2017 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2017, Cancun, Mexico, December 4-7, 2017

A Diagonal Calibration Method of Images Using a Rectangular Object in the case when the Optical Axis is Unknown

Takashi Ozeki† and Eiji Watanabe‡

†Department of Computer Science, Faculty of Engineering, Fukuyama University
1 Sanzo, Gakuen-cho Fukuyama, Hiroshima, 729-0292 Japan
‡Faculty of Intelligence and Informatics, Konan University
Kobe, 658–8501, JAPAN
Email: ozeki@fuip.fukuyama-u.ac.jp, e_wata@konan-u.ac.jp

Abstract—In this paper, we propose a diagonal calibration method of images when the optical axis is unknown. The method uses a rectangular object to calibrate images. By using the parallel lines, the orthogonality of corners of the object and the object's aspect ratio, we can estimate the camera angle and obtain a corrected image even if the image is cropped.

1. Introduction

If a poster on a bulletin board is taken by a digital camera, the captured image may be distorted diagonally because it is not taken from directly in front. So, many applications to correct images have been developed for pictures taken from a diagonal direction and taken from a tilted direction. In those methods, the image is often corrected based on the information of a deformed rectangular object in the image. The image is corrected by restoring the shape of a deformed rectangular object in the image [1]. Especially, it is assumed that the optical axis passes through the center position of the image [2]. However, such methods can't be used for cropped images since the optical axis is unknown. In this paper, we discuss the possibility of diagonal correction of images in the case when the optical axis is unknown.

2. Projection Model and Image Coordinates

In this paper, it is assumed that a rectangular object like a paper is included in the photographed image as shown in Fig. 1. In the projection model, as shown in Fig. 2, the

Figure 1: An Image including a Rectangular Object.

rectangular object is on the XY plane. Also, we set the coordinate system so that a corner point O of the quadrangle $OACB$ on a rectangular object is at the origin $(0, 0, 0)$. At this time, by rotating the image if necessary, four points O, A, C, B are relocated so that there are two vanishing points E, F those are intersections of extension of the line segment OA and the line segment BC and extension of the line segment OB and the line segment AC, respectively. Next, we set the Z axis in the depth direction. Also, we assume that the viewpoint $V(p, q, r)$ of the camera is on the front $r < 0$ and a plane of the corrected image on which the rectangular object is placed contains the origin $O(0, 0, 0)$. After that, we call the plane as the corrected plane. Even if such coordinate axes are set, generality of the problem can be maintained. The case when there are no two vanishing points is omitted in this paper.

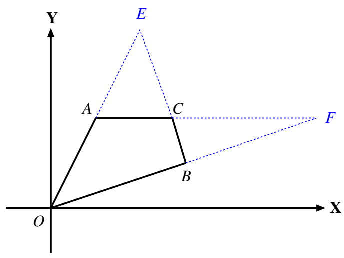

Figure 2: Setting Coordinates.

3. Correction Method when the Optical Axis is Known

Let O', A', C', B' to be four corner points of a rectangular object on the corrected plane as shown in Fig. 3. Here, the point A' is on the extension of the line segment VA. Other two points B' and C' are also on the extension of the line segment VB and VC, respectively. Firstly, we explain a relationship between the viewpoint V and four corner points O', A', C', B' of a rectangular object on the corrected plane using Fig. 3. Let k be the intersection line of two planes which contain three points V, O', A' and three points V, B', C', respectively. We use a proposition written in [2].

Proposition 1 *Let m and n be two different parallel lines in 3D space. Put planes containing the line m and n to be α and β, respectively. At this time, if two plane α and β aren't*

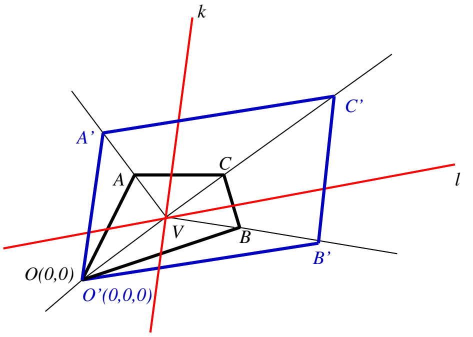

Figure 3: A Relationship between a Viewpoint V and a Rectangle $O'A'C'B'$.

parallel, the intersection line is also parallel to two lines m and n.

