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Relations between Early-warning Signals Based on Dynamical Network Markers and Network Structures

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Abstract—Early-warning signals for critical transitions have been investigated in several types of complex systems. For complex systems with network structures, an indicator based on dynamical network markers (DNM) was proposed and used to detect the early-warning signals of biological networks and foreign exchange markets. Since the DNM-based indicator can be applied to general complex systems with networks, it is important to understand how the indicator works from the viewpoint of network science. In the present paper, in order to clarify the features of the DNM-based indicator with respect to network structures, we detect the early-warning signals of a mathematical model with a network exhibiting a bifurcation, using the DNM-based indicator. The network is generated by a small-world network model, a scale-free network model, or a spatial network model. Through numerical experiments, we demonstrate that the DNM-based indicator can detect early-warning signals in all of the network structures investigated herein.

1. Introduction

Early-warning signals (EWSs) [1, 2] are important in predicting a critical transition from a desirable system state to an undesirable system state, e.g., a transition from a healthy state to a diseased state [1, 3] or a vegetative transition from a savanna to a treeless grassland [4]. From a mathematical point of view, an EWS indicates that the state of a system is close to a tipping point, i.e., a bifurcation point. Before and after a tipping point, the state of a system changes drastically through a critical transition.

Early-warning signals for critical transitions have been investigated in ecological networks, financial markets, and other complex systems [5]. There are many indicators for detecting EWSs, e.g., a recovery rate, autocorrelation at lag 1, and variance [2]. Chen et al. [3] found out that a node group in a network, called a dominant group or dynamical network biomarkers (DNBs), represents the dynamics of the network, and, therefore, comprise an EWS indicator. The DNB-based indicator consists of three statistics: the average of the standard deviations of the DNBs, SD_D, the average absolute value of the Pearson's correlation coefficients of the DNBs, PCC_D, and the average absolute value of the Pearson's correlation coefficients between the DNBs and the other nodes, PCC_O.

The DNB-based indicator was applied to the detection of the critical transitions in biological complex networks [3] and foreign exchange markets [6]. Since the concept of the DNB can be applied not only to biological systems but also to other complex systems in general, DNBs are also known as dynamical network markers (DNMs) [6]. Nakagawa et al. modified the original DNM-based indicator, as proposed in [3], to be applicable to complex networks under the conditions such that PCC_O takes a relatively large value due to noise or increases as a system approaches a critical transition [7]. They confirmed that the modified DNM-based indicator can detect the EWS of a mathematical model with a small-world network exhibiting a bifurcation. However, the applicability of the modified DNM-based indicator to mathematical models with other complex networks, e.g., scale-free networks and geometrical networks, has not yet been investigated. This is important, however, in order to evaluate the effectiveness of the modified DNM-based indicator.

In the present paper, we demonstrate that the modified DNM-based indicator can detect an EWS in three types of complex networks exhibiting a bifurcation. For this purpose, we use the May model [8] with complex networks as in [7]. As for the complex networks, we prepare a small-world network, a scale-free network, and a spatial network, which are generated by the Watts-Strogatz network model [9], the Barabási-Albert network model [10], and the geographical threshold graph model [11], respectively. Through numerical experiments, we demonstrate that the modified DNM-based indicator can detect an EWS in all of the network structures considered herein.

2. Methods

2.1. Early-warning signals based on dynamical network markers

A DNM-based indicator was first proposed to detect an EWS in biological complex networks [3]. The DNM is a dominant node group in a network that has the following features as the system approaches its tipping point: (i) The average of the standard deviations of each nodal state in the DNM, SD_D, drastically increases. (ii) The average absolute value of Pearson's correlation coefficients for each pair of nodal states in the DNM, PCC_D, increases. (iii) The average absolute value of Pearson's correlation coefficients between the nodal states of the DNM and the other nodes, PCC_O, decreases.

