2005 International Symposium on Nonlinear
Theory and its Applications (NOLTA2005)
Bruges, Belgium, October 18-21, 2005

A Simple Radial Basis ART Network: Basic Learning Characteristics and Application to Area Measurement

Masahide Ohki† and Torikai Hiroyuki‡, Toshimichi Saito‡

†,‡EECE Dept, Hosei University, Kajino-cho 3-7-2
Koganei-shi, Tokyo, 184-8584 Japan
Email: masahide@nonlinear.k.hosei.ac.jp, torikai@hosei.ac.jp, tsaito@hosei.ac.jp

Abstract—A simple *radial basis adaptive resonance theory network* (RBART) is proposed. Based on unsupervised learning, the RBART can approximate a set of input data by a set of circle-shaped categories. We clarify some generalities of learning characteristics, e.g., number of created categories and their total area are almost independ on distribution of inputs but depend on size of the input data set. We then propose that the RBART can be applied to robust measurement of area of given figures.

1. Introduction

In this paper we present a new *radial basis adaptive resonance theory* network (RBART). Based on a self-organizing unsupervised learning, the RBART can approximate a set of analog input data by a set of circle-shaped categories. We clarify some generalities of learning characteristics of the RBART, e.g., number of created categories and their total area are almost independ on distribution of inputs but depend on size of the input data set. We then propose that the RBART can be applied to robust measurement of area of given figures. Imagine a map including many dots that reflect population distribution, e.g., one dot may correspond to thousand people. The RBART accepts the non-uniformly distributed dots (i.e., population distribution) as inputs, and creates circle-shaped categories that cover the map. We show that area of the map can be measured by using the categories. Note that non-uniformly distributed inputs are not suited to some existing area measurement methods such as Monte Carlo method [1].

A variety of *adaptive resonance theory* (ART) systems have been proposed and their applications have been developed, e.g., data classification, information fusion, image processing, industrial design retrieval, and neural network design [2]-[11]. We note that application of an ART system to area measurement has not been considered sufficiently. The circle-shaped category can realize lower consumption of memory resources than other shapes of categories [2][3]. The circle-shaped category has been already employed in Hypersphere-ART [3], but our RBART has a simpler learning algorithm than other ART systems. Also the RBART has a new parameter that can control learning characteristics efficiently. Preliminary results along these lines can be found in [12].

2. Radial Basis ART Network

2.1. Definitions and Notations

We present a simple *radial basis adaptive resonance theory* network (RBART) defined as the followings. A 2-D input $\boldsymbol{I}$ is presented in a unit square

$$\boldsymbol{I} = \{(x, y) \mid 0 \le x \le 1, 0 \le y \le 1\} \quad (1)$$

In Fig.1(a), an input is indicated by a dot. The RBART has *categories* whose total number is denoted by n. The j-th category is associated with a weight vector

$$\boldsymbol{W}_j = (x_j, y_j, r_j), \quad j \in \{1, 2, \cdots, n\} \quad (2)$$

that represents a circle with center (x_j, y_j) and radius r_j as shown in Fig.1(a). Let d_j be the Euclidean distance between the input $\boldsymbol{I} = (x, y)$ and the category center (x_j, y_j):

$$d_j = \sqrt{(x - x_j)^2 + (y - y_j)^2}. \quad (3)$$

The RBART has a new *choice function* defined by

$$T(d_j, r_j) = d_j - kr_j, \quad -1 \le k \le 1 \quad (4)$$

where k is our original parameter, called *distance parameter*. Fig.1(b)-(d) illustrate the choice function T. The choice function T measures difference between the presented input $\boldsymbol{I}$ and a category $\boldsymbol{W}_j$. Same as other ART systems, the RBART has a *vigilance parameter*

$$0 \le \rho \le 1. \quad (5)$$

2.2. Learning algorithm

Fig.2 shows learning algorithm. As an initial input $\boldsymbol{I} = (x, y)$ is presented, we set $\boldsymbol{W}_1 = (x, y, 0)$ and $n = 1$. The RBART repeats the followings steps until all the inputs are presented.