From this proposition, the line which contains the line segment $O'A'$ and the line which contains the line segment $B'C'$ are parallel to the line k. Similarly, the line l which is the intersection of the plane contains three points V, A', C' and the plane contains three points V, O', B' are parallel to the line segment $A'C'$ and the line segment $O'B'$. Therefore, the plane containing the viewpoint V and two lines k and l is also parallel to the corrected plane which contains the rectangular object $O'A'C'B'$. Furthermore, two lines k and l are orthogonal at the viewpoint V since the quadrangle $O'A'C'B'$ is a rectangle.

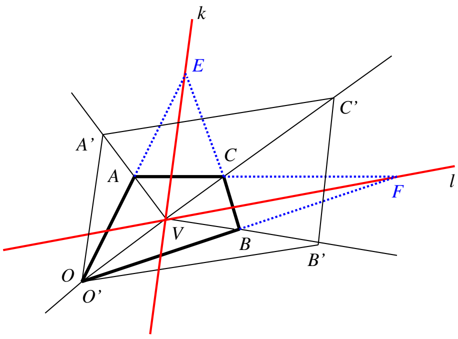

Figure 4: A Relationship between a Viewpoint V and two Vanishing Points E, F.

Next, we explain a relationship between a viewpoint V and two vanishing points E and F using Fig. 4. The vanishing point E is the intersection of two line containing the line segment OA and the line segment BC and it is included in the XY plane including the quadrangle $OACB$. Moreover, the vanishing point E is a point on the plane containing the triangle VOA. Therefore, it is also included in the plane containing the triangle $VO'A'$. Similarly, the vanishing point E is a point on the plane containing triangle $VB'C'$. As a result, the vanishing point E is a point on the intersection line k of two planes. In the same way, the other vanishing point F is a point on the intersection line l. Since two lines k and l are orthogonal each other, the angle of EVF is 90 degrees. Therefore, the viewpoint $V(p, q, r)$ exists on the spherical surface whose diameter is the line segment EF. Here, if the center of the image is the optical axis of the camera, we can obtain the values of p and q. So, it is also possible to determine the values of $r < 0$. From the coordinates of the viewpoint V and two vanishing points E, F, we can obtain the corrected plane containing the origin O using the fact that the corrected plane is parallel to the plane including three points V, E, F. Finally, we can obtain the rectangle $O'A'C'B'$ from the coordinates of the viewpoint V and the quadrangle $OACB$. This method has been already stated in [2].

4. Correction Method when the Optical Axis is Unknown

In the following, we will consider a correction method when it is not known whether the optical axis of the camera passes through the center of the image. If we don't know the optical axis of the camera, we can't decide where is the viewpoint on the sphere which has the diameter with two vanishing points E and F. Therefore, we add a condition that the aspect ratio of a rectangular object is known. For example, ordinary A4 papers has an aspect ratio of $1 : \sqrt{2}$.

We set the ratio to be $O'A' : O'B' = 1 : k$. Also, we denote $O(0,0)$, $A(a_1, a_2)$, $C(c_1, c_2)$, $B(b_1, b_2)$, $E(e_1, e_2)$, $F(f_1, f_2)$. Let $G(g_1, g_2)$ to be the center of the sphere where the viewpoint exists. Since the point G is the midpoint of two vanishing points E and F, we obtain

$$G\begin{pmatrix} g_1 \\ g_2 \end{pmatrix} = \begin{pmatrix} \frac{e_1+f_1}{2} \\ \frac{e_2+f_2}{2} \end{pmatrix}. \tag{1}$$

The spherical surface including the view point $V(p, q, r)$ with the line segment EF as the diameter is expressed by the equation:

$$(x - e_1)(x - f_1) + (y - e_2)(y - f_2) + z^2 = 0. \tag{2}$$

Also, we obtain the corrected plane including the origin O as follows

$$\begin{aligned} &(f_2 - e_2)rx + (e_1 - f_1)ry \\ &+ \quad \{(e_2 - f_2)p + (f_1 - e_1)q + e_1 f_2 - e_2 f_1\} z = 0 \end{aligned} \tag{3}$$

since it has a normal vector perpendicular to two line segments VE and VF.

