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A Coherent Ising Machine Applied to Asymmetric Traveling Salesman Problems

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Abstract– Effectiveness of high-speed combinatorial optimization method using Coherent Ising Machine (CIM) has been shown. Mutually connected neural network has been applied to the traveling salesman problem as an optimization algorithm. In order to improve the performance of the mutually connected neural network and run it with high-speed, the method of solving the combinatorial optimization problem by running mutually connected neural network on CIM has been proposed. In this paper, we aim to solve asymmetric traveling salesman problem (ATSP) in high speed by mapping the problem to the mutually connected neural network, which is difficult to obtain optimal solution by heuristic algorithms. Performance evaluations by simulation show that the optimum solution of ATSP can be obtained by the proposed method.

1. Introduction

A fast solution method of combinatorial optimization problems using Coherent Ising Machine (CIM) has been proposed [1] [2]. It is possible to obtain a good solution of the combinatorial optimization problem in a very short time and it has already been shown that CIM can obtain the good solution of the MAX-CUT problem which is one of the combinatorial optimization problem at high speed [3] For the MAX-CUT problem, high-speed and good approximate solutions are obtained even in a real machine of CIM [4] [5]. In Ref. [4], CIM with 100 spins can obtain an exact solution of the MAX-CUT problem at high speed, and Ref. [5] using the CIM with 2,048 spins, also shows the CIM can obtain the solution of the specific MAX-CUT problem with the computational time 70μs. In Ref. [5], CIM can get a solution at a very high speed and sufficiently high solution accuracy as compared with the simulated annealing.

We have already proposed a method using Hopfield-Tank Neural Network (HTNN) [6], which is a neural network with energy minimization properties, to solve combinatorial optimization problems in CIM [7]. HTNN and CIM have the same mutually connected network. Since coupling coefficient and bias of CIM can be determined through formulation of energy function by HTNN, combinatorial optimization problem can be solved by CIM. In Ref. [7], it is possible to optimize and find the optimal solution of Traveling Salesman Problem (TSP) and Quadratic Assignment Problem (QAP) using the CIM model.

Among traveling salesman problems, the problem that the cost between two cities depending on the visiting order of the city is called asymmetric traveling salesman problem (ATSP) and various applications to real society are known [8] [9]. It is known that it is difficult to find an optimal solution with a simple heuristic solution because the problem structure is more complicated than symmetric TSP [10]. Therefore, there is possibility that the CIM which find solution with high precision at high speed is effective.

In this paper, we aim to obtain an optimal solution of ATSP at high speed by CIM. We first define an energy function to solve ATSP using HTNN and derive the weights and threshold of HTNN as in Ref. [7]. Because of the obtained weights and threshold, we can determine mutual couplings and bias to operate on CIM. We also evaluate performance using computer simulation and clarify the performance of the proposed method.

2. Coherent Ising Machine

2.1. Coherent Ising Machine (CIM)

Coherent Ising Machine (CIM) is a system that can obtain minimum state (ground state) of the Ising model Hamiltonian at a high speed. Ising model is a model of a magnetic body composed of two states of upward and downward spins. The Hamiltonian of the whole mutual connected network is given by

$$H_{CIM} = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} J_{ij}\sigma_i\sigma_j + \sum_{i=1}^{N}\lambda_i\sigma_i \qquad (1)$$

where σ_i is the ith spin state ± 1, J_{ij} is the interaction strength between ith spin and jth spin, and λ_i is the influence of external magnetic field. It is known that many combinatorial optimization problems can be converted to Ising model [11].

In the CIM implemented using the phase of the optical parametric oscillator (OPO) as the spin, the OPO group networked by the optical transmission line often oscillates with a combination of phases that minimizes the loss of the whole network. Update formulations of spin state in CIM are given by

$$\frac{dc_{ij}}{dt} = (-1 + p - c_{ij}^2 - s_{ij}^2)c_{ij} + \sum_{k=1}^{N}\sum_{l=1}^{N}\xi_{ijkl}c_{kl} - \lambda_{ij} \qquad (2)$$

$$\frac{ds_{ij}}{dt} = (-1 - p - c_{ij}^2 - s_{ij}^2)s_{ij} + \sum_{k=1}^{N}\sum_{l=1}^{N}\xi_{ijkl}s_{kl} - \lambda_{ij} \quad (3)$$

Here, p is the pump rate, c_{ij} and s_{ij} are the signal amplitude of in-phase component and quadrature component, respectively. ξ_{ijkl} is the coupling coefficient, and λ_{ij} is the Zeeman term as a bias.

