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An Improved Formulation of Feature Values in Passive Reflectionless Transmission-Line Model Based on the Cochlea

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Abstract—A passive reflectionless transmission-line model well reproduces the physiological characteristics of the cochlea by adjusting the parameter values of that model. In the conventional method, it was possible to reproduce only the characteristics of the cochlea at the local position in the whole cochlea with the passive reflectionless transmission-line model, because the characteristics of the overall passive reflectionless transmission-line model could not be formulated. In this paper, we formulate the characteristics of the overall passive reflectionless transmission-line model and reproduce the characteristics of the whole cochlea in that model by the optimization technique.

1. Introduction

Various cochlear models have been proposed, and a transmission-line model is better candidate to reproduce the physiological characteristics of the cochlea. Among them, a passive reflectionless transmission-line model (hereinafter referred to as a passive model) can well reproduce cochlear passive characteristics [1] with small parameters by adjusting the values of model parameter [2].

Methods for qualitatively determining the parameter values of the passive model have been proposed [2, 3, 4]. However, we cannot quantitatively design the passive model through these methods. Therefore, we proposed an improved quantitative design method [5]. With this method, we succeed to reproduce the desired characteristics at some positions in cochlea. However, it is difficult for the improved method to reproduce the overall characteristics of cochlea.

Therefore, in this paper, we formulate the characteristics of the overall passive model for quantitatively determining the parameter values of the passive model that can reproduce desired overall characteristics. We use the Greenwood function [6] as a reference, which gives good approximations for the mammalian peak frequency. Then, we introduce parameter optimization methods based on the downhill simplex. In addition, we introduce two object functions for that optimization. Finally, we give design examples to evaluate validity of the proposed formulae.

2. A passive reflectionless transmission-line model

Figure 1 shows the passive model [2]. In Fig. 1, the bold line shows the passive reflectionless transmission-line and the equivalent circuit shows a minute part at distance x from the input voltage. In the equivalent circuit of Fig. 1, Parallel impedance $Z_p(x,\omega)$ is given by

$$Z_p(x,\omega) = j\omega L_p(x) + R_p(x) + \frac{1}{j\omega C_p(x)}, \tag{1}$$

where ω is the angular frequency of input voltage. The value of each circuit element in $Z_p(x,\omega)$ is changes exponentially with distance x, and these circuit elements can be written as

$$L_p(x) = L_0 e^{ax},\ R_p(x) = R_0 e^{-ax},\ C_p(x) = C_0 e^{ax}, \tag{2}$$

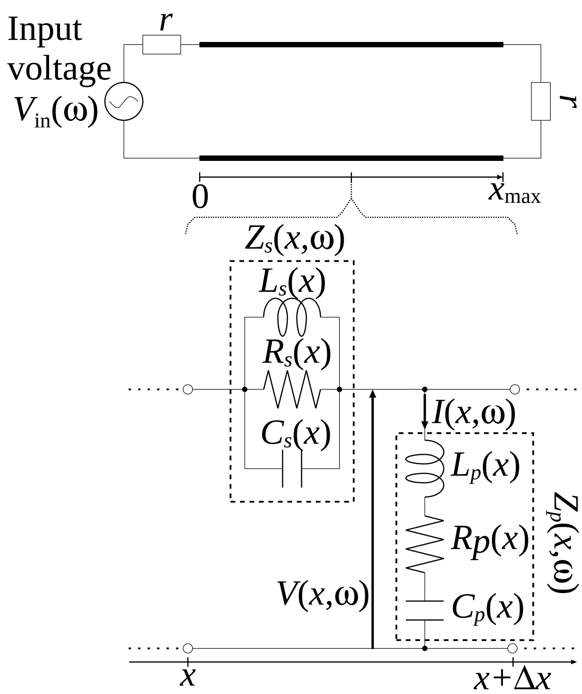

Figure 1: A passive reflectionless transmission-line model (passive model) [2].

where R_0, L_0 and C_0 are the value of circuit elements at distance $x = 0$ mm, and a is a positive constant.