Because the point $A'(a'_1, a'_2, a'_3)$ is the intersection of the line containing the line segment VA and the corrected plane of Eq. (3), we obtain

$$A'\begin{pmatrix} a'_1 \\ a'_2 \\ a'_3 \end{pmatrix} = \begin{pmatrix} (a_1 - p)t_a + p \\ (a_2 - q)t_a + q \\ -rt_a + r \end{pmatrix}. \tag{4}$$

Here, the equation

$$t_a = \frac{e_1 f_2 - e_2 f_1}{(e_2 - f_2)a_1 - (e_1 - f_1)a_2 + (e_1 f_2 - e_2 f_1)} \tag{5}$$

holds. Next, we set $OE = \alpha OA$ ($\alpha > 1$) and $OF = \beta OB$ ($\beta > 1$) in Fig. 2. Then, we have

$$E\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \alpha \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad (\alpha > 1) \tag{6}$$

and

$$F\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \beta \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \quad (\beta > 1). \tag{7}$$

Therefore, from Eq. (5), We obtain

$$t_a = \frac{\alpha}{\alpha - 1} > 1. \tag{8}$$

By the same way, we obtain

$$B'\begin{pmatrix} b'_1 \\ b'_2 \\ b'_3 \end{pmatrix} = \begin{pmatrix} (b_1 - p)t_b + p \\ (b_2 - q)t_b + q \\ -rt_c + r \end{pmatrix}. \tag{9}$$

Here, it holds

$$t_b = \frac{\beta}{\beta - 1} > 1. \tag{10}$$

From $O'(0,0,0)$ and the relation $O'A' = kO'B'$, we obtain

$$\begin{aligned} &\{(a_1 - p)t_a + p\}^2 + \{(b_2 - q)t_b + q\}^2 \\ = \;& k^2\{(b_1 - p)t_b + p\}^2 + k^2\{(b_2 - q)t_b + q\}^2. \end{aligned} \tag{11}$$

Therefore, if we set $M = (\alpha - 1)^2$ and $N = (\beta - 1)^2$, we obtain a sphere

$$\begin{aligned} &(N - k^2M)p^2 - 2(e_1N - k^2f_1M)p \\ +\;& (N - k^2M)q^2 - 2(e_2N - k^2f_2M)q \\ +\;& (N - k^2M)r^2 + (e_1^2 + e_2^2)N - k^2(f_1^2 + f_2^2)M \\ =\;& 0. \end{aligned} \tag{12}$$

4.1. In the case of $k^2 = \frac{N}{M}$

In this case, the sphere Eq. (12) becomes

$$\begin{aligned} &2(e_1 - f_1)p + 2(e_2 - f_2)q \\ -\;& (e_1^2 + e_2^2) + (f_1^2 + f_2^2) = 0. \end{aligned} \tag{13}$$

Here, if we assume

$$e_1 - f_1 = e_2 - f_2 = 0, \tag{14}$$

we obtain

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} // \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}. \tag{15}$$

This means OE // OF. Hence, there are no vanishing points and contradiction. So, the coefficients of p, q don't take zero at the same time and Eq. (12) becomes a plane m. Also, from Eq. (1) and Eq. (13), we understand that the center point G of the sphere (2) is on the plane m. Moreover, as shown in Fig. 5 the line segment EF of two vanishing points and the plane m are orthogonal at the point G. In conclusion, the sphere (2) and the plane (13) intersect and the viewpoint $V(p, q, r)$ exists on the great circle of that intersection.

