

Chaos-Based Multi-Carrier CDMA: Orthogonal Subspace Approach

G. Setti[†], G. Mazzini[†], R. Rovatti[‡]

[†]ENDIF - University of Ferrara, via Saragat 1, 44100 Ferrara - ITALY

e-mail: gsetti@ing.unife.it, g.mazzini@ieee.org

[‡]ARCES - University of Bologna, via Toffano 2/2, 40125 Bologna - ITALY

e-mail: rrovatti@arces.unibo.it

Abstract—Multi-code DS-CDMA systems assign more than one spreading sequence to each of the users. In these systems multiple access interference (MAI) is made of a synchronous and an asynchronous component. We propose an orthonormalization approach aiming at coupling the advantages of orthogonal code design in countering synchronous MAI with that of chaos-based codes in countering asynchronous MAI. Improvements with respect of classical orthogonal-i.i.d. sequences in terms of signal-to-interference ratio are demonstrated by simulations. The effect of quantization in sequence storing and processing is also taken into account.

1. Introduction

In the last 20 years a lot of work has been done regarding DS-CDMA. Starting from military application, DS-CDMA has gained an important role in commercial communications thanks to its VLSI implementation. DS-CDMA is actually the reference communication scheme for 3G system and its variations are the core for the further 4G systems.

In this very large framework, a very interesting and promising research field has been the definition and characterization of spreading sequences that achieve maximum effectiveness in reduce interference from non-useful links (by means of orthogonality related properties) and thus granting multi-access capabilities.

The design of spreading code has been considered by different authors with different approaches. With respect to standard basic characterization, focused on independent identically distributed (i.i.d.) spreading chip, further analysis has underlined that colored spreading sequence are able to optimize the system performance increasing the number users achievable by the system [1]. This is particular true for asynchronous environment, where relative phase and delays between a user and the interfering ones are completely random, as it happens in a mobile uplink channel.

Chaos theory has given a very simple and effective methods to select the optimal autocorrelation profile for the spreading sequences, allowing to reach the maximum performance increment of 15.4% in terms of user capacity. Such an improvement does not require any particular complexity cost, so the chaos-based approach is one of the most promising techniques.

With the upcoming 4G system, the multimedia service

gain a very important role and it is necessary to have the possibility to modulate the bandwidth of each users with a given granularity (see e.g. [2]). This paper analyzes a new technique which permits to give different basic channel flux to each users. The basic idea can be carried over from some 3G systems and it is to use independent spreading code for each users and for each elementary channel of each users. Inside a single user the different spreading codes are then orthogonalized in order to keep benefit from its intrinsic synchronous nature and to obtain a global benefit on the system performance.

So, our approach proposes to generate a number of chaos based spreading code able to cover all the users and all the fluxes for each users, and then to orthogonalize, by means of Gram-Schmidt procedure the fluxes inside the users. What is interesting, is that the orthogonalization procedure does not change the autocorrelation characteristics of the sequences.

The paper performs the analysis of this system and investigates the benefit by means of proper merit figures. The results are here obtained by means of simulation just to fix the idea of this novel technique, but further analytical analysis seems to be possible in order to full characterize the environment and the design criteria. Finally, the paper produces an investigation of the impact of the number of bits in the system implementation, showing that the orthogonalization procedure is not particularly sensitive to the number of bits used in the system, leaving open a low cost and low complexity implementation.

2. System Model and Performance

We assume that U users are present in the system and that each of them is assigned M spreading codes. The spreading codes are named $y^{u,m}$ with $u = 0, \dots, U - 1$ and $m = 0, \dots, M - 1$, u being the user index and m being the code index. Spreading sequences are real and periodic of period N that is also the spreading factor of the system. We indicate with $y_k^{u,m}$ (with $k = 0, \dots, N - 1$) the k -th chip of the m -th sequence of the u -th user. The energy of each spreading sequence is assumed to be $\sum_{k=0}^{N-1} (y_k^{u,m})^2 = 1$. We will also assume that the spreading sequences are generated by some sort of random symbol generator (each user independently of the other) and that we are interested in average performance also over all the possible generated

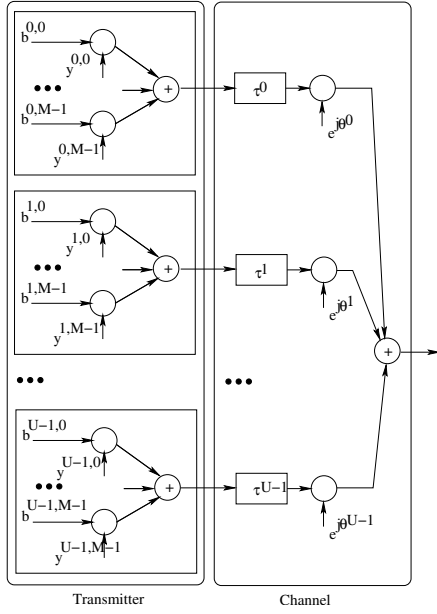


Figure 1: Low-Pass equivalent of a Multi-Code DS-CDMA system with U -users with M streams each

sequences.