When ξ_{ijkl} and λ_{ij} are derived from the objective function of the problem and p is adjusted, CIM converges autonomously and rapidly to a stable state where the Hamiltonian is the lowest, and the state of c_{ij} is the solution of combination optimization problem.

2.2. Optimization of Combinatorial Problem by Using Hopfield-Tank Neural Network (HTNN) and CIM

Combinatorial optimization method using Hopfield-Tank Neural Network (HTNN) [7] which solves combinatorial optimization problem mapping on neural network, is used for traveling salesman problem (TSP), quadratic allocation problem (QAP) and so on. We apply this optimization method to CIM which consists of mutual coupling of spin groups by OPO pulse.

The Hopfield-Tank Neural Network (HTNN) is a mutually connected neural network, and the output value X_{ij} of each neuron is represented by 1 or 0 depending on whether it exceeds the threshold θ_{ij} or not. The update formulation of states is given by

$$X_{ij}(t+1) = \begin{cases} 1 & \text{if} \ \sum_{k=1}^{N}\sum_{l=1}^{N} W_{ijkl} X_{kl}(t) - \theta_{ij} > 0 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

where W_{ijkl} is the weight of coupling between neurons. By updating the neuron according to the update formulation, HTNN decreases the energy function monotonically. Therefore, it is possible to search the solution by matching the energy function with the objective function of the optimization problem. The energy function of HTNN is given by

$$E_{HTNN} = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N} W_{ijkl} X_{ij} X_{kl} + \sum_{i=1}^{N}\sum_{j=1}^{N}\theta_{ij} X_{ij} \quad (5)$$

Since HTNN and CIM have the same mutually connected network structure, by matching the energy function of the target optimization problem with the energy function of HTNN, we determine interaction ξ_{ijkl} and the bias λ_{ij} which are components of CIM from the weight W_{ijkl} and the threshold θ_{ij} of HTNN. Therefore, we can solve various optimization problems by CIM when we derive coupling coefficient and bias required for updating states of CIM.

For example, Ref. [7] shows the method is applied to the Symmetric Traveling Salesman Problem (STSP) and Quadratic Assignment Problem (QAP), and indicates that it is possible to optimize the problems based on CIM model. In this paper, we focus on the asymmetric traveling salesman problem (ATSP) and show feasibility of high-speed optimization.

3. Asymmetric Traveling Salesman Problem

Traveling Salesman Problem (TSP) is a problem of finding the shortest path around all cities given the set of cities and the cost of moving between cities. In general, it is difficult to find exact solution of large scale TSP because the total number of combinations is $(N-1)!/2$, where N is the number of cities. In Ref. [7], it is confirmed that it is possible to find the optimum solution with high probability by the simulation using CIM model for 10 city TSPs.

Asymmetric traveling salesman problem (ATSP) is the general case of traveling salesman problem; the cost of distance between two cities varies according to the direction of movement. The total number of combinations of N city ATSP is $(N-1)!$, twice as large as STSP and it is difficult to find an optimal solution by heuristic algorithm. Therefore, in this paper, we consider optimization of ATSP on CIM to solve ATSP at high speed.

4. *N*-City ATSP Optimization on CIM

4.1. Application for HTNN

At first, we prepare a neuron of size $N \times N$ as shown in Figure 1 to derive the energy function of HTNN. We define a city index i and a visiting order index j in neuron. If the (i,j)th neuron fired ($X_{ij} = 1$), it is indicated that city i is the jth city visited in a tour.

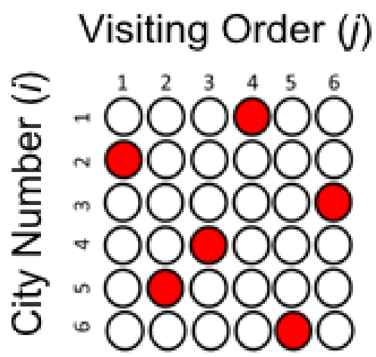

Figure 1 : HTNN neuron and example of firing pattern $(N = 6)$.

