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# Control of Chaotic Itinerancy Observed in Coupled Systems of One-Dimensional Gauss Maps by Switching Coupling

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**Abstract**—A mutually coupled Gauss map generates chaotic itinerancy even though that is a two-dimensional coupled map. In order to control interchange of each trajectory of maps, we proposed the method of switching the coupling strength and investigated the basin of attraction and the destination by the one-time mapping at the multiple coupling strength. As the result, we found the appropriate coupling strength for the proposed switching method to successfully interchange the trajectory of each map at any discrete time steps.

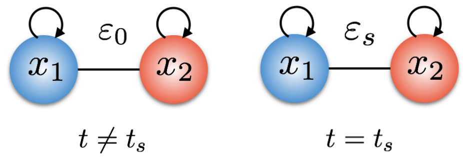



Figure 1: Mutually coupled Gauss map and switching coupling strength. $t_s$ represents the arbitrary discrete time step when the coupling strength changes.

## 1. Introduction

Dynamics of various phenomena observed in nature can be modeled by numerical equations including differential and difference equations. Although numerical models described by differential equations can reproduce the details of the complex phenomena, errors caused by numerical integration and huge time consuming tend to be problems. In this point of view, dealing with discrete time dynamical systems, which are generally described by difference equations, has an advantage of solving those problems. Furthermore, the complex properties associated with the systems can be represented in simple numerical models. Hence, analyzing the discrete time dynamical systems is efficient in terms of engineering applications [1, 2, 3]. The logistic map, which is known as one of the numerical models described by simple difference equations, was proposed to represent population growth by Verhulst [4], and has been well studied as a one-dimensional chaotic map by many researchers [5, 6, 7]. On the other hand, the Gauss map, which is also a one-dimensional chaotic map, has not been investigated in detail.

In our previous research, we proposed a mutually coupled Gauss map, and investigated the bifurcation structure of the fixed point and periodic point. In our proposed model, despite only a two-dimensional coupled map, we found that chaotic itinerancy, which is known as one of the characteristic phenomena observed in high-dimensional dynamical systems, appeared. At the system parameter where chaotic itinerancy occurred, each chaotic trajectory of each map unexpectedly interchanges. When we apply chaotic itinerancy observed in the proposed model to engineering fields, it is important that we intentionally interchange each chaotic trajectory when we need. In terms of the point, we choose the system parameter where interchange of each chaotic trajectory does not occur, and by causing switching coupling, namely, the method of changing the coupling strength at arbitrary time steps, we attempted to generate chaotic itinerancy on purpose. In this study, we show chaotic itinerancy we observed in our proposed model, and discuss the mechanism of the intentional interchange of the chaotic trajectory caused by switching coupling strength based on the initial value of each map.

## 2. Mutually coupled one-dimensional Gauss map

The dynamics of mutually coupled one-dimensional Gauss map is described by the difference equation given by

$$\boldsymbol{x}(t+1) = \boldsymbol{f}(t, \boldsymbol{x}(t)), \tag{1}$$

or, equivalently, the iterated map representing the discrete time dynamical system,

$$\boldsymbol{f} : \mathbb{R}^2 \to \mathbb{R}^2; \boldsymbol{x} \mapsto \boldsymbol{f}(t, \boldsymbol{x}), \tag{2}$$

where $\mathbb{R}$ is the set of real number; $\boldsymbol{x}$ and $\boldsymbol{f}$ indicate $(x_1, x_2)^\top$ and $(f_1, f_2)^\top$, respectively. The dynamics of the mutually coupled one-dimensional Gauss map dealt with in this study is described by

$$\begin{pmatrix} f_1(t,\boldsymbol{x}) \\ f_2(t,\boldsymbol{x}) \end{pmatrix} = \begin{pmatrix} \exp(-\alpha x_1^2) + \beta + \varepsilon(t)(x_2 - x_1) \\ \exp(-\alpha x_2^2) + \beta + \varepsilon(t)(x_1 - x_2) \end{pmatrix}, \quad (3)$$

where $\alpha$ and $\beta$ are the system parameters, and $\varepsilon(t)$ represents the function of the coupling strength between maps described by

$$\varepsilon(t) = \begin{cases} \varepsilon_0, & \text{if } t \neq t_s, \\ \varepsilon_s, & \text{if } t = t_s, \end{cases} \quad (4)$$

where $t_s$ is arbitrary discrete time steps. The system of Eq. (3) has reflection symmetry. As for

$$P := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (5)$$

it can be expressed as

$$\boldsymbol{f}(t, P\boldsymbol{x}) = P\boldsymbol{f}(t,\boldsymbol{x}). \quad (6)$$

Hereinafter, we set one of the system parameters to $\alpha = 12$. The values of $\beta$ and $\varepsilon(t)$ are controlled in order to generate the intentional interchange of each map.

