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Nonlinear Dynamics of Memristive Networks and Its Application to Reservoir Computing

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Abstract—Reservoir computing is one of the potent computational frameworks suitable for sequential data processing. Not only recurrent neural networks but also other physical systems and devices are available to construct a reservoir computing system. In this study, we focus on memristive networks consisting of coupled memristors for achieving physical reservoir computing. First, we present a mathematical model of memristive network circuits with any architecture and investigate its nonlinear dynamics. The dynamical response to input sequential data is also examined. Next, we deal with the problem of how to design memristive networks for better computational performance in a reservoir computing framework. Finally, we make a discussion toward device implementation of our system.

1. Introduction

Reservoir computing is a unified computational framework deriving from the two independently proposed models [1]: the echo state network [2] and the liquid state machine [3]. A reservoir computing system consists of a reservoir part, which transforms input sequential patterns into higher-dimensional spatiotemporal patterns, and the readout part, which is used to extract useful information from the spatiotemporal patterns. Although the reservoir part consists of recurrent neural networks in the original models, it is possible to realize the role of the reservoir using other physical systems with nonlinearity. The exploration of physical reservoirs is significant for effective hardware implementation of reservoir computing. For instance, physical reservoirs have been achieved with various systems including optoelectrical systems [4, 5], water in buckets [6], atomic switch networks [7, 8], soft robotics [9], coupled phase oscillators [10].

In this study, we deal with memristive reservoirs consisting of coupled memristors. One of the motivation to use memristors for reservoir computing is its history-dependent response to input voltage. This property is favorable for a reservoir because a reservoir should generate spatiotemporal patterns that depend on the temporal correlations of inputs for processing sequential data. A network of memristors is also expected to generate history-dependent high-dimensional spatiotemporal patterns in response to sequential voltage inputs. On the other hand, the memristors can be implemented with nano/micro-scale devices, and therefore, the integration of large-scale memristor networks is possible in an energy efficient fashion. So far some studies have shown the potential of reservoir computing based on memristor networks [11, 12, 13, 14, 15], but the nonlinear dynamics of these systems has not yet been fully investigated. To design the memristive reservoir for efficient computing, it is necessary to understand their nonlinear dynamics and control system parameters appropriately. For this purpose, we mathematically formulate the circuit equations of the memristive reservoir and analyze its nonlinear behavior. We also demonstrate that the memristive reservoir is useful for solving a classification problem.

2. Methods

The memristive reservoir computing system considered in this study is illustrated in Fig. 1. When the input sequential signals are given to the voltage sources in the memristive network, the voltages at the vertices and the currents on the edges in the network evolve with time. These spatiotemporal patterns are used to estimate the information on the input signals in the readout part.

Based on the new modified nodal analysis [16], we formulate the circuit equations for describing the dynamics of the memristive network. The system equations of the memristive network are described by the following differential-algebraic equations (DAEs) [17]:

$$E_m W(E_m^\top \mathbf{\Phi}_n) E_m^\top \frac{d\mathbf{\Phi}_n}{dt} + E_s \mathbf{I}_s = \mathbf{0}. \quad (1)$$

$$\frac{d\mathbf{\Phi}_n(t)}{dt} - \mathbf{v}_n(t) = \mathbf{0}, \quad (2)$$

$$E_s^\top \mathbf{v}_n(t) - \mathbf{V}_s(t) = \mathbf{0}, \quad (3)$$

where Eq. (1) represents the Kirchhoff's law, Eq. (2) corresponds to the Faraday's law, and Eq (3) indicates the con-

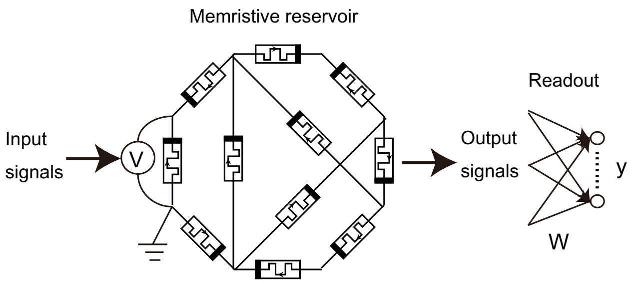

Figure 1: Schematic illustration of memristive reservoir computing in the case of ring topology.

Table 1: Variables and parameters in the memristive network model.

Variables	Meaning
N	Number of vertices
M	Number of memristive elements
S	Number of voltage sources
E_m	$N \times M$ incidence matrix for the connectivity of memristors
E_s	$N \times S$ incidence matrix for the connectivity of voltage sources
$\mathbf{v}_n = (v_1, \ldots, v_N)^\top$	Vector of node voltages
$\mathbf{\Phi}_n = (\Phi_1, \ldots, \Phi_N)^\top$	Vector of node fluxes
$\mathbf{V}_s = (V_1, \ldots, V_S)^\top$	Vector of source voltages
$\mathbf{I}_s = (I_1, \ldots, I_S)^\top$	Vectors of edge currents

straint that a source voltage is the same as the voltage difference between its end nodes. The major variables and parameters are described in Table 1.

