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A bio-inspired pattern dynamics of power flow in consensus networks

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Abstract—Management of distributed power storages in networked systems could be useful technology for robust and flexible power supply to distributed loads. Such a mechanism are inherent in biological systems. Specifically, functions such as adaptation of power supply patterns according to frequency of exchanging and concentrating power among network nodes are applicable to engineering systems. In this study, we consider a model of power flow in network systems by using consensus dynamics which is compatible with charge dynamics in electronic circuits. We discuss how to implement the adaptation function and power exchange in the power packet dispatching system.

1. Introduction

Distributed power sources and their autonomous energy management in a networked system are expected to be robust and flexible in multiple layers of power networks ranging from isolated system to large scale power grids. One of promising managing techniques for those distributed power sources is the power packet dispatching system in which a discretized power is transmitted in a power network with the information of the destination node [1]. Recently, power packets dispatching systems has been implemented by electric circuits and discussed in view of its possible applications in isolated systems such as robots [2]. A simple mathematical model for exchanging power packets in a switching network has been discussed by using the consensus dynamics [3]. Along the research line of implementation of the power packets into open isolated systems, bio-inspired approaches can be useful, since biological systems are open networked systems in terms of power flow in the system and distributed power storages. In addition, biological functions such as quickness of power supply to frequently used loads with a limited amount of power, and sustainability of power storages are effective in the perspective of engineering systems.

In this study, we discuss a bio-inspired energy management systems for the power packets by using the consensus network model. Specifically, we focus on pattern dynamics of power supply route in a network as well as power consumption at network nodes. Regarding power supply to a specific node, enforcing connections used frequently for the node is efficient for quick power supply to the node. Also, we consider shared power supply among few nodes to extend the battery life of the distributed power storages.

2. Model

Power packet dispatching systems has a mixer for generating power packets and a router for forwarding the generated packets to a destination load in a power network. In this study, we consider two types of network. One is real electric circuit network and the other is a virtual network for simulating the real network. The real network corresponds to the electric circuits for power packets so that nodes of the real network represent mixers and routers. As a network topology of the real network, we assume a tree of a directed graph, which implies that there is no loop in the network. The root and the nodes of the tree correspond to the mixer and the routers, respectively. The leaf corresponds to a load or the router attached to a load.

On the other hand, the virtual network has a structure of a star graph. The center node of the star corresponds to the mixer, and the other nodes corresponds to the routers or loads. The edges of the star correspond to routing paths in the real network from the mixer to the router or loads. If the real network does not include loops, then the number of edges in the star is equal to the number of nodes minus one in the real network.

We consider edges not only in the star but also additional edges between two leaf nodes except the center node. The connection between two leaf nodes of the star implies that electric charges can be transmitted between the two corresponding routers in the real network. It is important how to design those connection among leaf nodes for efficient power transmission. That is to say, if one node has less power, then other nodes gather power to the less power node in an efficient way regardless of the power line from the power souce. It should be noted that there is a power storage at each node for buffering electric charge to send power to some other nodes which have less charges.

2.1. Power dynamics between nodes

In order to represent the power dynamics in the virtual network, we consider the consensus dynamics between one node and its neighboring nodes [4]. We consider the routers i is connected to the router j by the resistor (resistance $R_{j,i}$) and the router i has a capacitor (capacitance C_i). The dynamics of the voltage of the router i, denoted by V_i is as follows.

$$\dot{V}_i = \sum_{j \in N_i} \frac{1}{C_i R_{j,i}} (V_j - V_i), \tag{1}$$

where N_i is the set of neighboring nodes to i. If the capacitance C_i of all nodes are the same as C. The dynamics of charge at node i, denoted by q_i can be written as:

$$\dot{q}_i = \sum_{j \in N_i} \frac{1}{CR_{j,i}} (q_j - q_i). \tag{2}$$

If the $CR_{j,i}$ takes the value 1 or ∞, the dynamics of (2) is equivalent to the consensus dynamics [5]. According to the eq. (2), the dynamics of node i depends only on its neighboring nodes, which means that the network dynamics is locally and autonomously determined. There is no central controller in the network.

As for modeling the transmission of the power packets, we additionally introduce switching topology to the consensus network system governed by (2). The power packet is generated by the switch in electric circuits and only one packet can be sent simultaneously in a single power line. Therefore, we consider switching connection in the virtual network where one node can only connect to at most one node at one moment. Here we assume that all the nodes in the virtual network are able to connect to any other nodes, namely the network is complete graph. This assumption is natural because physically all the nodes can be connected in the real network.