## 3. Method

Let us summarize our method of switching coupling strength in order to intentionally generate interchange of the trajectories and how to investigate the basin of attraction.

### 3.1. Method of switching coupling strength

To intentionally interchange each trajectory of $x_1$ and $x_2$, we proposed a method of switching coupling strength between the maps at arbitrary discrete time steps $t_s$ as shown in Fig. 1. We set the values of system parameters to $\alpha = 12$ and $\beta = -0.505$ in order not to generate unexpected interchange between two chaotic trajectories. The coupling strength was set to $\varepsilon_0 = -0.086$ when $t \neq t_s$, and set to $\varepsilon_s$ at $t_s$. At $t = t_s$, after each map was updated by Eq. (3) with the coupling strength $\varepsilon_s$, it was reset to $\varepsilon_0 = -0.086$.

### 3.2. Investigation of the basin of attraction

Depending on the values of $x_1$ and $x_2$, the method mentioned above can not necessarily result in interchange of each trajectory. Whether the method successfully works or not is associated with the basin of attraction. Namely, investigating which set of initial values leads to each trajectory is important to understand the mechanism of interchange of each trajectory. We set the initial value of $x_1$ and $x_2$ in the range of $[-0.55, 0.55]$ by intervals of 0.001, and iterated 1000 time steps by Eq. (3). Then, based on the values of $x_1$ and $x_2$, we classified the waveforms observed in the coupled Gauss map into two types. Let us name those two types as Type A and B. Type A is that the average value of $x_1$ is greater than that of $x_2$; Type B is opposite to Type A, namely, the average value of $x_1$ is smaller than that of $x_2$.

## 4. Result

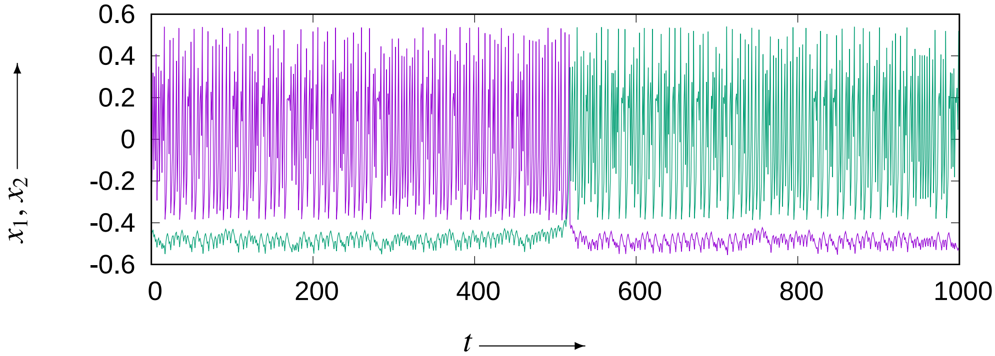
Figure 2 shows the waveforms observed in the mutually coupled Gauss map. When $\beta = -0.504$, the interchange occurred at approximately $t = 520$ without any parameter change as shown in Fig. 2(a). At such parameter settings, we cannot control when interchange occurs. According to the reason, we set the parameter value to $\beta = -0.505$ so that such unexpected interchange does not appear. When we set $\beta = -0.505$, depending on sets of initial values of $x_1$ and $x_2$, two types of the waveforms were observed. Figures 2(b) and (c) show waveforms of Type A and B, respectively. We focused on swapping Type A and B when we needed to switch them by controlling the coupling strength.