We denote the conductance of the individual memristors (called memductance) by W. For the linear drift model of the memristor [18] which we employ in this study, the memductance is described as follows [19]:

$$W(\Phi) = \frac{dq(\Phi)}{d\Phi} = \frac{1}{\sqrt{M(w(0))^2 - 2a\Phi}}, \quad (4)$$

where Φ denotes the magnetic flux and q denotes the charge. The constant a is determined by the property of the device as follows:

$$a = \frac{\mu_v R_{\rm on}(R_{\rm off} - R_{\rm on})}{D^2}, \quad (5)$$

where μ_v represents the average ion mobility, $R_{\rm on}$ represents the large resistance in the undoped region, $R_{\rm off}$ represents the small resistance in the doped region, and D represents the length of the memristive device.

The DAEs in Eqs. (1)-(3) are numerically integrated using the solver ode15i operating on the software package Matlab 2016 [20].

We employ the temporal sequence of the electric currents on the network edges as the high-dimensional spa-

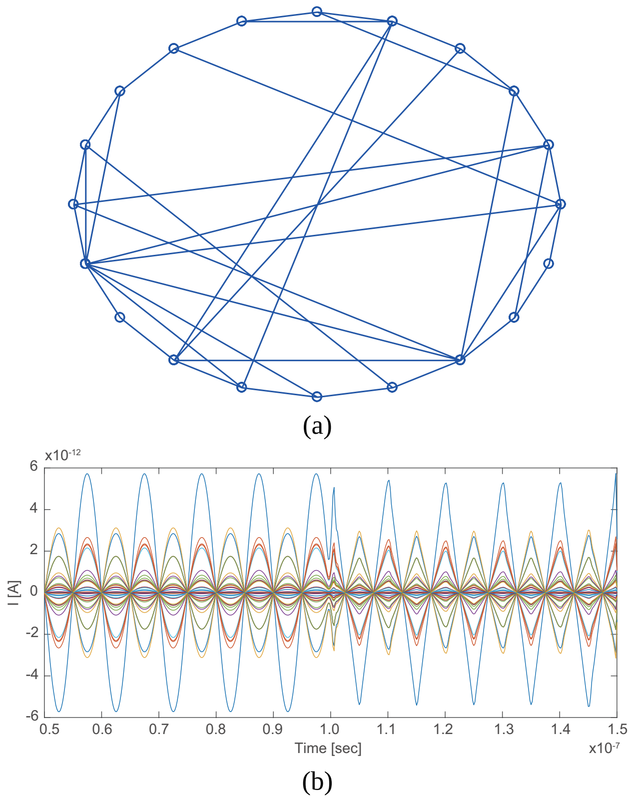

Figure 2: (a) An example of the structure of the memristive network with $N = 20$ and $M = 40$. Each blue line contains the memristor element. (b) The time courses of the electric currents on the network edges. The sinusoidal inputs are given in the first half of the duration and the triangular inputs are given in the second half.

tiotemporal patterns produced by the memristive reservoir. We randomly choose L edges and represent the scaled currents as $x_l(t)$ for $l = 1, \ldots, L$. In the readout part, these reservoir outputs are transformed into K network outputs as follows:

$$y_k(t) = f\left(\sum_{l=1}^{L} w_{kl} x_l(t)\right), \quad \text{for } k = 1, \ldots, K \quad (6)$$

where w_{kl} denotes the weight coefficient and $f(h) \equiv (1 + \exp(-h))^{-1}$ is the sigmoid-type activation function.

3. Results

3.1. Nonlinear dynamics

First we generate the network structure with $N = 20$ and $M = 40$ as illustrated in Fig. 2(a). This structure is similar to the small-world network [21]. The length of the memristor device and the ion mobility rate are set at $D = 1.0 \times 10^{-8}[m]$ and $\mu_v = 2.5 \times 10^{-6}[m^2 s^{-1} V^{-1}]$ for all the memristive elements. To consider the variability of device properties inevitable in nano/micro-scale systems, we set $R_{on} = 3.33 \times 10^7 \times (1 + u_k)[\Omega]$, $R_{off} = 3.33 \times 10^{10} \times (1 + u_k)[\Omega]$, $w_0 = 0.5 \times D(1 + u_k)$, where u_k represents a random number taken from the uniform distribution with range $[-u, u]$. We set the variability parameter u at $u = 0.1$ to allow 10% mismatch of the device properties.