Moreover, we introduce a node connection rule by taking into account that each node demands some fixed amount of charge. The connection rule is as follows.

1. Each node has its own target amount of charge.
2. If a node i has less amount of charge than its target amount, set $\Delta_i < 0$.
3. If a node i has more amount of charge than its target amount, set $\Delta_i > 0$.
4. One node with $\Delta < 0$ can connect with other node with $\Delta > 0$, vice versa.
5. The number of connection of one node at most one.

Once connection is built between nodes, those nodes transmit electric charge by the consensus dynamics.

2.2. Muscle-like storage model

In this study, we consider bio-inspired adaptation in the structure of network connection together with aging of storage at nodes. First, we consider muscle-like power storage model. In general, skeletal muscles distributed in the body consume glucose as a power source and make a movement. Moreover, muscle is able to store glucose in itself. Due to this biological mechanism, the muscle has two similar point to the power packet system. One point is that storing glucose in a muscle is similar to a buffer of charges in the routers. The other point is that muscle movement is similar to a putting load on the router. In addition, glucose is like a power packet. Thus, it is possible to interpret network of skeletal muscles as a power packet network system where each node has a power storage.

Furthermore, skeletal muscles can adapt to their amount of use, namely if one muscle is used frequently, then the muscle can gather a large amount of power. Following this adaptive feature, we consider a rule for distributing power packets depending on the frequency of using each network connection. By this adaptive rule, we intend to achieve two functions. One is quick supply of power to a target node through frequently used network connection. The other is that power supply is shared among a few nodes in order to extend the power storages' life of the network. To this end, we assume adaptiveness to connection of power network so that frequently used connections are enhanced and other connections are removed.

2.3. Degree of adaptation: Restoring time

In order to investigate the adaptiveness of the bio-inspired network quantitatively, we introduce a degree of adaptation. To measure the degree, we do numerical simulations of the virtual networks by the following four steps.

1. *Initial power distribution* : First, all the power is at the center node and is distributed to the other nodes by randomly switching consensus protocol. The power is distributed homogeneously.
2. *Power supply to load* : One node lose its power due to a load, called load node. Then, other nodes supply power to the load node by random switching consensus protocol. During the random switching in the current step and the previous step, frequently used connections are recorded. Other connections are cut off. This is an adaptation step.
3. *Power storage enhancement* : By the recorded connections, distributed power is re-distributed to one of two end nodes of each recorded connection. The power is distributed heterogeneously. There is no power loss at the load node.
4. *Power relaxation* : Re-distributed power is again homogeneously distributed by random switching consensus protocol with recorded connections. There is no power loss at the load node.

In this study, we assume that power source is attached only to the center node of the star, i.e. the mixer, and only one load is attached to one fixed node where power is lost. By this assumption, the considering network is an open isolated system. In the power relaxation step, we measure the

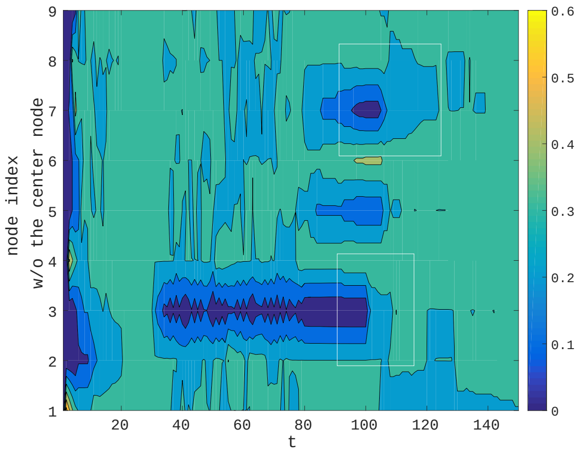

Figure 1: Spatio-temporal pattern of charge dynamics for the four steps. The color bar indicates amount of charges.

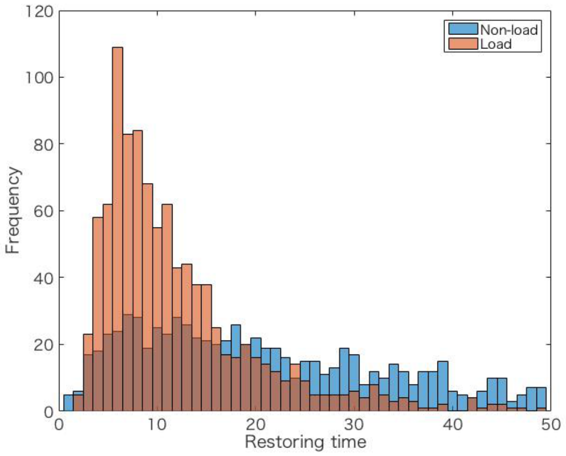

Figure 2: Histogram of the restoring time with regards to the load node (Orange) and a non-load node (Blue). The number of trials is 1000.

relaxation time at each node from the heterogeneous network state to the homogenous network state. We call the time for relaxation at each node as the restoring time. It should be noted that the restoring times at different nodes are different each other owing to the adapted connections. We determine the restoring time at each node by the time when the state value of node go beyond some threshold.