In Fig. 1, the characteristics impedance r is independent of distance x and frequency ω [7], and it is given by

$$r \equiv \sqrt{Z_s(x,\omega)Z_p(x,\omega)}. \tag{3}$$

As a result, the series impedance $Z_s(x,\omega)$ in Fig. 1 can be written as

$$Z_s(x,\omega) = \frac{r^2}{Z_p(x,\omega)}. \tag{4}$$

Also, we define propagation constant $\gamma(x,\omega)$ of the passive model as

$$\gamma(x,\omega) \equiv \sqrt{\frac{Z_s(x,\omega)}{Z_p(x,\omega)}} = \frac{r}{Z_p(x,\omega)}. \tag{5}$$

With Eq. (5), the voltage $V(x,\omega)$ and the shunt current $I(x,\omega)$ at distance x in Fig. 1 are given by

$$V(x,\omega) = V(0,\omega)\exp\int_0^x \gamma(y,\omega)\,\mathrm{d}y, \tag{6}$$

$$I(x,\omega) = \frac{V(x,\omega)}{Z_p(x,\omega)}, \tag{7}$$

where $V(0,\omega)$ is the voltage at distance $x = 0$ mm. Therefore, we define the transfer function at distance x as

$$F(x,\omega) \equiv \frac{I(x,\omega)}{V(0,\omega)} = \frac{1}{Z_p(x,\omega)}\exp\int_0^x \gamma(y,\omega)\,\mathrm{d}y. \tag{8}$$

From Eqs. (1), (2), and (5), the characteristics of transfer function of the passive model is determined by adjusting the parameter values of a, L_0, R_0, C_0, and r.

In addition, the gain $g(x,\omega)$ at distance x can be written as

$$g(x,\omega) = 20\log|F(x,\omega)|. \tag{9}$$

3. Formulation of feature values in the passive reflectionless transmission-line model

Figure 2 shows the gain characteristics with respect to the frequency with parameter values of $a = 0.288$, $L_0 = 2.385\times10^{-7}$, $R_0 = 1.5$, $C_0 = 2.132\times10^{-7}$, and $r = 5$ as in [8]. In Fig. 2, the gain has a peak, i.e., the maximum gain, at a particular distance. The frequency at which the gain peaks for a particular distance is referred to as the "peak frequency $f_{\mathrm{peak}}(x)$." Beyond the peak frequency $f_{\mathrm{peak}}(x)$, the gain rapidly decreases. From the result, the passive model gain $g(x,\omega)$ peaks at a high frequency where distance x is small, and peaks at a low frequency where x is large.

From [9], to formulate the peak frequency characteristics in the passive model, the peak frequency $f_{\mathrm{peak}}(x)$ can be derived from

$$\left.\frac{\partial|F(x,\omega)|}{\partial x}\right|_{\omega=2\pi f_{\mathrm{peak}}(x)} = 0. \tag{10}$$

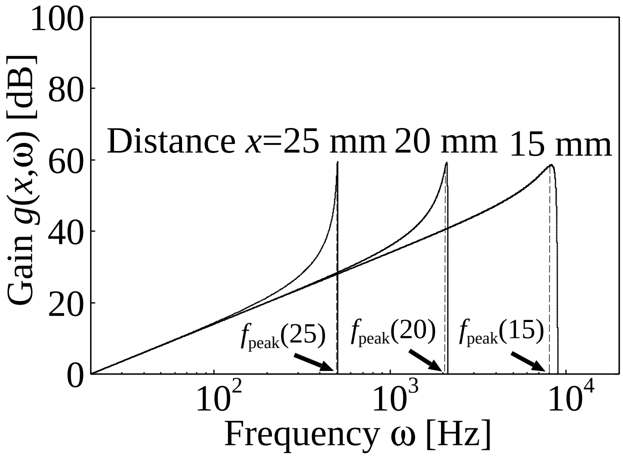

Figure 2: The frequency vs. gain characteristics of the passive model. The dashed line in the figure depict peak frequency $f_{\mathrm{peak}}(x)$.

From Eq. (10), the peak frequency $f_{\mathrm{peak}}(x)$ can be written as

$$f_{\mathrm{peak}}(x) = \mu(x)f_{\mathrm{res}}(x). \tag{11}$$

In Eq. (11), the resonance frequency $f_{\mathrm{res}}(x)$ and a coefficient function $\mu(x)$ are given as

$$f_{\mathrm{res}}(x) = \frac{1}{2\pi\sqrt{L_0C_0}}e^{-ax}, \tag{12}$$

$$\mu(x) = \sqrt{\frac{-\nu(x)+\sqrt{\nu(x)^2+4}}{2}}, \tag{13}$$

where

$$\nu(x) = \frac{rR_0C_0}{aL_0}e^{-ax}. \tag{14}$$

Also, we need a definition of Q value for the gain characteristics as in [10]. Figure 3 shows an example of the gain characteristics with respect to the frequency at distance $x = 23$ mm. In the figure, $g_{\max}(x)$ is the maximum value of the gain $g(x,\omega)$ at the peak frequency $f_{\mathrm{peak}}(x)$. We newly introduce lower frequency $f_{\mathrm{L}}(x)$ and higher frequency $f_{\mathrm{H}}(x)$ to define quality factor $Q_{10}(x)$ at which $g(x,\omega) = g_{\max}(x) - 10$ dB, and $f_{\mathrm{L}}(x) < f_{\mathrm{H}}(x)$ as shown in Fig. 3. Using these frequencies, we define quality factor $Q_{10}(x)$ as

$$Q_{10}(x) \equiv \frac{f_{\mathrm{peak}}(x)}{f_{\mathrm{H}}(x) - f_{\mathrm{L}}(x)}. \tag{15}$$

