Dynamics regularization with tree-like structures

Rochelle Nieuwenhuis[1] and Madoka Kubota[2] and Morris. R. Flynn[3]
and Masayuki Kimura[4] and Takashi Hikihara[4] and Vakhtang Putkaradze[1,*]

1: Department of Mathematical and Statistical Sciences, University of Alberta
Edmonton, AB, T6G 2G1 Canada
2: Fuji Electric, Power Electronics Technology Development Center
3: Department of Mechanical Engineering, University of Alberta Edmonton, AB, T6G 2G8 Canada
4: Department of Electrical Engineering, Kyoto University Katsura, Kyoto, 615-8510, Japan
∗ Corresponding author: putkarad@ualberta.ca

Abstract—We explore energy harvesting by a fully nonlinear, tree-structured resonator and its response to a broadband forcing of the branches. It is shown that the dynamics leads to the regularization, or narrowing of the spectrum, of vibration towards the stem. We present evidence that branched structures can be highly efficient in energy harvesting compared with alternative broadband harvesting methods, such as oscillators connected in parallel, if the forcing applied to the structure is sufficiently broadband.

Introduction Solar photovoltaic arrays or wind turbines can be used to generate energy on large scales, or, alternatively, can be used to replace or supplement battery power for autonomous equipment operating in remote locales. However, batteries are inconvenient for long-term use, particularly in cold climates where they lose their power quickly. On the other hand, photovoltaic arrays do not perform properly when snow-covered or when solar radiation is weak, e.g. in northern climates during winter months. Finally, wind turbines operate most efficiently only within a narrow band of wind speeds, outside of which the amount of electricity produced plummets. In light of these difficulties, there has been considerable interest in energy harvesting from freely-available ambient mechanical motion. Harvesters of this sort are normally based on a set of objects positioned on a flexible piezoelectric element [1]; they convert the motion of the underlying substrate (e.g. vibration) into the deformation of a piezo element and thereby produce energy. Recent work related to *vibro-wind* energy generation explores the combined dynamics of a chain of such harvesters due to fluid-structure interactions [2] and also the oscillations induced by vortex shedding behind arrays of flexible structures [3].

Reported estimates of energy efficiency of energy harvesting devices vary wildly, due in part to the different ways that the efficiency is defined. For devices based on mechanical oscillators coupled with piezo elements, the efficiency ranges from fractions of a percent to a few percent [4], decreasing for broadband forcing. Recent measurements at the extreme end of impact forcing also suggest a peak efficiency of about 1% [5]. The power output of piezo-electric elements increases with frequency while the efficiency tends to decrease [6], therefore piezo-electric elements tend to operate within an optimal frequency band. Within this band, piezo-based harvesting devices remain a viable option for energy harvesting; studies have demonstrated a power output of $\sim 100\,\mu\mathrm{W/cm^3}$ for $\sim 100\,\mathrm{Hz}$ frequencies [7, 8]. Other studies showed generation at even lower frequencies $\sim 100\,\mathrm{mW}$ at $\sim 50\,\mathrm{Hz}$ [9].

Most research into piezo-electric harvesters has been motivated by the miniaturization of devices, which have corresponding high natural frequencies. At much lower frequencies, such as $\sim 1\,\mathrm{Hz}$, direct piezoelectric generation is inefficient, and two-stage devices have been suggested [10]. The principle of operation for these devices is the “up-scale” of the input frequency. This is achieved using moving internal resonator(s) and yields higher frequencies that are then suitable for harvesting by piezoelectric devices. In this vein, strong nonlinearity, bi-stability and chaotic responses remain topics of active investigation, see *e.g.*, [11, 12]. The case of energy harvesting from incoherent, random forces is known more generally as *broadband harvesting* and is the topic of a recent comprehensive review by [13]. Of particular relevance to this study is the arrangement of resonators in parallel or series for the composite harvesting device [14, 15].

For purposes of increasing harvesting efficiency, we suggest a “regularization” or narrowing of broad-band forcing using a simple, tree-like mechanical structure. An important consequence of our approach is that the band-narrowing is achieved due to the structure of the system itself, without requiring external control mechanisms. Such band-narrowing is useful both from the view of piezoelectric generators and electromagnetic induction. In the former case, piezoelectric design can be fine-tuned to the frequency output of the mechanical structure. In the latter case and for larger-scale applications involving, say, wave or wind motion, one can envision e.g. a flywheel-type drive, producing alternating current (AC) of rather narrow frequency. Such current can later be easily converted with a transformer to a desired voltage. This is in contrast to the direct current (DC) generated using broadband harvesting structures. Because producing electric power from regular motion is a relatively straightforward task, this study focuses almost exclusively on the band-narrowing capabilities of tree-like structures, which we consider the key next step in advancing vibration-harvesting technologies. Although the associated quantitative details will be shown to be nontrivial, the conceptual foundation is simple. Consider, for instance, a tree swaying in the wind. Whereas the motion of the branches are irregular, the movement of the trunk is typically much more regular. Unfortunately, energy extraction from real trees is difficult: recent investigations [16, 17] of life-sized trees yielded modest power generation i.e. 44 mW from the trunk and several Watts for the motion of entire (several m tall) tree including the top branches. This rather disappointing result follows from the comparatively small deviations of the tree trunk from its equilibrium position, an evolutionary adaption presumably meant to minimize the likelihood of trunk fracture during violent wind storms. We therefore construct artificial tree-like structures with characteristics that are in some sense opposite to real trees, providing potentially large amplitude displacements and an efficient regularization of the forcing.