Step 1. Category choice: Present an input $\boldsymbol{I} = (x, y)$, and calculate the choice function $T(d_j, r_j)$ for each category. A category having the minimum choice function is chosen and is indexed by J:

$$T(d_J, r_J) = \min_j \{T(d_j, r_j)\}. \quad (6)$$

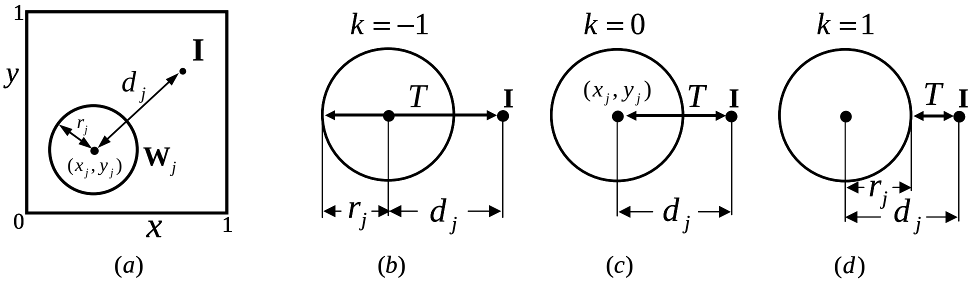


Figure 1: (a) Input $\boldsymbol{I}$ and circle-shaped category $\boldsymbol{W}_j$. (b)-(d) Choice function $T(d_j, r_j) = d_j - kr_j$.

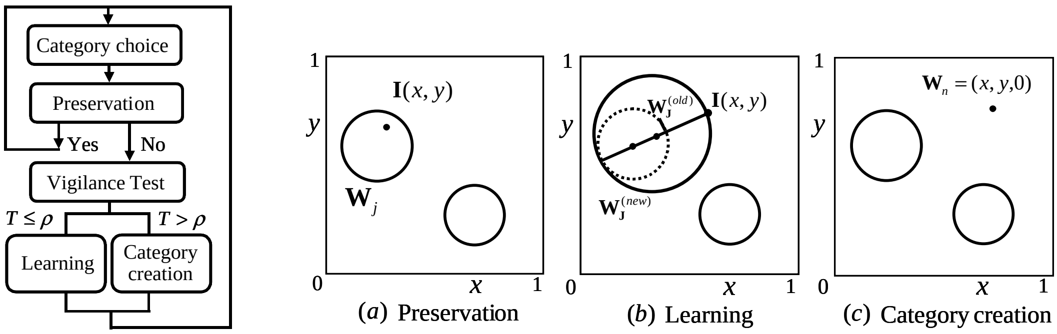


Figure 2: Learning algorithm

Step 2. Preservation: If the input $\boldsymbol{I}$ exists in a category, the category is preserved as shown in Fig.2(a). In this case go to *Step 1*. Otherwise go to *Step 3*.

Step 3. Vigilance Test: If the chosen category satisfies a vigilance criterion

$$T(d_J, r_J) \le \rho, \tag{7}$$

then a resonance occurs and go to *Step 4*. Otherwise go to *Step 5*.

Step 4. Learning: Update the chosen category $\boldsymbol{W}_J$ such that the updated category $\boldsymbol{W}_J^{(new)} = (x_J^{(new)}, y_J^{(new)}, r_J^{(new)})$ includes the original category $\boldsymbol{W}^{(old)} = (x_J^{(old)}, y_J^{(old)}, r_J^{(old)})$ and the input $\boldsymbol{I} = (x, y)$ as shown in Fig.2(b). The update is described by

$$\begin{aligned} x_J^{(new)} &= \tfrac{1}{2}(r_J^{(old)}(\tfrac{x_J^{(old)}-x}{d_J}) + x_J^{(old)} + x), \\ y_J^{(new)} &= \tfrac{1}{2}(r_J^{(old)}(\tfrac{y_J^{(old)}-y}{d_J}) + y_J^{(old)} + y), \\ r_J^{(new)} &= \sqrt{(x_J^{(new)} - x)^2 + (y_J^{(new)} - y)^2}. \end{aligned} \tag{8}$$

Go to *Step 1*.