To calculate $Q_{10}(x)$, we need estimated $f_{\mathrm{L}}(x)$ and $f_{\mathrm{H}}(x)$ by numerical simulations for the frequency vs. gain characteristics like Fig. 3. However, we want to calculate $Q_{10}(x)$ without numerical simulations. Therefore, from the observations on Fig. 3, we assume that $Q_{10}(x)$ is high when $f_{\mathrm{peak}}(x) \approx f_{\mathrm{res}}(x)$, while $Q_{10}(x)$ is low when $f_{\mathrm{peak}}(x) \ll f_{\mathrm{res}}(x)$. In other words, from Eq. (11), $Q_{10}(x)$ is high when $\mu(x) \approx 1$, while $Q_{10}(x)$ is low when $\mu(x) \approx 0$. This implies

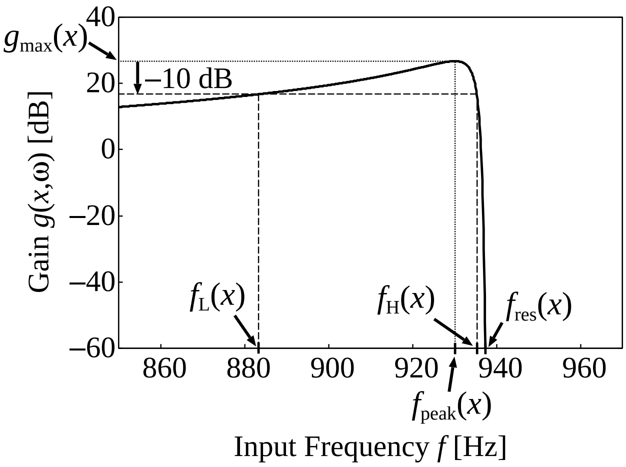

Figure 3: An example of the frequency vs. gain characteristics with definitions of the low frequency $f_L(x)$ and the high frequency $f_H(x)$. In the figure, the dotted line shows a relationship between the maximum gain $g_{max}(x)$ and the peak frequency $f_{peak}(x)$,and the dashed line shows a relationship between the gain $g_{max}(x)$ − 10 dB and the lower frequency $f_L(x)$ or the higher frequency $f_H(x)$.

that the coefficient function $\mu(x)$ has a certain relationship with the quality factor $Q10(x)$. To find out the relationship, we made the coefficient function $\mu(x)$ vs. the quality factor $Q_{10}(x)$ plots as shown in Fig. 4. From the figure, we obtained a regression equation as

$$Q_{10}(x) \cong 10^{\left[\{5(1-\mu(x))\}^{-1/4}-1\right]}. \tag{16}$$

4. Optimization of the parameter values

We propose a method for quantitatively determining the parameter values of the passive model that can reproduce desired overall characteristics. We use the Greenwood function as a reference, which gives good approximations for the mammalian peak frequency. The Greenwood function is given by

$$f_G(x) = A\left\{10^{B(1-x/x_{max})} - K\right\}, \tag{17}$$

where A, B, and K are constants, and x_{max} is the total length of cochlea as shown in Fig. 1. From [6], $A = 165.4$, $B = 2.1$, $K = 0.88$, and $x_{max} = 35$ mm. Also, we use the quality factor $Q_G(x)$ with the Greenwood function as a reference from [11], which is given by

$$Q_G(x) = 2.5 + 0.5 \cdot f_G(x_{max} - x) \times 10^{-3}. \tag{18}$$

To obtain the parameter values that provide a better match between the passive model and two references which are the Greenwood function $f_G(x)$ and the quality factor

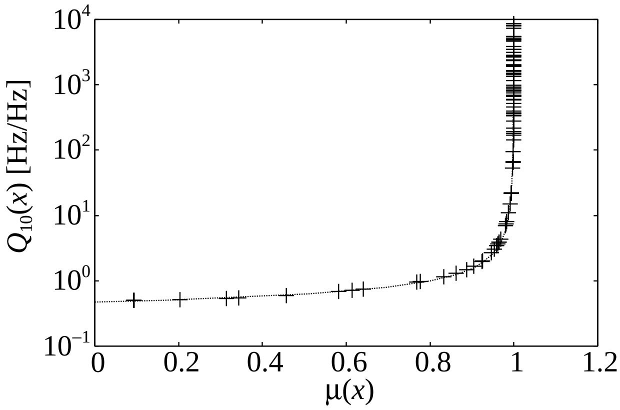

Figure 4: The characteristics of the coefficient function $\mu(x)$ vs. the quality factor $Q_{10}(x)$. The dotted lines show Eq. (16). The cross symbols in the figure depict 100 sets of values of $\mu(x)$ and $Q_{10}(x)$ when the parameter values of the passive model are $R_0 \leq 1$, $C_0 \leq 10^{-7}$, and $r \geq 10$.