When sinusoidal and triangular waves signals are given to the voltage source, the memristive reservoir produces spatiotemporal dynamics as shown in Fig. 2(b). The nonregular network topology and the variability of individual memristors contribute to mapping the 1-dimensional input signal to high-dimensional spatiotemporal patterns.

3.2. Classification problem

Let us consider a classification of two-class waveform patterns including sinusoidal (class 1) and triangular (class 2) waveforms. For each class, P sample data are prepared. The sample input are the waveform sequences with length T. The jth waveform signal ($j = 1, \ldots, P$) in the two classes are represented as follows:

$$V_j^{(1)}(t) = V_0 \sin(\omega_j t), \quad (7)$$

$$V_j^{(2)}(t) = V_0 \cdot \mathrm{triangular}(\omega_j t), \quad (8)$$

for $t = 1, \ldots, T$. The voltage amplitude is fixed at constant V_0 and the angular frequency of the jth waveform is set at $\omega_j = \omega_0(1 + r_j)$ where r_j is randomly taken from the uniform distribution with range $[-r, r]$.

The number of neurons in the readout part is set at $K = 2$. The sample output for class i ($i = 1, 2$) signal is given by

$$d_k^{(i)}(t) = \begin{cases} 1 & \text{for } k = i, \\ 0 & \text{for } k \neq i, \end{cases} \quad (9)$$

for $t = 1, \ldots, T$.

For the jth input pattern, the network output is denoted by $y_{k,j}^{(i)}(t)$ for class $i = 1, 2$, neuron $k = 1, 2$, and $t = 1, \ldots, T$. Using the standard linear regression method, we train the weight coefficients w_{lk} such that the error

$$\sum_{k=1}^{2} \sum_{j=1}^{P} |y_{k,j}^{(i)} - d_k^{(i)}| \quad (10)$$

is minimized.

We obtain the weights using 50 training samples for each class and evaluate the classification accuracy using 50 testing samples for each class. Figure 3 shows the classification accuracy plotted against the number of reservoir outputs, L. We see that a small number of reservoir outputs is not effective and multiple reservoir outputs are necessary for high classification accuracy. The three plots with different marks correspond to the different values of the voltage of the input signal, V_0. We observe that the accuracy is increased with V_0. When V_0 is sufficiently small, the V-I curve is almost linear. As V_0 is increased, the nonlinearly tends to increase in the V-I relationship. Therefore, it is

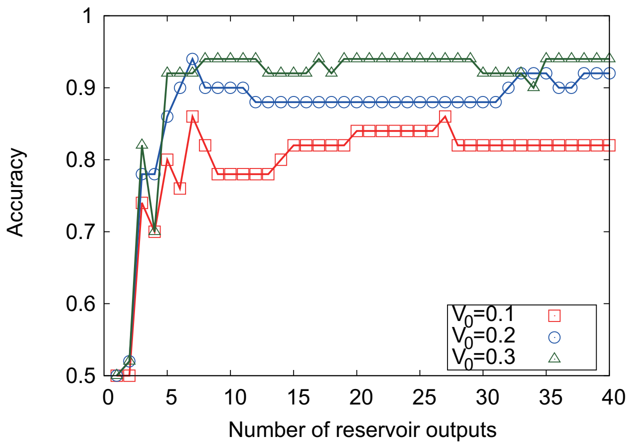

Figure 3: The classification accuracy is plotted against the number L of reservoir outputs for $V_0 = 0.1$, 0.2, and 0.3.

suggested that the nonlinearity of the memristive reservoir influences the computational performance of the memristive reservoir system.

4. Summary

We have formulated the circuit equations governing the dynamics of memristive networks for reservoir computing. Through the numerical simulation of the model, we have demonstrated the spatiotemporal patterns produced by the memristive reservoir. In the application to the waveform classification problem, we have confirmed that the nonlinearity of the memristive reservoir is significant for the computational ability of the reservoir computing system. We have used the ideal memristor model in this study, but the V-I curve in the real memristive device would have a more distorted and noisy V-I curve. It remains to be elucidated how such realistic properties of memristive devices affects the computational performance. It is also an important issue to reveal the effect of network topology on the computational performance of the memristive reservoir system. Through addressing these issues, the hardware implementation of memristive reservoirs could be explored for energy efficient information processing.

Acknowledgments

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References

[1] Benjamin Schrauwen, David Verstraeten, and Jan Van Campenhout. An overview of reservoir computing: theory, applications and implementations. In *Proceedings of the 15th European Symposium on Artificial Neural Networks.*, pages 471–482, 2007.

[2] Herbert Jaeger. The ”echo state” approach to analysing and training recurrent neural networks-with an erratum note. *Bonn, Germany: German National Research Center for Information Technology GMD Technical Report*, 148:34, 2001.

[3] Wolfgang Maass, Thomas Natschläger, and Henry Markram. Real-time computing without stable states: A new framework for neural computation based on perturbations. *Neural Computation*, 14(11):2531–2560, 2002.