Step 5. Category creation: Let $n = n + 1$ and create a new category as shown in Fig.2(c):

$$\mathrm{W}_n = (x, y, 0). \tag{9}$$

Go to *Step 1*.

After the learning, the number of created categories is denoted by N.

2.3. Basic Learning Dynamics

Let us consider basic learning dynamics based on the input set in Fig.3(a): 1000 inputs (dots) are distributed randomly and uniformly in the circle with center $(x, y) = (0.5, 0.5)$ and area 0.5. We refer to this input set as *input set (a)*. The input set (a) can be regarded as a simplified version of the circle-in-the-box problem [3][13]. Let the figure in which the inputs are given be referred to as *input figure*. In the case of Fig.3, the input figure is the circle with area 0.5. It can be seen that the number n of categories increases as more inputs are presented. We have confirmed that 1000 inputs are enough for convergence of the number n of categories as shown in Fig.3(b). Hence we fix the number of inputs to 1000 hearafter.

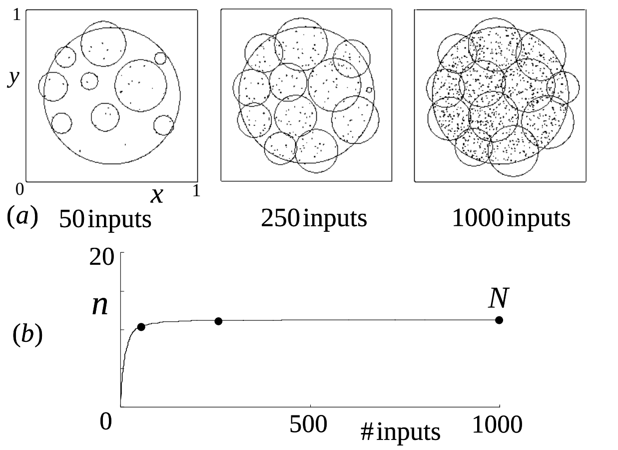
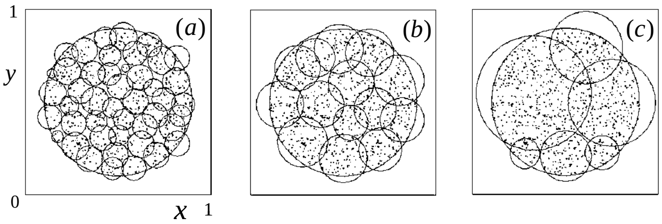


Figure 3: (a) Basic learning dynamics. $a = 0.3$ and $\rho = 0.15$. (b) Convergence characteristics of the number n of created categories. N is the number of categories after the learning (after 1000 inputs are presented).



	(a)	(b)	(c)
Distance Parameter k	−1	0	1
# Categories N	45	14	6
Total Area Rate A	1.30	1.34	1.55

Figure 4: Learning results for the input set (a). The vigilance parameter is $\rho = 0.15$.

3. Robust Learning and Application to Area Measurement

Fig.4 shows some learning results for the input set (a). In order to characterize the results, we consider the following measures.

1) Number N of categories after the learning.

2) Total area rate

$$A = \frac{1}{S_I} \sum_{j=1}^{N} \pi r_j^2 \qquad (10)$$

where $S_I = 0.5$ is the area of the input figure.

3) Co-efficient of variance CV_A of the total area rates.

N and A are given by averages for 100 trials, and CV_A is given by co-efficient of variance for the 100 trials. The measures for Fig.4(a)-(c) are shown in the table of Fig.4. In this paper we fix the vigilance parameter to

$$\rho = \rho_0 = 0.15. \qquad (11)$$

In Fig.5, characteristics of the measures for the distance parameter k are shown.

Next, let us consider learning characteristics for four kinds of input sets (a)-(d) shown in Fig.6(a)-(d). Let distribution of the inputs be referred to as *input distribution*. The input sets (a)-(d) in Fig.6(a)-(d) can be characterized as shown in the table of Fig.6. Fig.5 show learning characteristics for the input sets (a)-(d). We can confirm the following.