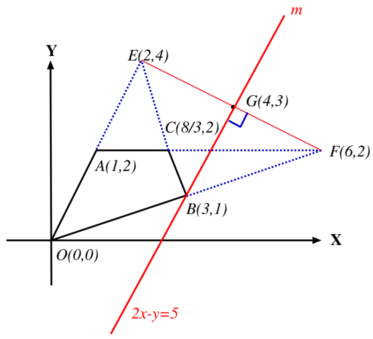

Figure 5: A Sample in the case of $k^2 = \frac{N}{M}$.

4.2. In the case of $k^2 \neq \frac{N}{M}$

In this case, the sphere Eq. (12) becomes

$$\begin{aligned} &\left(p - \frac{Ne_1 - k^2Mf_1}{N - k^2M}\right)^2 + \left(q - \frac{Ne_2 - k^2Mf_2}{N - k^2M}\right)^2 + r^2 \\ &= \frac{k^2MN}{(N - k^2M)^2}EF^2. \end{aligned} \tag{16}$$

Therefore, the center point H of this sphere is expressed by

$$H\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} \frac{Ne_1 - k^2Mf_1}{N - k^2M} \\ \frac{Ne_2 - k^2Mf_2}{N - k^2M} \\ 0 \end{pmatrix}. \tag{17}$$

This means that the point divides two points E, F into N versus k^2M externally and it also exists on the line of the XY plane including line segment EF.

If we set each radius of two spheres (2) and (16) to be R_1, R_2, we obtain $R_1^2 = \frac{EF^2}{4}$, $R_2^2 = \frac{k^2MN}{(N-k^2M)^2}EF^2$ and

$$HG^2 = \frac{(N + k^2M)^2}{4(N - k^2M)^2}EF^2. \tag{18}$$

Therefore, it holds

$$HG^2 = R_1^2 + R_2^2. \tag{19}$$

Hence, as shown in Fig. 6, two spheres form a right angle at the intersection point I on the XY plane. The plane m

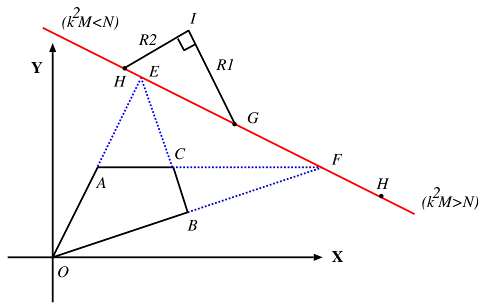

Figure 6: A Relationship between Two Spheres.

which contains the intersection of two spherical surfaces is obtained from two spherical Eq. (2) and Eq. (16) as follows:

$$\begin{aligned}(N+k^2M)*(f_1-e_1)x+(N+k^2M)*(f_2-e_2)y\\ +\quad N(e_1^2+e_2^2-e_1f_1-e_2f_2)\\ -\quad k^2M(f_1^2+f_2^2-e_1f_1-e_2f_2)=0.\end{aligned} \quad (20)$$

From this equation, the view point $V(p,q,r)$ exists on a circle where a sphere (2) and a sphere (16) intersect. Here, the plane m and the line of the XY plane including the line segment EF as follows:

$$(f_2-e_2)x-(f_1-e_1)y+e_2f_1-e_1f_2=0 \quad (21)$$

intersect at right angles and the intersection point J is expressed by

$$J\begin{pmatrix} j_1 \\ j_2 \end{pmatrix}=\begin{pmatrix} \frac{Ne_1+k^2Mf_1}{N+k^2M} \\ \frac{Ne_2+k^2Mf_2}{N+k^2M} \end{pmatrix}. \quad (22)$$

It divides two vanishing points E,F into N versus k^2M internally as shown in Fig. 7.

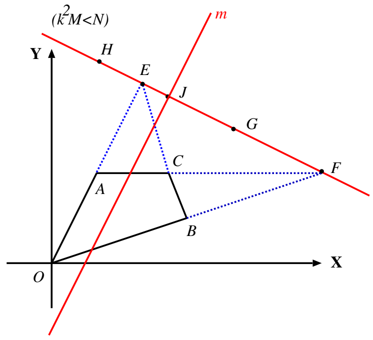

Figure 7: A Relationship between Points E, F, G, H, J.