However, as for the final condition, PCC_O may take a relatively large value compared with SD_D and PCC_D depending on the dynamical system or noise effects. In such a case, the original DNM-based indicator [3],

$$\tilde{I}_c = \frac{SD_D\, PCC_D}{PCC_O}, \tag{1}$$

does not work well because $\tilde{I}_c$ does not take a large value before critical transitions when PCC_O becomes relatively large. In order to detect the EWS of such a system, Nakagawa et al. [7] proposed a modified DNM-based indicator, as follows:

$$I_c = SD_D\, PCC_D. \tag{2}$$

2.2. May model with a network

In order to describe a dynamical system, we used the May model [8], which expresses the dynamics of a continuous population of a single species as follows:

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = r_i x_i \left(1 - \frac{x_i}{K}\right) - \frac{c x_i^2}{x_i^2 + h^2}, \tag{3}$$

where x_i is the ith population (the state variable) of N populations, $r_i = r_0 - H\delta_i$ is the birthrate of the ith population with positive constant parameters r_0 and H and random numbers δ_i between 0 and 1, K is the carrying capacity, and c and h take constant positive values. Equation (3) assumes the logistic growth model $r_i x_i (1 - x_i/K)$ as the birth term and the Holling's type III consumption function $c x_i^2 / \left(x_i^2 + h^2\right)$ as the death term.

In order to introduce the interactions between populations x_i and x_j to Eq. (3), a migration term is added [7]:

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = r_i x_i \left(1 - \frac{x_i}{K}\right) - \frac{c x_i^2}{x_i^2 + h^2} + D \sum_{j=1}^{N} a_{ij} \left(x_j - x_i\right), \tag{4}$$

where D is the positive constant intensity of interactions between populations, and $A = \{a_{ij}\}$ is an adjacency matrix, i.e., when the ith and jth populations interact, then $a_{ij} = 1$; otherwise $a_{ij} = 0$.

2.3. Network models

For the expression of the interactions A, we prepared three network structures generated by the Watts-Strogatz network model [9], the Barabási-Albert network model [10], and the geographical threshold graph model [11], which realize small-world networks, scale-free networks, and spatial networks, respectively. These network structures are controlled by parameters p_{WS}, p_{BA} and p_{GEO}, respectively.

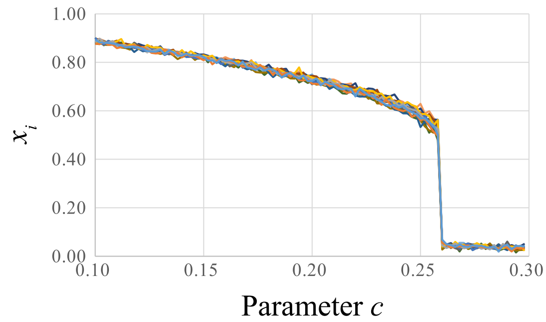


Figure 1: Bifurcation diagram of Eq. (4) for the Watts-Strogatz network ($N = 200$ and $p_{WS} = 0.1$). The solid lines indicate the results for randomly selected x_i. The horizontal axis indicates the bifurcation parameter c. Equation (4) has a bifurcation at approximately $c = 0.26$.

3. Numerical experiments

3.1. Numerical model setting

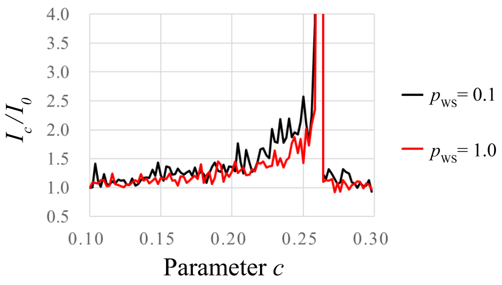
We simulated a dynamical system based on Eq. (4). The number of state variables N was fixed at 200, and other parameters were given as $r_0 = 1.0$, $H = 0.1$, $K = 1.0$, and $D = 0.4$. Then, c was selected as the bifurcation parameter of this system. Equation (4) was converted to a stochastic differential equation. The converted equation was solved using the Euler-Maruyama method [12]. For the generation of the network structures, we prepared the network models as follows: $p_{WS} = \{0.1, 1.0\}$, $p_{BA} = \{1, 100\}$, and $p_{GEO} = \{0.0, 50.0\}$.