[4] Laurent Larger, Miguel C Soriano, Daniel Brunner, Lennert Appeltant, Jose M Gutiérrez, Luis Pesquera, Claudio R Mirasso, and Ingo Fischer. Photonic information processing beyond turing: an optoelectronic implementation of reservoir computing. *Optics express*, 20(3):3241–3249, 2012.

[5] Seiji Takeda, Daiju Nakano, Toshiyuki Yamane, Gouhei Tanaka, Ryosho Nakane, Akira Hirose, and Shigeru Nakagawa. Photonic reservoir computing based on laser dynamics with external feedback. In *International Conference on Neural Information Processing*, pages 222–230. Springer, 2016.

[6] Chrisantha Fernando and Sampsa Sojakka. Pattern recognition in a bucket. In *European Conference on Artificial Life*, pages 588–597. Springer, 2003.

[7] Henry O Sillin, Renato Aguilera, Hsien-Hang Shieh, Audrius V Avizienis, Masakazu Aono, Adam Z Stieg, and James K Gimzewski. A theoretical and experimental study of neuromorphic atomic switch networks for reservoir computing. *Nanotechnology*, 24(38):384004, 2013.

[8] Adam Z Stieg, Audrius V Avizienis, Henry O Sillin, Renato Aguilera, Hsien-Hang Shieh, Cristina Martin-Olmos, Eric J Sandouk, Masakazu Aono, and James K Gimzewski. Self-organization and emergence of dynamical structures in neuromorphic atomic switch networks. In *Memristor Networks*, pages 173–209. Springer, 2014.

[9] Kohei Nakajima, Helmut Hauser, Rongjie Kang, Emanuele Guglielmino, Darwin G Caldwell, and Rolf Pfeifer. A soft body as a reservoir: case studies in a dynamic model of octopus-inspired soft robotic arm. *Front. Comput. Neurosci*, 7(10.3389), 2013.

[10] Toshiyuki Yamane, Yasunao Katayama, Ryosho Nakane, Gouhei Tanaka, and Daiju Nakano. Wave-based reservoir computing by synchronization of coupled oscillators. In *Neural Information Processing*, pages 198–205. Springer, 2015.

[11] Manjari S Kulkarni and Christof Teuscher. Memristor-based reservoir computing. In *2012 IEEE/ACM International Symposium on Nanoscale Architectures (NANOARCH)*, pages 226–232, 2012.

[12] Jens Bürger and Christof Teuscher. Variation-tolerant computing with memristive reservoirs. In *Proceedings of the 2013 IEEE/ACM International Symposium on Nanoscale Architectures*, pages 1–6. IEEE Press, 2013.

[13] Jens Bürger, Alireza Goudarzi, Darko Stefanovic, and Christof Teuscher. Hierarchical composition of memristive networks for real-time computing. In *Nanoscale Architectures (NANOARCH), 2015 IEEE/ACM International Symposium on*, pages 33–38. IEEE, 2015.

[14] Jens Bürger, Alireza Goudarzi, Darko Stefanovic, and Christof Teuscher. Computational capacity and energy consumption of complex resistive switch networks. *AIMS Materials Science*, 2(4):530–545, 2015.

[15] Cory Merkel, Qutaiba Saleh, Colin Donahue, and Dhireesha Kudithipudi. Memristive reservoir computing architecture for epileptic seizure detection. *Procedia Computer Science*, 41:249–254, 2014.

[16] Wei Fei, Hao Yu, Wei Zhang, and Kiat Seng Yeo. Design exploration of hybrid cmos and memristor circuit by new modified nodal analysis. *IEEE Transactions on very large scale integration (VLSI) systems*, 20(6):1012–1025, 2012.

[17] Gouhei Tanaka, Ryosho Nakane, Toshiyuki Yamane, Seiji Takeda, Daiju Nakano, Shigeru Nakagawa, and Akira Hirose. Waveform classification by memristive reservoir computing. In *International Conference on Neural Information Processing*. Springer, 2017.

[18] Dmitri B Strukov, Gregory S Snider, Duncan R Stewart, and R Stanley Williams. The missing memristor found. *nature*, 453(7191):80–83, 2008.

[19] Nathan R McDonald, Robinson E Pino, Peter J Rozwood, and Bryant T Wysocki. Analysis of dynamic linear and non-linear memristor device models for emerging neuromorphic computing hardware design. In *Neural Networks (IJCNN), The 2010 International Joint Conference on*, pages 1–5. IEEE, 2010.

[20] MATLAB. *version 9.0 (R2016a)*. The MathWorks Inc., Natick, Massachusetts, 2016.

[21] Duncan J Watts and Steven H Strogatz. Collective dynamics of ‘small-world’ networks. *nature*, 393(6684):440–442, 1998.