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Dynamical Features of Cellular Automata Rules in Describing Digital Sound

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Abstract—In the present paper, we treat a novel method in describing digital sound data by rule sequences of cellular automata. The method can give an errorless coding for all the data. In addition, except for a few data, the method can simultaneously provide a compressive coding. In realizing compressive coding for all the data perfectly, it is important to study dynamical features of the rule of cellular automata in describing digital sound data. From computer experiments, it is shown that a certain two-rules set has a specific feature in giving compressive coding.

1. Introduction

Recently, a novel description method of digital data based on cellular automata (referred as CA hereafter) has been proposed [1]-[7], which successfully gives errorless coding. For almost data, simultaneously, the method enable us compressive coding. In addition, the coding shows attractor dynamics analogous with conventional nonlinear dynamical systems. The attractor dynamics means that even if starting from any initial condition of CA, original data can be recovered by applying the coding of rule sequences of CA. The attractor dynamics is one of most important features of the data description related with noise robustness.

The method is based on the idea that there exist a certain deterministic rules behind complex dynamics even if the system has a large degree of freedom. In proving our idea, as complex dynamics we focus on real digital sound data (pronounced words and music CD (compact disk) data), as deterministic rules on rules of CA. Our results suggest that we succeed to extract rules behind sound data by means of rule sequences of CA [1]-[7].

Cellular automata are applicable in modeling various natural phenomena [8, 9]. The dynamical properties are investigated in details [10]. Our study is strongly related with "rule dynamics" proposed by Aizawa and Nagai [11].

From the viewpoint of rule dynamics, we can translate time development of sound data into time development of rule sequences of CA. The rule sequences would reflect certain dynamical features of sound data. In order to investigate the possibility, it is important to investigate dynamical features of the rules. In addition, if we succeed to give the least amount of data by our coding of rule sequences without reproducing error, the rule sequences could represent dynamical complexity of data.

Therefore, our purpose of the present paper is to investigate dynamical features of rules in describing digital sound data by evaluating three quantities. From our studies, we would show the features of a certain specific two-rules set in describing digital sound data.

2. Basic Idea for Describing Digital Sound Data by means of CA

Let up present one-dimensional CA with two states and three neighbors cellular automata (referred as 1-2-3 CA hereafter). The variable $\{a_i^t \mid a_i^t = 0 \text{ or } 1,\ i = 1, \cdots, N\}$ represent the state of the i-th cell in the chain at time step t. The state of the i-th cell at time step $t+1$, a_i^{t+1}, is determined by the states of itself and those of the neighboring two cells at time step t so that updating rule can be represented as

$$a_i^{t+1} = f(a_{i-1}^t, a_i^t, a_{i+1}^t), \tag{1}$$

where a function f is called a transition function which updates the state of a_i^t to a_i^{t+1}. In order to specify updating rule, we introduce,

$$\begin{aligned} f(0,0,0) &= f_0 \quad f(0,0,1) = f_1 \\ f(0,1,0) &= f_2 \quad f(0,1,1) = f_3 \\ f(1,0,0) &= f_4 \quad f(1,0,1) = f_5 \\ f(1,1,0) &= f_6 \quad f(1,1,1) = f_7, \end{aligned} \tag{2}$$

where $f_i = 0$ or 1 $(i = 0, \cdots, 7)$. By choosing each of f_i to be 0 or 1, we can determine a certain specified rule.

Furthermore, defining the rule number as

$$\begin{aligned} n &= 2^0 f_0 + 2^1 f_1 + 2^2 f_2 + 2^3 f_3 \\ &\quad + 2^4 f_4 + 2^5 f_5 + 2^6 f_6 + 2^7 f_7, \end{aligned} \tag{3}$$

Table 1: Time steps until generating all the possible 2^{16} bit-patterns by applying two-rules set. The number in the bracket represents the Wolfram's class.

rule #1	rule #2	Time steps
15 (2)	240 (2)	16
60 (3)	195 (3)	16
85 (2)	170 (1)	16
102 (3)	153 (3)	16
150 (3)	165 (3)	18
102 (3)	195 (3)	19
90 (3)	105 (3)	19
90 (3)	180 (1)	27

one can name each n "the specified rule number" from 0 to 255 (totally $2^8 = 256$ rules). If we assign the two state of each cell 1(0) to a black(white) small dot, respectively, the time development of a certain initial state in 1-2-3 CA gives bit-pattern sequences.

