular frequency of localized modes, which is normalized by ω_{U}, with respect to the overlap. As mentioned above, the frequency becomes a peak at 37% overlap. It implies that the localized mode becomes narrowest at 37% overlap because the frequency difference between ω_{U} and the localized mode is positively–related to the localization [7]. Indeed, S_{n} becomes minimum at 37% overlap which is shown in Fig. 11(a). However, S_{x} monotonically decreases with respect to the overlap. The reason is that even though S_{n} increases as the overlap becomes greater than 37%, the distance between adjacent coils becomes smaller. The effect of the distance decreasing is larger than that of the S_{n} increasing. Therefore, the S_{x} shows the monotonic decreasing.

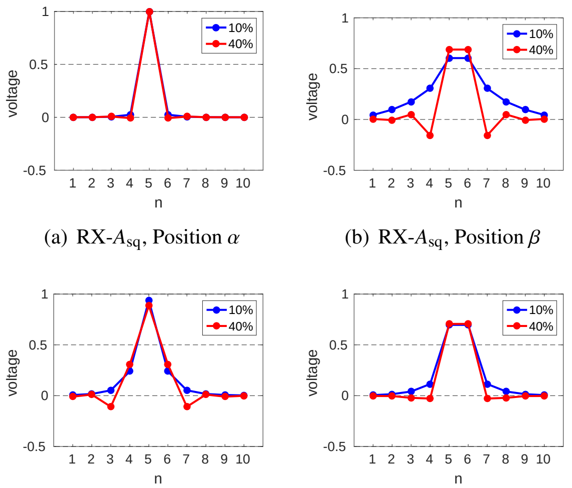
When the receiving coil is A_{sq}, the angular frequency is quite different between the position α and β. On the other hand, the angular frequency is only slightly changed if the receiving coil is B_{sq}. This suggests that the receiving coil should be larger than the transmitting coils because the frequency of the power source can be fixed wherever the receiving coil is placed. If the frequency of the localized mode depends on the position of the receiving coil, the frequency of the power source should be controlled by detecting the receiving coil. It will be complicated for realization.

The shapes of the localized modes labeled (a), (b), (c), (d) in Figs. 9 are shown in Figs. 10. The localized modes labeled (a) is strongly localized comparing with the other cases. However, the localized modes (b) show broad shapes. Thus, the localized modes caused by RX-A_{sq} will greatly change its localization with respect to the position of it. This feature is not desired for wireless power transfer applications. On the other hand, the localized modes (c) and (d) show almost the same distribution. It implies that



(a) RX-A_{sq} (b) RX-B_{sq}

Figure 9: Angular frequency



(a) RX-A_{sq}, Position α (b) RX-A_{sq}, Position β

(c) RX-B_{sq}, Position α (d) RX-B_{sq}, Position β

Figure 10: Amplitude distribution of localized mode

power transfer efficiency will not depend on the position of the receiving coil.

5. Conclusion

In this paper, we have focused on localized modes in a one–dimensional resonant circuit array consisting of overlapped square coils. The amount of overlap strongly affects the frequency dependence and the shape of the localized modes with respect to the position of the receiving coil. As a result, by considering two different receiving coils, the size of the receiving coil should be larger than that of the transmitting coils. In addition, by considering the effective coupling coefficient, it was revealed that there exists an optimal overlap for wireless power transfer. In the future, we will try to introduce the power source, the load, and the parasitic resistance to the circuit equation and will do experiments.

Acknowledgments

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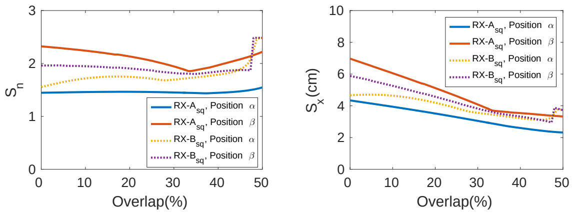

(a) S_{n} (b) S_{x}

Figure 11: Localized strength

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