$Q_G(x)$, we define the object function E_{sum} as

$$E_{sum} = \sum_{n=0}^{N}\Big(w_1 E_f(n/N \cdot x_{max}) + w_2 E_Q(n/N \cdot x_{max})\Big), \tag{19}$$

where w_1 and w_2 are weights, $E_f(x)$ and $E_Q(x)$ are associated with the peak frequency and the quality factor, respectively. In Eq. (19), $E_f(x)$ and $E_Q(x)$ are defined as

$$E_f(x) = \log\left(\left|\frac{f_{peak}(x) - f_G(x)}{f_G(x)}\right| + 1\right), \tag{20}$$

$$E_Q(x) = \left|\frac{Q_{10}(x) - Q_G(x)}{Q_G(x)}\right|. \tag{21}$$

Next, we employ the downhill simplex (DS) method to determine the parameter values. In the following simulations, the total number of trials are 1,000 with 10,000 iterations in each trial. In addition, the number of simplex in DS is equal to 30.

Figure 5 and 6, respectively, show the peak frequency characteristics and the quality factor characteristics with respect to distance x, which are obtained from the optimized parameter values through DS. The results in Fig. 5 confirm that the peak frequency obtained with optimized parameter values match with the Greenwood function $f_G(x)$. In contrast, in Fig. 6, the quality factor $Q_{10}(x)$ qualitatively matches the curve of the $Q_G(x)$ characteristics.

5. Conclusions

We formulate the peak frequency and Q value characteristics of the overall passive reflectionless transmission-line model and reproduce these characteristics of the whole cochlea in that model by the optimization technique. We

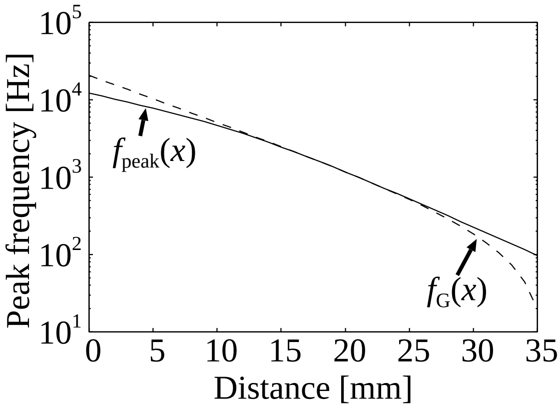

Figure 5: Peak frequency resulting from DHS using the object function E_{sum} (solid line), and that of the Greenwood function $f_{\mathrm{G}}(x)$ (dashed line) with respect to distance x.

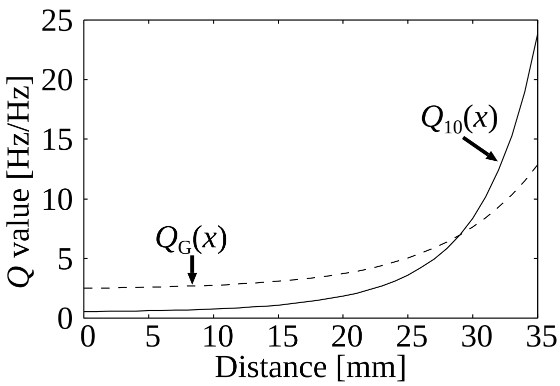

Figure 6: Q value resulting from DHS using the object function E_{sum} (solid line), and that of the Q value function $Q_{\mathrm{G}}(x)$ (dashed line) with respect to distance x.

determine the parameter values of the passive reflectionless transmission-line model based on parameter optimizations. We used the downhill simplex method for optimization technique. For this optimization, we demonstrated the effectiveness of the proposed method through design examples. From the simulations, we found that the downhill simplex method is preferable to match peak frequency characteristics. Also, the quality factor $Q_{10}(x)$ obtained with optimized parameter values qualitatively match the curve of the $Q_{\mathrm{G}}(x)$ characteristics.

As future work, we will improve the peak frequency characteristics at low and high frequencies as well as the Q value characteristics by improvements in the object function. We will use the best effective of optimization method by comparing with other methods.

Acknowledgments

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