In modern technology, on the other hand, sound data (analog signals) are usually treated as digital signals both in time and their amplitude. For instance, a music CD (compact disc) contains sound data recorded as digital signals taken under sampling frequency, 44.1 kHz and the amplitude quantization, 16 bits. Therefore, time development of the digital sound data gives bit-pattern sequences.

Our idea about describing digital sound data by 1-2-3 CA are stated as follows:

- $\mathbf{a}^t = \{a_i^t | i = 1, \ldots, 16\}$ at time step t of 1-2-3 CA can be regarded as "an amplitude of quantized sound signals in binary coding" taken with appropriately sampling frequency.
- Time development of digital sound data ($\mathbf{a}^t \Rightarrow \mathbf{a}^{t+1}$) can be generated by applying rule to $\mathbf{a}^t$, that is, $\mathcal{R} \circ \mathbf{a}^t = \mathbf{a}^{t+1}$, where $\mathcal{R}$ is a certain rule sequence.

Developing our idea, it was discovered that only two rules, which are appropriately chosen from all the $2^8 = 256$ rules in 1-2-3 CA, are sufficient to generate all the patterns consisting of 16 bits starting from an arbitrary initial pattern of them [2, 3, 5]. See the Ref. [2, 5] about detailed method how to extract rule sequences consisting of two rules.

3. Evaluation Quantities

3.1. Generation Ability of Bit-Pattern Sequences

In order to achieve errorless coding completely, the two-rules set, which can generate all the possible 2^{16} bit-patterns within the application time steps of the rule set as less as possible, is practical. Therefore, we evaluate the least application time steps in generating all the possible 2^{16} bit-patterns for all the possible pairs of two-rules, ${}_{256}C_2$.

Result is given in Table 1, where the rule sets giving the least application time steps and typical ones are presented. The least application time steps is 16 time steps for four sets, (#15, #240), (#60, #195), (#85, #170) and (#102, #153).

In this paper, based on the result, we define generation ability in generating bit-patterns as

$$\text{ability}_{\text{generation}} = \frac{\text{number of states reproduced within} N_{\text{max}}}{2^{N_{\text{max}}}}, \quad (4)$$

where N_{max} denotes the maximum application time steps of rules. It should be noted that in the case of $N_{\text{max}} = 16$, the quantity of ability$_{\text{generation}}$ corresponds to the ratio how many bit-patterns are generated of all the 2^{16} bit-patterns.

3.2. Compressibility

Compressibility is one of features of the coding, which, in a sense, is related with complexity of data. In our coding, time development of bit-pattern sequences of sound data is described by time development of rule sequences. Thus, at each time step, (i) 1 bit × (length of rule sequence) and (ii) extra bits which indicate data section are necessary to describe the coding data. Thus, total amount of coding data is described by

$$\text{total amount of coding} = \sum_{t=1}^{T} (\log_2 N_{\text{max}} + L_t) + 16, \quad (5)$$

where $\log_2 N_{\text{max}}$ represents the extra bits, L_t is the length of rule sequences at time step t and T is the total length of original data. The 16-bits indicate the initial configuration of CA.

In final, the compressibility is defined as,

$$\begin{aligned} &\text{compressibility} \\ &= \frac{\text{total amount of coding data}}{\text{total amount of original data}} \\ &\simeq \frac{\log_2 N_{\text{max}} + \text{(averaged length of rule sequences)}}{16}, \end{aligned} \quad (6)$$

3.3. Description Ability

In our coding process, if the correct bit-pattern is not obtained within N_{max}, then at the time step, we give up the description and employ the original bit-pattern instead of the rule sequence as the code. Thus, for smaller N_{max}, the time step when the data could not be generated by the two-rules set increases. In other words, the completeness in describing digital data is also one of features of the two-rules set only. Here, the completeness means how many times steps the original data is described by the two-rules set.

In this paper, the completeness is evaluated by description ability as follows:

$$\text{ability}_{\text{description}} = \frac{\text{data length described by rule-set}}{\text{total data length}}. \tag{7}$$

The value of 1 means that for all the time steps, data is described by the two-rules set only.

4. Computer Experiments

4.1. Experiment Condition

In computer experiments, two kinds of signals are employed:

1. 150 music data which are taken from four classic and one Japanese pops music CD. All the intervals of the data are 1 second.

2. 600 pronounced words which are taken from 15 different kinds of word pronounced by both 20 men and 20 women. The data are made by ATR (Advanced Telecommunication Research Institute International) in Japan and are supplied as commercial database.

The music data are digitized under the sampling frequency 44.1 kHz and the amplitude at each time step quantized into 16 bits, the pronounced words 22.05 kHz , 16 bits, respectively. The data format of music is that of commercial music CD.

