2017 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2017, Cancun, Mexico, December 4-7, 2017

Initial value estimation for moving intrinsic localized modes in nonlinear coupled oscillators

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Intrinsic localized modes (ILMs), which are also called as discrete breathers (DBs), are known as energy localized states in nonlinear coupled oscillators caused by nonlinearity and spatially discreteness[1, 2]. Since it has been discovered in the Fermi-Pasta-Ulam (FPU) lattice[1], many efforts have been dedicated to understand properties of ILMs especially for standing ILMs which are pinned at a certain site of lattice. Recently, moving ILMs have also attracted researchers attention because of the possibility of carrying kinetic energy in crystals or artificial nano-structures. However, moving ILMs are not sufficiently investigated because of the difficulty of analysis. One of the reasons for it is that initial value estimation for moving ILMs are not established. Therefore, in this work, several methods used for creating moving ILMs in previous studies are evaluated by lifetime and velocity for clarifying what method is the best and what kind of method we can propose in the future.

A mixed lattice,

$$\begin{aligned}\frac{\mathrm{d}^2u}{\mathrm{d}t^2} &= -\alpha_1 u_n - \alpha_2(2u_n - u_{n-1} - u_{n+1}) \\ &- \beta_1 u_n^3 - \beta_2(u_n - u_{n-1})^3 - \beta_2(u_n - u_{n+1})^3,\end{aligned} \quad (1)$$

is focused on in this work. This model is based on the micro-mechanical cantilever array in which ILMs are identified experimentally[3, 4]. When the on-site coefficients α_1 and β_1 are zero, the lattice corresponds to the FPU lattice. On the other hand, the nonlinear Klein-Gordon (NKG) lattice can be represented by eliminating the nonlinear interaction terms, namely $\beta_2 = 0$. The lattice of Eq.(1) is thus called the mixed lattice in this work.

In the mixed lattice, two kinds of standing ILMs exist. One is Sievers-Takeno (ST) mode and the other is Page (P) mode. In this work, the ST-mode is used for initial value estimation for moving ILMs.

For initial value estimation, three existing methods, the direct method, the heuristic method, and the eigenvector method, are evaluated in this work. The direct method is the most simple way to create moving ILM[5]. The initial displacement $\boldsymbol{u}(0)$ is estimated by analytical approximation solution of ILM, namely $\boldsymbol{u}(0) = a_0(\ldots, 0, -1/2, 1, -1/2, 0, \ldots)^{\mathrm{T}}$, where a_0 is amplitude of ST mode. Initial velocities $\boldsymbol{v}(0) = \epsilon(\ldots, 0, 1, 0, -1, 0, \ldots)^{\mathrm{T}}$ is added to start the ILM moving. The velocity of the created moving ILM will positively relate to ϵ.

The heuristic method is similar to the direct method but it requires a numerically rigorous ILM solution for the initial displacement, namely $\boldsymbol{u}(0) = \boldsymbol{u}_{\mathrm{ST}}$. The initial velocity estimation is the same as in the direct method, $\boldsymbol{v}(0)$ is heuristically estimated.

The eigenvector method[6] uses velocity part of the pinning mode eigenvector for estimating the initial velocities. This method requires a standing ILM solution and its eigenvalues and eigenvectors. No heuristic estimation is included in this method.

In addition to the previous methods mentioned above, we introduce new methods for the initial value estimation. One is the spline interpolation method. Several numerically rigorous solutions of moving ILMs are necessary for this method, and they are used for the spline interpolation. The numerically rigorous solution can be obtained at certain velocities by using Newton-Raphson method for Poincaré map. By using spline interpolation, a moving ILM having arbitrary velocity can be obtained. The other method is based on the wavelet function. This method is similar to the direct method because no information on existing standing ILMs are required. Phenomenologically, moving ILM has very similar envelope to wavelet function. This is the reason why the wavelet function is used to estimate initial value of moving ILMs in this work.

For evaluating the methods, the relationship between the proper velocity and the lifetime of moving ILMs are used. As a result, the eigenvector method and the spline interpolation method show a good performance to estimate initial values of moving ILMs. These method can create relatively fast moving ILMs and the created ILMs survive for a long period. In the presentation, we will give more details about estimating initial values of moving ILMs.

Acknowledgments

This work was partially supported by The Kyoto University Foundation.

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