Next, we define constraint and objective function. The constraints necessary for ATSP are two points, (I) Visiting same city only once, and (II) Only one city visiting at the same time, which are represented by the following energy functions.

$$E_1 = \sum_{i=1}^{N}\left(\sum_{j=1}^{N} X_{ij} - 1\right)^2, \tag{6}$$

$$E_2 = \sum_{j=1}^{N}\left(\sum_{i=1}^{N} X_{ij} - 1\right)^2. \tag{7}$$

Since purpose of ATSP is to minimize the length of tour around the city, objective function of ATSP E_3 is given by

$$E_3 = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} X_{ij}(d_{ik}X_{k,j+1} + d_{ki}X_{k,j-1}) \tag{8}$$

Therefore, the energy function of ATSP weights each term with parameters A, B, C and is expressed as follows,

$$E_{ATSP} = AE_1 + BE_2 + CE_3. \tag{9}$$

Weight of HTNN W_{ijkl} and threshold of HTNN θ_{ij} can obtained by comparing with derived E_{ATSP} in Eq. (9) and energy function of HTNN E_{HTNN} in Eq. (5) as follows,

$$W_{ijkl} = -A\delta_{ik}(1-\delta_{jl}) - B\delta_{jl}(1-\delta_{ik}) - C(d_{ik}\delta_{l,j+1} + d_{ki}\delta_{l,j-1}), \tag{10}$$

$$\theta_{ij} = \frac{-(A+B)}{2}, \tag{11}$$

where δ_{ij} is the Delta function. δ_{ij} takes 1 only when $i = j$. Otherwise, it takes 0.

4.2. Optimization Method by Using CIM

We consider implementing HTNN on CIM. We must convert the output implementation as in Eq. (12) because output of HTNN is {0, 1} and output of CIM is {-1, 1},

$$\hat{X}_{ij} = 2X_{ij} - 1. \tag{12}$$

We use $\hat{X}_{ij}$ to derive the following formulation, ±1 HTNN.

$$E_{\pm 1HTNN} = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N}\hat{W}_{ijkl}\hat{X}_{ij}\hat{X}_{kl} + \sum_{i=1}^{N}\sum_{j=1}^{N}\hat{\theta}_{ij}\hat{X}_{ij}, \tag{13}$$

$$\hat{W}_{ijkl} = \frac{W_{ijkl}}{2}, \tag{14}$$

$$\hat{\theta}_{ij} = \theta_{ij} - \sum_{k=1}^{N}\sum_{l=1}^{N}\frac{W_{ijkl}}{2}, \tag{15}$$

Weight of ±1 HTNN $\hat{W}_{ijkl}$ and threshold of them $\hat{\theta}_{ij}$ are compared with Hamiltonian formulation of CIM (1). The mutual coupling ξ_{ijkl} and the external magnetic field term λ_{ij} are defined as

$$\begin{aligned}\xi_{ijkl} &= W_s\hat{W}_{ijkl} = \frac{W_sW_{ijkl}}{2} \\ &= \frac{W_s}{2}\{-A\delta_{ik}(1-\delta_{jl}) - B\delta_{jl}(1-\delta_{ik}) \\ &\quad - C(d_{ik}\delta_{l,j+1} + d_{ki}\delta_{l,j-1})\}.\end{aligned} \tag{16}$$

$$\begin{aligned}\lambda_{ij} &= T_s\hat{\theta}_{ij} = T_s\left(\theta_{ij} - \sum_{k=1}^{N}\sum_{l=1}^{N}\frac{W_{ijkl}}{2}\right) \\ &= \frac{T_s}{2}[-(A+B) - \sum_{k=1}^{N}\sum_{l=1}^{N}\{-A\delta_{ik}(1-\delta_{jl}) \\ &\quad - B\delta_{jl}(1-\delta_{ik}) - C(d_{ik}\delta_{l,j+1} + d_{ki}\delta_{l,j-1})\}].\end{aligned} \tag{17}$$

where W_s and T_s are scaling parameters.

5. Performance Evaluation by Simulation

We evaluated performance of the proposed method using ATSP problem of 10 cities. We create 10-city problem by deleted some cities from the TSPLIB benchmark problem. Table 1 shows the problem structure.

Table 1: Problem structure of 10-city ATSP

Departure City Index \ Destination City Index	1	2	3	4	5	6	7	8	9	10
1	0	26	82	65	100	147	134	69	117	42
2	66	0	56	39	109	156	140	135	183	108
3	43	57	0	16	53	100	84	107	155	85
4	27	41	62	0	97	144	131	96	144	69
5	109	135	161	174	0	47	34	54	102	67
6	157	171	114	130	60	0	40	114	162	127
7	143	169	132	148	34	31	0	88	133	101
8	95	121	177	160	54	101	88	0	48	53
9	79	105	161	144	91	138	125	37	0	37
10	42	68	124	107	67	114	101	27	75	0

Table 1 shows the distance between cities as a matrix. Since self-coupling does not exist in HTNN, then diagonal elements are all zero in this paper. This problem is mapped on the CIM, and the spin state is updated based on the model of CIM (Eqs. (2) and (3)), and the final route is determined from the firing states of the spins. To determine the firing states, we use the judgment method [13] which treats the upper N number of neurons as firing, only the upper 10 pulses of the value are judged as firing.