Each user is entitled to send M bits simultaneously, each associated with a different spreading codes. If g is the rectangular chip pulse of duration T , the low-pass equivalent of the signal transmitted by the u -th user is

$$s^u(t) = e^{j\theta^u} \sum_{m=0}^{M-1} \sum_{j=-\infty}^{\infty} b_j^{u,m} \sum_{k=0}^{N-1} y_k^{u,m} g(t - kT - jNT - \tau^u)$$

where τ^u and θ^u are the random delay and phase associated to the u -th user and $b_j^{u,m}$ is the j -th bit transmitted over the m -th sequence of the u -th user (see Figure 1). Delays are assumed to be independent one of the other and uniformly distributed in $[0, NT]$, phases are assumed to be independent on of the other and uniformly distributed in $[-\pi, \pi]$. Bits are assumed to be antipodal quantities, independent each from the others when either u , m , or j changes.

We concentrate our analysis on simple correlation-based receivers interested in decoding a single bit stream from a single user and we assume that the system is completely symmetric as far as the power assignment is concerned. With this we may assume to decode $b_0^{0,0}$ assuming that the corresponding waveform is synchronized at the receive, i.e., that $\tau^0 = \theta^0 = 0$. We do that by correlating the incoming signal $\sum_{u=0}^{U-1} s^u(t)$ with the signal $\sum_{k=0}^{N-1} y_k^{0,0} g(t - kT)$ and taking a hard decision on the sign of the output of the correlator.

Assuming that the performance of the receiver is mainly limited by multiple access interference (MAI), the three main components at the output of the correlator are

- the useful component with unit energy

- the asynchronous MAI due to the $(U - 1)M$ sequences of the users other than the 0-th that have random delays and phases
- the synchronous MAI due to the $M - 1$ sequences of the same users that are unknown to this very “local” receiver.

The asynchronous MAI is a random variable since it depends on delays and on spreading sequences. The synchronous MAI is also a random variable since it depends on spreading sequence assignment. For both of them we will assume a Gaussian approximation that, since they are both zero-mean, leaves us with the task of evaluating their variance.

Once that these two variances are known, the average bit error probability of the system can be estimated as

$$\text{BER} = \frac{1}{2} \text{erfc} \sqrt{\frac{1}{2(\sigma_{\text{asynch}}^2 + \sigma_{\text{synch}}^2)}}$$

where we have exploited the fact the the useful component as unit energy.

Due to the independence between bits, each of the variance of the asynchronous MAI is proportional to the number of streams it considers. Hence, there must be two quantities R and Q such that

$$\text{BER} = \frac{1}{2} \text{erfc} \sqrt{\frac{1}{(U - 1)MR + Q}}$$

where R characterizes asynchronous MAI and Q characterizes synchronous MAI.

As far as asynchronous MAI is concerned, a well-established path [1] leads to recognize that, as long as the interfering sequence can be thought as independent of the useful sequence, R depends on the autocorrelation $A_k = \mathbf{E}[y_0^{u,m} y_k^{u,m}]$ of the sequences themselves by means of

$$R = \frac{2}{3N} + \frac{4}{3N^3} \sum_{k=1}^{N-1} (N - k)^2 A_k^2 + \frac{(N - k + 1)(N - k)}{2} A_k A_{k-1}$$

Synchronous MAI depends on the correlation between different sequences assigned to the same users, i.e.

$$Q = \mathbf{E} \left[\left(\sum_{m=1}^{M-1} \langle y^{0,0}, y^{0,m} \rangle \right)^2 \right]$$

If the sequences assigned to the same user are independent one of the other we may write $Q = (M - 1)Q'$ with

$$Q' = \frac{2}{N} + \frac{4}{N^2} \sum_{j=1}^{N-1} A_j^2$$

In general, the M sequences assigned to a user are not independent and no synthetic feature can be highlighted deciding performance.

3. Chaos-Based Spreading

When chaos-based spreading is considered [3], the symbols y_k^u are taken from the quantization of the trajectory of a discrete time chaotic dynamical system.

Formally speaking we will assume that a state-update function $M : [0, 1] \mapsto [0, 1]$ exists giving rise to the trajectory $x_{k+1} = M(x_k)$ starting from an initial condition x_0 . We will assume that the corresponding dynamical system can be classified as “mixing” (see e.g. [4]) so that it can be effectively analyzed and designed considering the statistical features of a collection of trajectories with non-vanishing measure.