5. Determining the Viewpoint on the Circle

In both cases of subsection 4.1 and 4.2, there is a viewpoint on a circle. Actually, if it holds $r<0$, the viewpoint can be any point on the obtained circle since corrected rectangles by using those viewpoints become all similar. The reason is that if each point X on the photographed image is set to $OX=sOA+tOB$ where $0\le s<\alpha$ and $0\le t<\beta$, the point becomes on the corrected plane as follows:

$$O'X'=\frac{s\beta(\alpha-1)}{\alpha\beta-t\alpha-s\beta}O'A'+\frac{t\alpha(\beta-1)}{\alpha\beta-t\alpha-s\beta}O'B'. \quad (23)$$

Therefore, the point X' is not related to the coordinates of the viewpoint $V(p,q,r)$ but the relative position in the corrected plane is determined only by the values of s,t,α,β. Therefore, for example, we can determine a viewpoint V as

$$V\begin{pmatrix} p \\ q \\ r \end{pmatrix}=\frac{1}{N+k^2M}\begin{pmatrix} Ne_1+k^2Mf_1 \\ Ne_2+k^2Mf_2 \\ -k\sqrt{NM}\cdot EF \end{pmatrix}. \quad (24)$$

Fig 8 shows a cropped image of the Fig. 1 and corrected images by using our method.

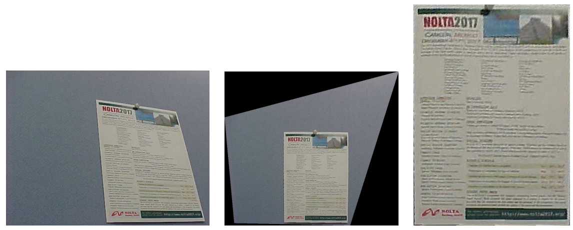

Figure 8: A Cropped Image and Corrected Images.

6. Relationship with Projective Transformation

Using the following projective transformation, it is possible to correct distorted images if the coordinates for more than four points by the transformation are known [3].

$$\begin{pmatrix} X \\ Y \end{pmatrix}=\frac{1}{h_{31}x+h_{32}y+h_{33}}\begin{pmatrix} h_{11}x+h_{12}y+h_{13} \\ h_{21}x+h_{22}y+h_{23} \end{pmatrix} \quad (25)$$

In this transformation, a line l_v expressed by $h_{31}x+h_{32}y+h_{33}=0$ is including two vanishing points. Also, a line l_y shown by $h_{11}x+h_{12}y+h_{13}=0$ is including the line segment OA and a line l_x expressed by $h_{21}x+h_{22}y+h_{23}=0$ is including the line segment OB. By projective transformation, the line l_y is projected on the Y axis and the line l_x is projected on the X axis. Also, the line l_v is projected to a line at infinity.

7. Conclusions

In this paper, we discussed the possibility of diagonal correction of cropped images. If there are two vanishing points, a viewpoint can be determined for all convex quadrangles and a diagonal calibration is possible under the condition that there is a rectangular object in the image and the aspect ratio is known,

Acknowledgments

This work was supported by JSPS KAKENHI Grant Number JP16K00499.

References

[1] Y. Takezawa, M. Hasegawa and S. Tabbone, "Camera-Captured Document Image Perspective Distortion Correction Using Vanishing Point Detection Based on Radon Transform," *ICPR2016*, pp. 3957–3963, 2016.

[2] T. Ozeki and E. Watanabe, "A Visual Position Calibration Method of Images on a Table by a Rectangle Paper -Second report-," *The Memoirs of the Faculty of Engineering Fukuyama University*, vol. 35, pp. 143–pp.147, 2012.

[3] R. Szeliski, "Computer Vision Algorithms and Applications," *Springer UK*, 2011.