Property: The characteristics of the number N of categories and the total area rate A do not very much depend on the input sets (a)-(d). That is, the learning of the RBART is robust against difference of the input figure and the input distribution.

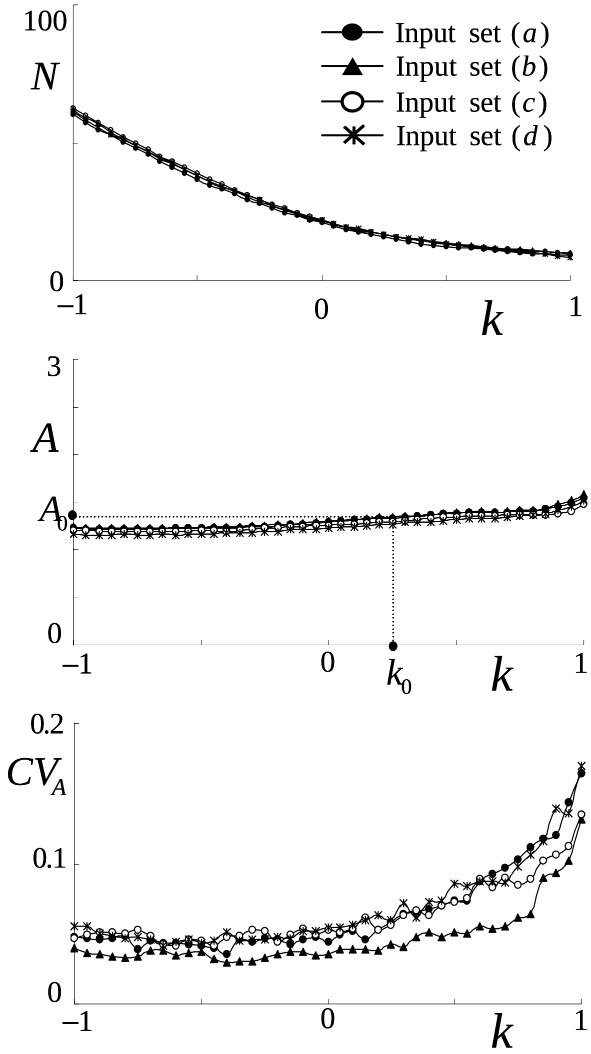


Figure 5: The characteristics of the measures for the distance parameter k.

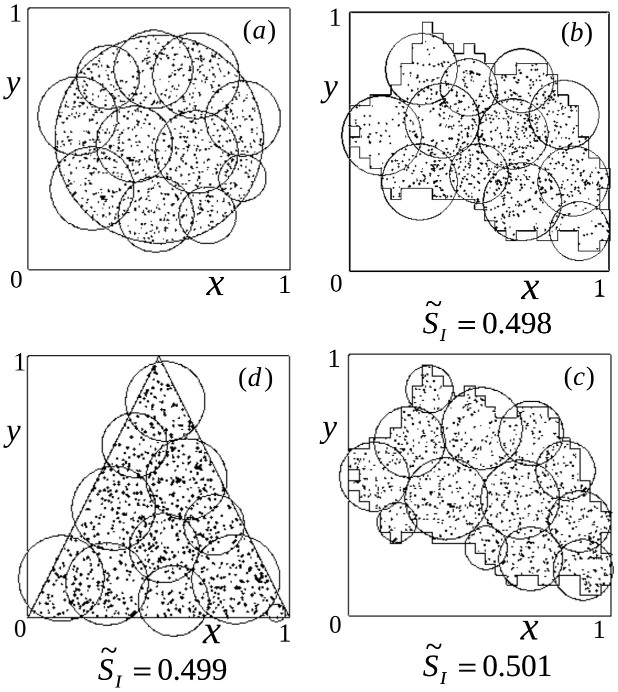
Based on this property, we propose application of the RBART to area measurement. In order to consider this application, we fix the distance parameter to

$$k = k_0 = 0.3. \qquad (12)$$

Let us regard the input set in Fig.6(b) as a given data: the input figure is a map of Saitama Prefecture in Japan and the input distribution corresponds to a virtual population distribution, e.g., one dot may correspond to thousand people. As shown in Fig.6(b), the RBART creates the categories that cover the map. Let us define total area S of the categories:

$$S \equiv \sum_{j=1}^{N} \pi r_j^2. \qquad (13)$$

In the case of Fig.6(b), the total area is $S = 0.70$. As shown in Fig.5, the total area rate A is almost identical for the input sets (a)-(d). Hence we represent the total area rates for the