2.4. Aging coefficient

In the viewpoint of physical limitation of usage time of buttery, we define a longevity of the power storage at a node as follows. With a natural assumption, a power storage is deteriorated or aged as the time of consensus processes increases, namely the node at both ends of connection in the switching network at one moment ages by some amount. Accordingly, all the network ages depending on the number of connections at one moment. As a criterion of network aging, we define aging coefficient by the speed of aging. The state of aging of a network at time t is denoted by $g(t)$. $g(t)$ is recursively described as

$$g(t) = g(t-1) + \sum_i k(i,t), \tag{3}$$

where $k(i,t)$ is the degree at node i at time t. Then, we define the aging coefficient A as

$$A = (g(t_0 + T) - g(t_0))/NT, \tag{4}$$

where T is observing time and N is the number of edges after the adaptation.

3. Numerical Simulation

Figure 1 shows spatio-temporal patterns of charge dynamics of a 10-node virtual network, where interaction of charges among nodes is governed by switching consensus dynamics (2). The period of initial power distribution is shown in $0 < t \leq 30$. Then, the adaptation step is in $30 < t \leq 80$. The node where the amount of charge is very small is the load node. In the period of $80 < t \leq 100$, power storage enhancement is observed at some nodes. Finally, charges are relaxed in $100 < t$. We focus on the two node for checking the restoring time at the period of the charge relaxation. One is the load node like the node #3 in Fig. 1, and the other is the node where amount of charge is very small in the power enhancement period like the node #7.

Figure 2 shows the histogram of the restoring time with regards to the load node and the non-load node. We did the numerical simulation for 1000 trials. As can be seen, the restoring time of the load node is shorter than that of the non-load node. This result implies that the recorded connections obtained in the adaptation period facilitate quick power supply to the load node. In other words, there are more connections to the load node than other nodes, which is confirmed numerically.

Figure 3 shows histogram of the aging coefficient for 1000 trials of numerical simulations. We compare two cases. The first case is that power supply is centralized to the load node. The second case is that there is one non-load node which shares power supply with the load node. The average aging coefficient in the first case shown by the blue bars is larger than that in the second case shown by the orange bars. This result implies that the shared power supply is better than the centralized one in terms of longevity of buffers. However, the restoring time in the shared case is slightly longer than the centralized case. This is a natural trade-off due to sharing power supply. Moreover, the number of adapted connections in the shared case is larger than that in the centralized case.

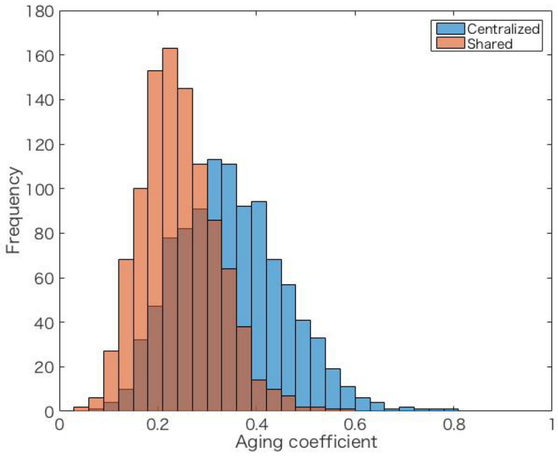

Figure 3: Histogram of the aging coefficient in the case of shared power supplying to the load node (Orange) and centralized power supplying (Blue). The number of trials is 1000.

4. Summary

In this study, we have numerically investigate the power packet dispatching system that is modeled by the consensus dynamics in the network with switching topology. In addition, we have introduced a bio-inspired adaptive mechanism in the rule of selecting and removing network connections. As a result, the following two characteristics have been found. i) Frequently used power supply route to a load in the consensus network is adaptively remained and the adapted network can gather power to the load node more quickly than other nodes. ii) Sharing power supply with the load node makes the battery life of the whole network long.

As for the future work, general theory for the relation between the network adaptation and the speed of power supply and battery aging is required. Moreover, it should be investigated how the adaptive mechanism is applied in a large size network which incorporates time delay.

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