For each sequence, an initial condition x_0 is drawn independently and the corresponding trajectory x_0, \dots, x_{N-1} is then passed through an antipodal quantization function $Q : [0, 1] \mapsto \{-1, +1\}$, yielding $y_k = Q(x_k^u)$.

Moreover, we will concentrate on a particular family of maps called (n, t) -tailed shifts [5] that is well-known to give the degrees of freedom needed to obtain the statistical behaviors in which we are interested. These maps are defined, for $t < n/2$, as $M(x) = (n-t)x \bmod \frac{n-t}{n} + \frac{t}{n}$ if $0 \leq x < \frac{n-t}{n}$, while $M(x) = t(x - \frac{n-t}{n}) \bmod \frac{t}{n}$ otherwise. From [5] we know that one can design (n, t) -tailed shifts to feature any rate $r = -t/(n-t)$ of exponential decay in correlation functions.

As long as we adopt an antipodal quantization function, we have that [5] $A_k = r^{|k|}$. It can be proved that $(n, 0)$ -tailed shifts are able to generate i.i.d. sequences.

4. Subspace orthonormalization

What we propose is to use a chaos-based set of UM independent spreading sequences, divide it into U subsets of M sequences. Each subset is then orthonormalized and assigned to a user.

The rationale behind this procedure is that, when N is large enough, sequences are generated approximately orthogonal. The orthonormalization procedure is applied to subsets of the codes ensemble. This is expected to eliminate synchronous interference while altering minimally the sequences themselves that should approximately maintain an autocorrelation profile that is known to reduce asynchronous interference.

5. Numerical Results

A C simulator has been written to test the system and compute the average performance. The system has been stressed with 1000000 trials for each interesting configuration. The cases with $N = 16, 32$ and 64 have been considered obtaining similar performance. Only case $N = 32$ is reported in the following for the sake of conciseness.

The considered system has been loaded with three different user configurations, all related to the equivalent load $UM = 12$. In the Figures 2 3 and 4 the case with $U =$

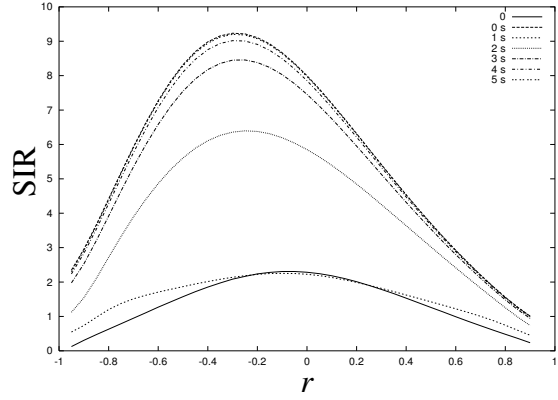


Figure 2: System performance with $N = 32$, $U = 2$ and $M = 6$.

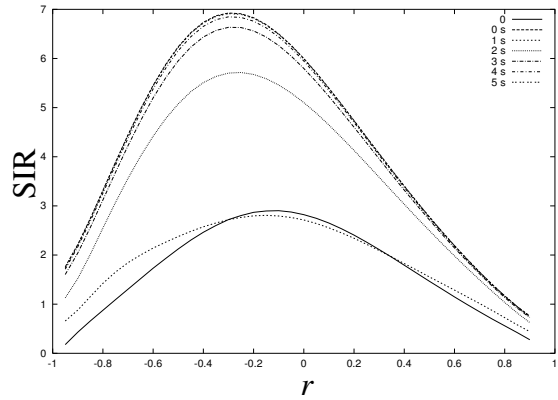


Figure 3: System performance with $N = 32$, $U = 3$ and $M = 4$.

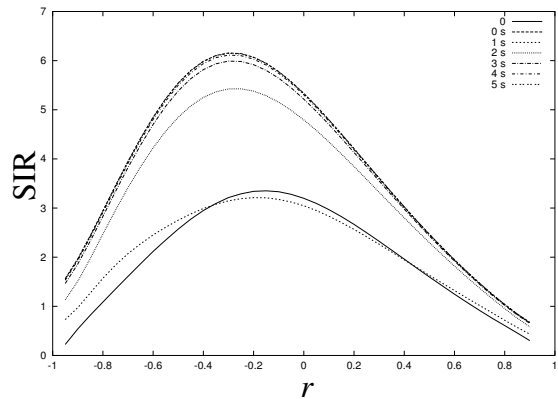


Figure 4: System performance with $N = 32$, $U = 4$ and $M = 3$.