Input set	Input figure	Area S_I of Input figure	Input distribution
(a)	Circle	0.5	Uniform
(b)	Map of Saitama Pref.	0.5	Gaussian
(c)		0.5	Uniform
(d)	Triangle	0.5	Uniform

Figure 6: (a)-(d) are learning results for the input sets (a)-(d) in the table. $k = 0.3$ and $\rho = 0.15$. (b) is used as an example data for applications to area measurement.

input sets (a)-(d) by the total area rate for input set (a) as the following.

$$A_0 \equiv 1.41 \tag{14}$$

which is the total area rate for the inputset (a) and the parameter case $(k, \rho) = (k_0, \rho_0)$. Then the area S_I of the given map in Fig.6(b) can be approximated by

$$\tilde{S_I} = \frac{S}{A_0}. \tag{15}$$

In the case of Fig.6(b), the approximated area is $\tilde{S_I} = 0.498$, i.e., the true area $S_I = 0.5$ can be measured with small error. Property clarifies that the area measurement by the RBART is robust against non-uniformly distributed inputs. Note that non-uniformly distributed inputs are not suited to some existing area measurements methods such as Monte Carlo method [1]. The area measurement is also possible for uniformly distributed inputs as shown in Fig.6(c) and (d). As shown in Fig.5 the co-efficient of variances CV_A for the 100 trials are low. Hence the area measurement is robust for trials.

4. Conclusions

We have presented the novel RBART which has the following properties. The RBART has the circle-shaped category that consumes lower memory resources than other category shapes. The RBART has the simpler learning algorithm than other ART systems. The learning characteristics is robust against input distribution. Based on this property, we have proposed the applications of the RBART to the area measurement. We have shown the parameter setting methods suitable for the applications. Future problems include: (a) consideration of the applications to real data; (b) development of a hardware RBART; and (c) development of a supervised version of the RBART, like ARTMAP[4].

References

[1] D. E. Knuth, vol.2, 3rd edn., Addison Wesley, 1998.

[2] G.A.Carpenter, S.Grossberg, and D.B.Rosen, Neural Networks, vol.4, pp.759-771, 1991.

[3] G.C. Anagnostopoulos and M. Georgiopoulos, Proc. of IJCNN, Vol.6, pp.59-64, 2000.

[4] O. Parsons and G.A. Carpenter, Neural Networks, Vol.16, pp.1075-1089, 2003.

[5] A.M. Waxman, D.A. Fay, B.J. Rhodes, T.S. McKenna, R.T. Ivey, N.A. Bomberger, V.K. Bykoski and G.A. Carpenter, Proc. Int. Conf. on Information Fusion, pp.1-8, 2002.

[6] T.P. Caudell, S.D.G. Smith, R. Escobedo and M. Anderson, Neural Networks, Vol.7, No.9, pp.1339-1350, 1994.

[7] S. Vasilic and M. Kezunovic, IEEE Tran. Power delivery (in press), 2004.

[8] D. Chralampidis, T. Kasparis and M. Georgiopoulos, IEEE Trans. NN, Vol.12, No.5, pp.1023-1036, 2001.

[9] Shie-Jue Lee and Chun-Liang Hou, IEEE Trans.NN, vol.13, pp.1308-1321, 2002.

[10] G.C. Anagnostopoulos and M. Georgiopoulos, Proc. IJCNN, Vol.2, pp.1221-1226, 2001.

[11] G.C. Anagnostopoulos and M. Georgiopoulos, Neural Networks, Vol.15, pp.1205-1221, 2002.

[12] M. Ohki, H. Torikai and T. Saito, Proc. of ICCNS, p83, 2004.

[13] R.O. Duda and P.E. Hart, Wiley, New York, New York, 1973.