When the bias parameter T_s and the pump rate p are varied with respect to the parameter W_s, we show the changes in the optimal solution arrival rate as the performance evaluation result in Fig. 2. We set parameters, $A = B = 1.0$, and $C = 0.18$.

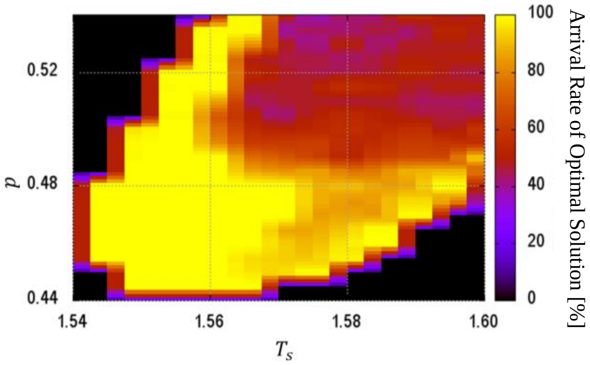

Figure 2: Arrival rate of optimal solution 10-city ATSP ($W_s = 1.66$).

Then we show the time series of spins. It turns out that c_{ij} is in a stable state at about 2.0ms.

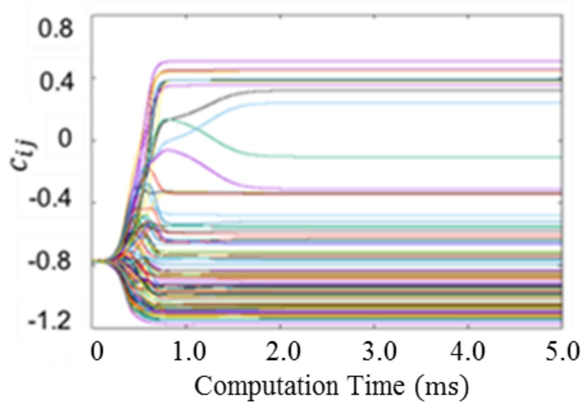

Figure 3 : Time series of in-phase component in 10-city ATSP ($A = B = 1.0, C = 0.18, W_s = 1.66, T_s = 1.57, p = 0.47$) .

Finally, the changes of Hamiltonian are shown in Figure 4. This figure shows that the Hamiltonian gradually decreases with state update and that the minimum value of the Hamiltonian is obtained when the network becomes stable.

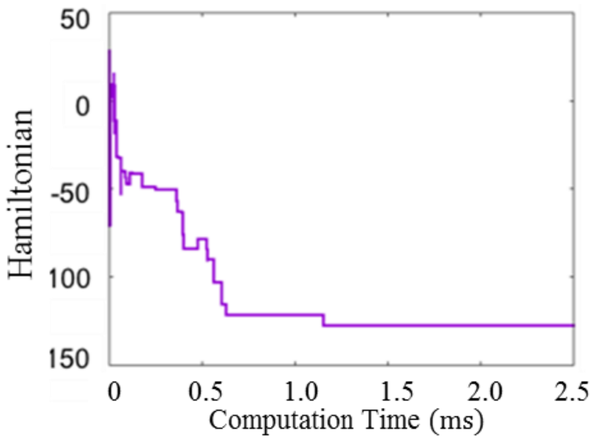

Figure 4 : Time series of Hamiltonian in 10-city ATSP.

6. Conclusion

In this paper, we propose an application method of Coherent Ising Machine (CIM) for the asymmetric traveling salesman problem (ATSP). We formulated the energy function of HTNN using the method of Ref. [7] and derived mutual coupling and external magnetic field necessary for CIM formulation. Finally we performed performance evaluation by computer simulation using model of CIM and showed that CIM can obtain exact solution of ATSP

In future, we aim to solve ATSP by CIM of actual machine. In addition we will apply it to various real problems such as optimization of wireless communication which require high speed because CIM is a method capable of seeking solutions at a very high speed and high performance.

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