2, $M = 6$; $U = 3, M = 4$ and $U = 4, M = 3$ are reported, respectively. Each Figure gives signal-to-interference ratio (SIR) as a function of the correlation parameter r , by varying the number of bits used to represent the spreading sequence at the output of the orthogonalization process. The case labelled with "0" and plotted with continuous line accounts for system without orthogonalization and an infinite number of bits to represent the spreading chip. The dashed lines consider the orthogonalization process. The orthogonalized cases are reported by ranging the bit used to represent the spreading sequences from 1 to 5 bits, with label "1s", "2s", "3s", "4s" and "5s". The case "0s" is related to the orthogonalized system with an infinite number of bits to represent the spreading chip.

Note that for any combination of the system parameters the case without orthogonalization process has performance analogous to the case with orthogonalization performed with a single bit. This is due to the fact that original sequences are quantized with two values and a single bit is enough to represent that. The differences are due to the orthogonalization process which attempt to change the bit attribution while the strict quantization effect attempts to resort the original value but the two effects imply difference in the final spreading sequences.

On the other hand, when the number of bits used to quantized the spreading chip increases, the SIR increases and a gain using the orthogonalization process appears. Note how just the use of 2 bits gives in all the considered configuration considerable improvements. The improvement increases monotonically with the number of bits used to quantize and a saturation effect is just evident by going from 4 to 5 bits, indicating that a low complexity system may be used to reach the optimal performance sustainable from the proposed approach.

Another interesting remarks is regarding the optimization of the performance by changing the autocorrelation profile of the original spreading sequences, by varying the r parameter. The maximum of the curves change with the number of bits used for the quantization. As the number of bits decreases, the orthogonalization effects reduces and the position of the SIR maximum shifts the the right, i.e. towards the classical optimality for synchronous environment for which i.i.d. sequences are optimal.

Finally, note that with the constant load UM the performance increases as M increases. This is due to the reduced contribution of sequence self-interference inside the same user. On the contrary, the maximum tend to shift right due to the increase of general synchronism of the whole system.

Table 1 reports the optimal correlation parameter r for different system configurations, the $b = 0$ rows corresponding to the absence of quantization. In this case, the optimal r does not depend on U and M and, as N increases, appears to tend to $r = -2 + \sqrt{3} = -0.268$ that is the optimal correlation parameter in the non-multicode case (see e.g. [1]). When antipodal sequences are employed ($b = 1$), te larger the N the more i.i.d. are the optimal sequences after

N	b	$U = 2$	$U = 4$	$U = 3$
		$M = 6$	$M = 3$	$M = 4$
16	1	-0.216	-0.250	-0.263
32	1	-0.109	-0.153	-0.182
64	1	-0.072	-0.106	-0.133
16	2	-0.241	-0.268	-0.282
32	2	-0.242	-0.264	-0.272
64	2	-0.244	-0.262	-0.268
16	0	-0.307	-0.307	-0.307
32	0	-0.285	-0.285	-0.285
64	0	-0.276	-0.276	-0.276

Table 1: The optimal correlation parameter r for different system configurations

subspace orthogonalization. When $b = 2$ both trends are present and depend on U and M .

A further direction of investigation, especially when the number of quantization bits are large enough to avoid consideration regarding their effects, is related to an analytical evaluation of the performance and on the maximum achievable with respect to the system parameters.

6. Acknowledge

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References

- [1] G. Mazzini, R. Rovatti, G. Setti, "Interference minimization by autocorrelation shaping in asynchronous DS-CDMA systems: chaos-based spreading is nearly optimal", IEE Elect. Lett., vol. 35, pp. 1054-1055, 1999
- [2] Dong In Kim, E. Ossain, V.K. Bhargaba, "Dynamic rate adaptation based on multi-dimensional multi-code DS-CDMA in cellular wireless networks", IEEE Trans. Comm., vol. 51, n. 2, pp. 247-260, 2003
- [3] G. Setti, G. Mazzini, R. Rovatti, S. Callegari, "Statistical Modeling of Discrete-Time Chaotic Processes: Basic Finite-Dimensional Tools and Applications", Proc. IEEE, vol. 90, pp. 662-690, 2002
- [4] A. Lasota, M. C. Mackey, *Chaos, Fractals, and Noise*, Springer-Verlag, 1994
- [5] R. Rovatti, G. Mazzini, G. Setti, "A Tensor Approach to Higher-Order Expectations of Quantized Chaotic Trajectories - Part I: General Theory and Specialization to Piecewise Affine Markov Systems," IEEE TRANSACTION ON CIRCUITS AND SYSTEMS - I, vol. 47, pp. 1571-1583, 2000