Then, we investigated the basin of attraction in order to understand what set of initial values brought $x_1$ and $x_2$ to Type A and B waveforms as shown in Fig. 3(a). White and black color indicate the sets of the initial value of $x_1$ and $x_2$ which generate Type A and B waveforms, respectively. Each coordinate value at the corners of the red rectangles is the combination of the maximum and minimum values of $x_1$ and $x_2$ which we obtained by iterating the coupled map for 10000 times with removing the transient state at $\varepsilon_0 = -0.086$ and $\beta = -0.505$. Note that when $x_1$ and $x_2$ have same initial value, namely, they are on the diagonal from the left bottom to the right up in Fig. 3(a), each map oscillates in-phase, hence it behaves neither Type A nor B.

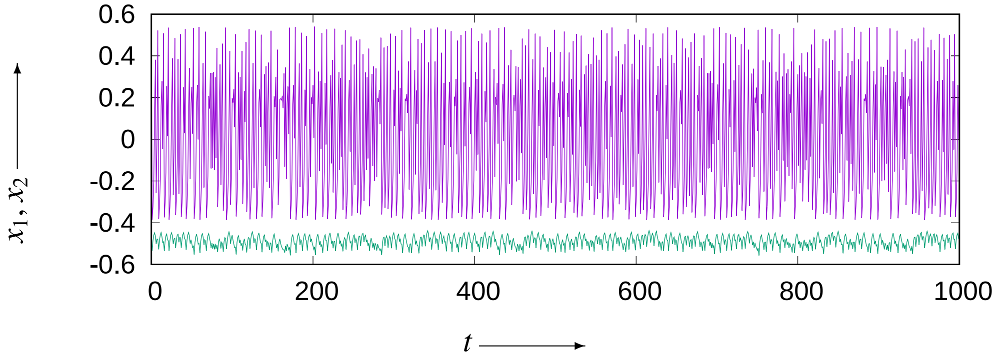
Figure 3(b) shows the result of the one-time mapping from each set of $x_1$ and $x_2$ on the grid points with interval of 0.001 in the landscape rectangle at each $\varepsilon_s$. If the set of $x_1$ and $x_2$ would be on the white region in Fig. 3(a) and it was mapped to the point on the black region after one-time iteration, we can see they could successfully interchange their trajectories. Each color filled region as shown in Fig. 3(b) indicates the destination by one-time mapping from the sets of the initial values inside the landscape rectangle. When $\varepsilon_s = 2.6$, the sets of $x_1$ and $x_2$ have been mapped from inside of the landscape rectangle to the portrait rectangle, which means interchange of each trajectory was successfully conducted at $\varepsilon_s = 2.6$ as shown in Fig. 4.

## 5. Conclusion

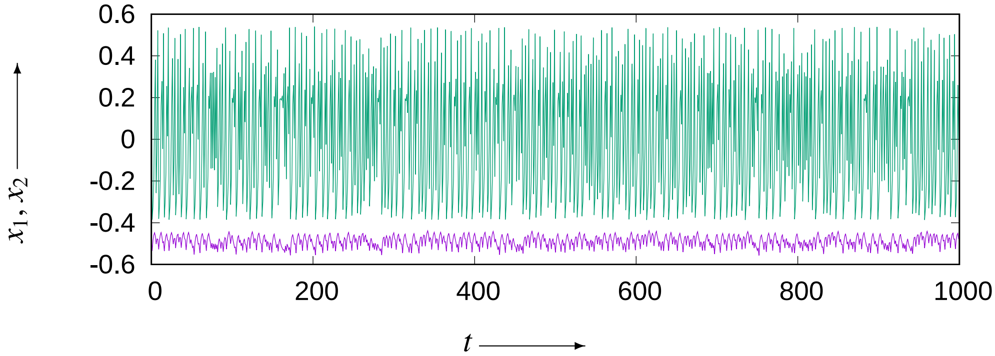
The mutually coupled Gauss map we proposed generates chaotic itinerancy even though that is a two-dimensional coupled map. In order to control the interchange of each chaotic trajectory of the map, we proposed the method of switching coupling strength. To understand the mechanism about whether intentional interchange works well or not in specific coupling strength, we investigated the basin of attraction and the destination from each set of initial values by the one-time mapping at each coupling strength. As the result, we found the appropriate coupling strength to successfully interchange the trajectory of each map at any discrete time steps.