For each network, we chose m nodes as the DNM that can strongly contribute to the increase of I_c. In our problem setting, since we used the mathematical model of Eq. (4), we calculated the eigenvalues and the eigenvectors of the system to identify the contributing nodes in the model. The top m components in the eigenvector of the maximum eigenvalue were selected as the DNM of the system. This process corresponds to the screening of nodes from experimental data. We chose $m = 20$ components, which corresponds to 10% of the total number of nodes N.

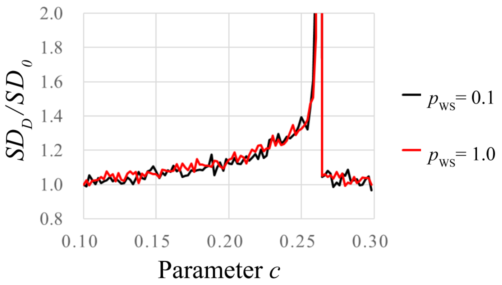
Figure 1 shows an example of the bifurcation diagram of Eq. (4) for the Watts-Strogatz network model with $p_{WS} = 0.1$. Equation (4) with the parameter values we selected has a bifurcation point at approximately $c = 0.26$.

3.2. Results

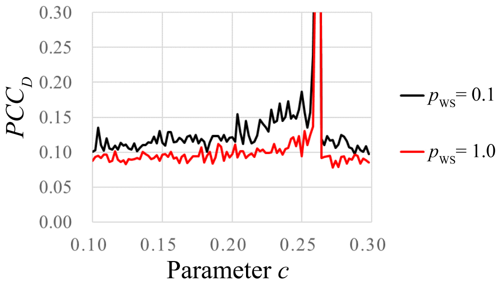
Figures 2-4 show the DNM-based indicator I_c and its components, SD_D and PCC_D for the models of the Watts-



(a) I_c/I_0

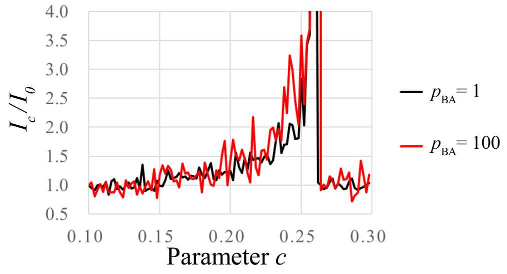


(b) SD_D/SD_0

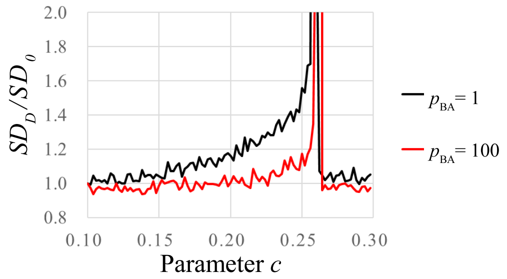


(c) PCC_D

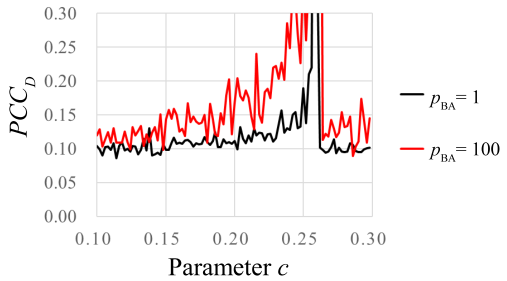
Figure 2: Numerical results of Eq. (4) for the Watts-Strogatz network. (a) I_c/I_0, where I_0 is the value of I_c at $c = 0.1$. (b) SD_D/SD_0, where SD_0 is the value of SD_D at $c = 0.1$. (c) PCC_D. The horizontal axis indicates the bifurcation parameter c.



(a) I_c/I_0



(b) SD_D/SD_0



(c) PCC_D

Figure 3: Numerical results of Eq. (4) for the Barabási-Albert network. (a) I_c/I_0. (b) SD_D/SD_0. (c) PCC_D.

Strogatz network, the Barabási-Albert network, and the geographical threshold graph, respectively.