In this paper, we use the term "trees" in the generalized sense, both for structures possessing a branched structure analogous to real trees and, alternatively, for coupled sequences of oscillators which exhibit a tree-like dynamical diagram. The former type is more useful for large-scale, low-frequency harvesting e.g. by wind or waves whereas the latter is more relevant for miniaturized harvesting devices.

The details of the theory and experiments for tree-like structures, pertinent to the dynamics of tree-like structures, are described in the upcoming paper [18], which is available from the authors upon request. In this paper, we focus exclusively on the power generation from the tree-like structure under a broadband forcing, which was not reported earlier.

Energy harvesting efficacy of the tree-like design vs parallel oscillator arrangement. For completeness of the exposition, we shall briefly review the theoretical predictions on energy harvesting presented in [18] and make a comparison between the energy-harvesting efficiency of the branched and parallel designs. The parallel design consists of 7 oscillators with different resonant frequencies. The tree-like design consists of 7 nonlinear springs arranged in a tree-like structure of a stem (level 0), having 2 branches of level 1, with 2 branches at level 2 at each of the first level branches (7 oscillators in total). This will allow us to clarify the advantages and disadvantages of our design, as compared with the classical nonlinear oscillator designs established in the literature. We model each branch as a nonlinear oscillator, with the adjacent branches coupled using a linear force, and utilize the simplest harvesting model of energy produced, dependent on the effective resistance of the harvesting device.

Results from solving the equations of motion from [18] are presented in Figure 1. The efficacy of harvesting of the branched design, shown in Figure 1 a, compared to its parallel counterpart, shown in panel (b), increases as the forcing becomes more and more broadband. When the forcing is fully broadband, the branched structure harvests more than 100 times as much energy as the parallel design. On the other hand, when the forcing contains less than approximately 30 distinct frequencies, the parallel assembly offers superior performance.

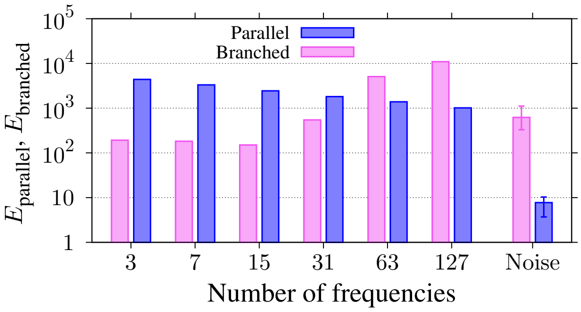

Figure 1: Energies harvested by the branched design E_{branched} and parallel assembly E_{parallel}, from [18].

Energy production from a tree-like structure

To confirm that our experimental model is capable of extracting energy from broadband forcing, a limited series of experimental trials were also conducted inside a closed-loop wind tunnel having a volume of approximately 350 m^3. A schematic of the wind tunnel is presented in figure 1 of Bourguignon, Johnson & Kostiuk (1999) [19]. The test section measured 1.22 m tall by 2.44 m wide and contained a viewing window so that digital images of the tree could be collected as described below (note that it was impractical to make measurements of the acceleration for experiments conducted inside the wind tunnel). A variable-speed 150 kW DC motor was used to drive the wind tunnel fan, which measured 3 m in diameter. Although it was therefore possible to dynamically adjust the wind speed, we found that the response time was slow owing to the inertia of the fan and the large volume of air that had to be accelerated or decelerated. Accordingly, dynamic forcing was achieved by fixing the wind speed and raising and lowering a barrier consisting of a 119 cm wide × 53 cm tall wooden board mounted

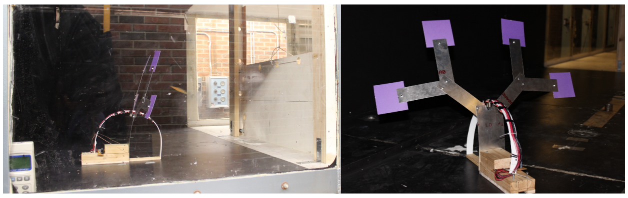

Figure 2: Pictures of the tree-like structure in wind tunnel. Left: the tree-like structure and flow barrier. Right: detail of the tree. The plastic semicircle contains coils with a circuit and LED for energy harvesting.

perpendicular to the oncoming wind along a railing system placed 97 cm in front of the tree – see figure 2. The maximum wind speed as measured using the Sper Scientific vane probe anemometer was approximately 15 m/s whereas the vertical position of the barrier was manually adjusted using a rope-and-pulley system.