(a) Waveforms, $\beta = -0.504$

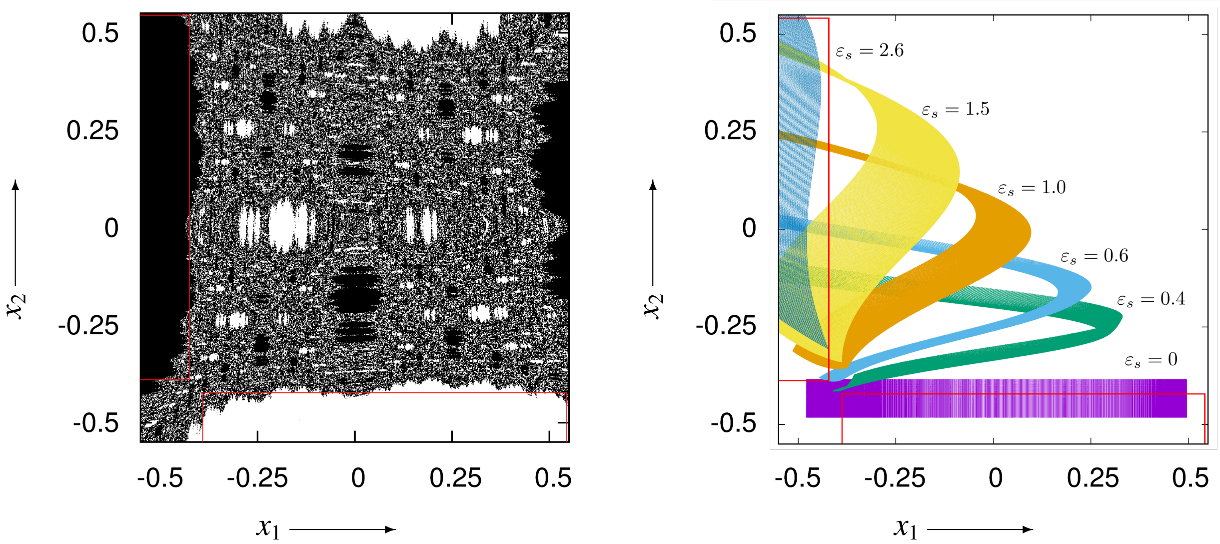


(b) Type A, $\beta = -0.505$



(c) Type B, $\beta = -0.505$

Figure 2: Waveforms of the mutually coupled Gauss map at $\alpha = 12, \varepsilon_0 = -0.086$. In (a), the interchange of two maps unexpectedly occurs at $\beta = -0.504$; (b) shows Type A waveforms in which the average value of $x_1$ is greater than that of $x_2$; (c) represents Type B waveforms. The trajectories of $x_1$ and $x_2$ are swapped as compared with Type A.




(a) Basin of attraction (b) One-time mapping

Figure 3: Relationship between initial values and observed phenomena.

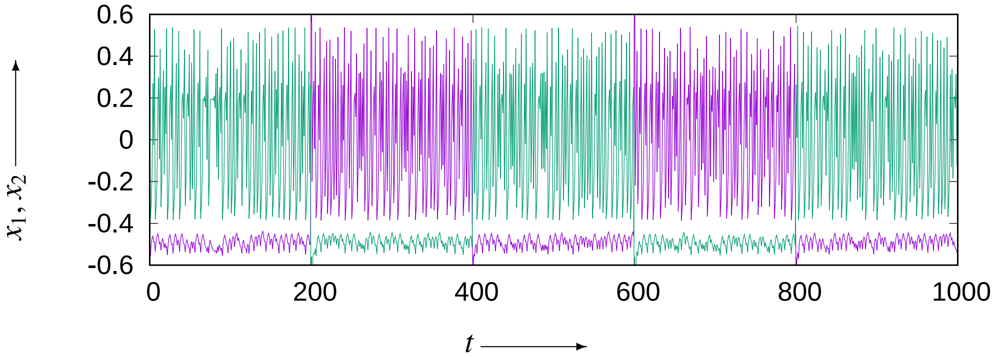



Figure 4: Result of controlling interchange of each trajectory by switching the coupling strength from $\varepsilon_0 = -0.086$ to $\varepsilon_s = 2.6$ at $t_s = 200n, n = 1, \ldots, 4$ and $\beta = -0.505$.

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