For the boundary condition, fixed boundary condition is employed through the experiments, that is, the boundary cell at the left of the 16th cell takes 1 and the boundary cell at the right of the 1st cell takes 0. In our method, the fixed boundary condition is important. If periodic boundary condition was applied, we could not give errorless coding.

4.2. Results

From our computer experiments, the two-rules set of (#90, #180) gives highest compressible coding without reproducing error. Several examples are presented in Table 2. In the case of $N_{\max} = 16$, except for several JPOP's, we can succeed to give compressive coding. For smaller vale of $N_{\max}$, compressibility decreases because of decreasing description ability.

In the present paper, we focus on the problem why the two-rules set of (#90, #180) can give high compressible coding. We compare the three quantities, generation ability, compressibility and description ability, for the (#90, #180) with those for the other sets as shown in Table 3. In the four sets, (#15, #240), (#60, #195), (#85, #170) and (#102, #153), with the highest generation ability, our method fails to give compressive coding. In the case of complete description, by considering the extra bits of $\log_2 N_{\max} = 4$, the rule set with smaller than 12 averaged length of rule sequences could give compressive coding. In the four sets, the averaged length takes than 12, then we fails to give compressive coding.

In the rule set of (#90, #180), on the other hand, the average length is quite short and the description ability is high even though the generation ability is much smaller than others. Thus, the rule set of (#90, #180) is appropriate in describing the time development of sound data. In order to achieve high compressibility, it is necessary to find out the rule set with similar properties to (#90, #180), shorter averaged length and higher description ability. This is future work of ours.

Finally, we investigate dynamical features of the sets from the viewpoint of Wolfram's classification [10]. Result is given in the number of the bracket in Table 1. We can observe that mostly one of the two rules tends to belongs to the class 3, which corresponds to chaotic state. Thus, we expect that chaotic rule could generate the variety of bit-pattern sequences, corresponding to introducing divergent dynamics.

5. Conclusions

In this paper, we investigate dynamical features of the two-rules set of CA in describing digital sound data by evaluating (i) the bit-patterns generation ability, (ii) compressibility and (iii) description ability of the coding. Our results are as follows:

- In the rule set of (#90, #180), the average time is quite short and the description ability is high even if the generation ability is much smaller than others. The rule set is special one in reproducing the time development of sound data.

- Mostly one of the two rules tends to belongs to the class 3, which corresponds to chaotic state. Thus, chaotic rules could play important roles in leading to the variety of bit-pattern sequences.

Usual compressive methods of data description are based on FFT (Fast Fourier Transformation). The compressibility of them is much higher than ours, however they associate unavoidable loss of information. In our method, one can obtain complete description of data.

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Table 2: Compressibility of the two-rules set (#90, #180).

	N_{max} = 4			N_{max} = 8			N_{max} = 16		
	pocket	sports	knife	pocket	sports	knife	pocket	sports	knife
woman 1	0.953	1.045	1.06	0.846	0.987	1.021	0.802	0.936	0.943
woman 2	0.940	1.002	1.066	0.839	0.940	1.027	0.795	0.907	0.944
man 1	0.996	1.039	1.064	0.874	0.968	0.999	0.822	0.915	0.916
man 2	1.041	1.060	1.077	0.914	0.976	1.026	0.848	0.914	0.942
	sample1	sample2	sample3	sample1	sample2	sample3	sample1	sample2	sample3
JPOP	1.118	1.116	1.118	1.128	1.117	1.124	1.013	0.998	1.008

Table 3: Evaluation results of the three quantities for N_{max} = 16. The "generation" represents the generation ability and the "description" represents the description ability. The "averaged length" denotes the averaged length of rule sequence at each time step It should be noted that in the evaluation of the averaged length, we exclude the cases that the method fails to generate original data within N_{max}.

rule #1	rule #2	generation	pocket(woman 2)			JPOP		
			compressibility	averaged length	description	compressibility	averaged length	description
15	240	1.00	1.152	13.79	1.00	1.147	12.40	1.00
60	195	1.00	1.129	13.85	1.00	1.146	14.34	1.00
85	170	1.00	1.163	14.11	1.00	1.170	14.72	1.00
102	153	1.00	1.209	14.91	1.00	1.212	15.39	1.00
150	165	0.919	1.153	11.87	0.84	1.154	12.55	0.88
102	195	0.89	1.147	10.38	0.77	1.153	12.70	0.89
90	105	0.89	1.147	12.52	0.89	1.154	12.58	0.88
90	180	0.53	0.795	7.62	0.93	1.013	10.75	0.86

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