Experimental images were collected using a Canon Digital Rebel T3i camera having a video frame rate of 30 fps. The camera was located approximately 1.3 m from the outer branch nearest the camera and was oriented so that, in plan view, the camera, tree and barrier formed an approximate right-angle triangle. In every fifth frame we measured the coordinates associated with reference points at the top of the stem and the outer and top branches nearest the camera.

For purposes of energy conversion via electromagnetic induction, six cylindrical rare earth magnets each of diameter 1.8 cm and height 0.3 cm were placed on either side of the top of the stem, i.e. between the connection points with the two primary branches – see figure 2. As the tree oscillated to and fro, the magnets passed directly below six coils consisting of approximately 800 turns of 30 gauge magnetic wire. The coils each had a diameter of 3.7 cm and thickness of 0.6 cm and were attached to the lower surface of a 2.5 cm wide plastic strip that arced over the central trunk. Along its arc, the plastic strip had a length of 66.0 cm and its ends were fixed 16.5 cm away from the bottom of the central trunk.

In order to rectify the electrical current, each of the coils described above was wired to a diode bridge. Bridges were connected in parallel to three 2200 μF capacitors, followed by a 7.5 V zener diode. As shown in the circuit diagram of figure 3 (top panel), these components were in turn connected to a 0.25 W 100 Ω resistor and a 3.2 V 20 mA red LED. The latter was illuminated when electromagnetic induction resulted in a voltage $\geq$ 3.2 V and thus the power generated was larger than 60mW. Under the applied forcing, energy generation and capacitor storage provided a continuous source of power to the diode.

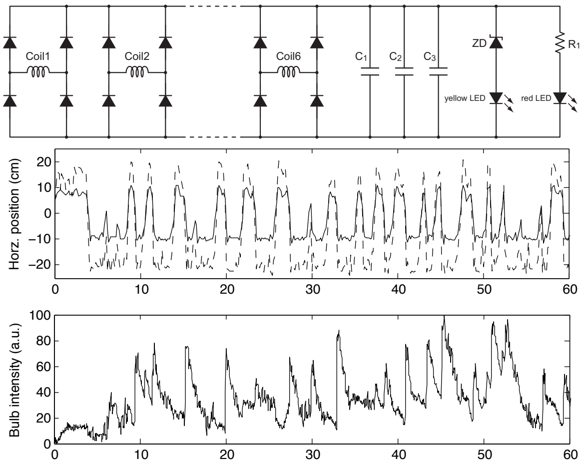

Figure 3: Top: The circuit diagram used in conjunction with the tree-like energy harvesting device shown in Figures 2. Middle: Horizontal deviations of the stem (solid line) and top branch (dashed line) from their respective equilibrium values. Bottom: LED brightness as measured using a Canon Digital Rebel T3i camera. In the middle and bottom panels, the horizontal axis variable is time, t, measured in seconds.

The middle and top panels of figure 3 show, respectively, timeseries of the horizontal deviation of the stem and top brach from their equilibrium positions and timeseries of the LED illumination. Note that in the latter case the diode brightness measured by the digital camera is a nonlinear function of the electrical current and so should not be interpreted as a direct measurement of the power produced by the energy harvester. Another interesting feature of the harvester is that the peak LED brightness does not correspond to the maximum stem acceleration, which is due to the presence of the capacitors in the circuit.

The data of figure 3 confirm that our tree-like energy harvester is capable of power generation in excess of 60 mW continuously over a prolonged period. Of course, if the forcing is steady then a wind turbine would offer superior performance to our energy harvester. Similar comments apply in the case of a narrow, well-defined frequency band, in which other power generation methods are expected to exceed the efficiency the tree-like device. However, if the forcing is highly broadband, the harvesting efficacy of our device is superior to others suggested in the literature. Therefore, this simple and robust harvesting device is expected to provide enough power to operate modern electronic sensor and positioning devices. Such applications are highly useful in circumstances where standard sources of stand-alone power, such as solar voltaic cells, are not applicable, for example, for devices working in prolonged darkness.

Conclusion and further directions Of course, a multi-level structure exhibiting branching has many resonances and thus increased response to broadband forcing. However, the regularization of the vibration at each level cannot be explained solely by a linear analysis. The key to the regularized vibration is the interaction of nonlinearity and multiple coupled resonances, as is explained in [18]. More studies are needed to optimize the parameters of structured tree-like design for broadband energy harvesting applications.

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