- Watts-Strogatz network

  Figure 2(b) shows that SD_D increased before the bifurcation point, regardless of p_{WS}. From Fig. 2(c), PCC_D with $p_{WS} = 0.1$ is larger than that with $p_{WS} = 1.0$. The diffrence of PCC_Ds with $p_{WS} = 0.1$ and 1.0 near the bifurcation point was relatively large compared with that far from the bifurcation point. This difference leads to the difference between I_c with $p_{WS} = 0.1$ and 1.0 in Fig. 2(a).

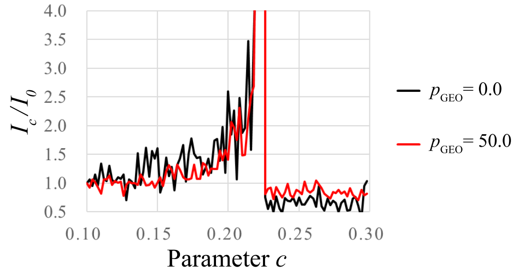
- Barabási-Albert network

  When $p_{BA} = 1$ in Figs. 3(b) and 3(c), SD_D gradually increased as c approached the bifurcation point, whereas PCC_D rapidly increased near the bifurcation point. When $p_{BA} = 100$, SD_D took large values only near the bifurcation point whereas PCC_D gradually increased with c. Since SD_D and PCC_D exhibited a complementary relationship, I_c with both $p_{BA} = 1$ and 100 increased before the bifurcation point.

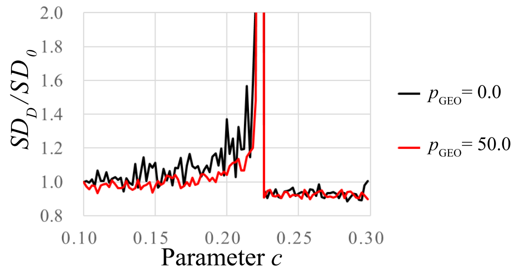
- Geographical threshold network

  SD_D and PCC_D gradually increased toward the bifurcation point, regardless of p_{GEO} in Figs. 4(b) and 4(c). There were large differences between the PCC_D values for $p_{GEO} = 0.0$ and $p_{GEO} = 50.0$. However, since the trends of PCC_D for different p_{GEO} along c were similar, I_c/I_0 became similar in both cases.

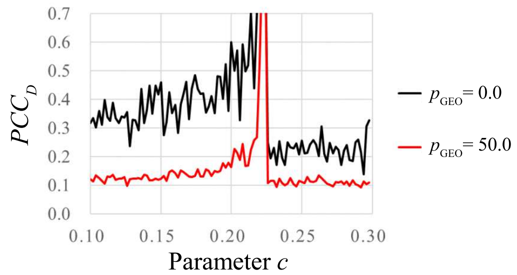
In all of the numerical results shown in Figs. 2(a), 3(a), and 4(a), I_c increased when the system state approached the bifurcation point. This suggests that I_c is a robust indicator of the EWS with respect to the network structures we investigated.



(a) I_c/I_0



(b) SD_D/SD_0



(c) PCC_D

Figure 4: Numerical results of Eq. (4) for the geographical threshold network. (a) I_c/I_0. (b) SD_D/SD_0. (c) PCC_D.

4. Conclusions

The present paper dealt with the DNM-based indicator I_c for detecting an EWS of the mathematical model with three types of complex networks exhibiting a bifurcation. In order to clarify the relations between the DNM-based indicator and the network structures, we prepared the May model with networks generated by the Watts-Strogatz network model, the Barabási-Albert network model, and the geographical threshold graph model. Through numerical experiments, I_c could detect an EWS for all of the network models. Then, the standard deviations, SD_D, and/or the Pearson's correlation coefficients of DNM, PCC_D took large values before the bifurcation point. Since I_c is the product of SD_D and PCC_D, I_c increased regardless of the network model.

An issue of interest is whether I_c can detect the EWS of other complex networks. It may be helpful to clarify the relations between the DNM-based indicator and network measures [13] that characterize network structures, e.g., the average path length, the average clustering coefficient, and the modularity.

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