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Numerical and experimental investigation of basin for a DC bus system with delayed feedback control

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Abstract—The present paper investigates a transient stability of a DC bus system with delayed feedback control for a step type variation of DC load. The transient stability of the DC bus system depends on the size of basin of its equilibrium point. It is numerically and experimentally shown that the delayed feedback controller can increase the size of basin.

1. Introduction

Considerable research on alternating-current (AC) power-grid networks has been done in the field of nonlinear science [1, 2]. On the other hand, direct-current (DC) power systems have been gaining attention because of rapid progress in DC sources (e.g., solar PVs and fuel cells) and DC loads (e.g., information equipment). The DC power systems are now expected as a future power transmission style [3–5]. However, as the power electronic converters, which are used everywhere in DC power systems, behave as constant power loads (CPLs), these converters cause the destabilization of DC bus line voltage [6]. In the field of power electronics, numerous research have focused on suppression of the destabilization [7–10].

Delayed feedback control has been popular as a method for stabilizing unstable periodic orbits or unstable equilibrium points embedded within chaotic systems [11, 12]. This control has the following advantages: its control law does not require the location of the orbits or the equilibrium points; its signal converges on zero after stabilization; the orbits or the equilibrium points can be tracked even if their locations are slowly moved due to varied system parameters [12–14]. Our previous paper analyzed dynamics of a simple DC bus system from a viewpoint of bifurcation theory, and showed that delayed feedback control can suppress the destabilization in bus line by stabilizing an equilibrium point (i.e., an operating point) [15].

In practical DC bus systems, the power consumption of CPLs must be considered to be varied in time due to users' demands. A variation of power consumption causes a movement of the equilibrium point. As a delayed feedback controller has a potential to track such equilibrium point, our previous report [16] investigated its tracking performance by using frequency domain analysis. Although a basin of attraction of the equilibrium point is strongly related to transient stability of a DC bus system with time-varying consumption, the previous report did not consider the basin.

The purpose of the present paper is to evaluate the transient stability on the basis of the size of basin. First, we numerically demonstrate that the delayed feedback controller can improve the transient stability of a DC bus system in the case that its DC power consumption is varied as a step function. Second, we numerically and experimentally show that the size of basin becomes larger owing to the delayed feedback controller.

2. DC bus system with time-varying CPL

A simplified DC bus system with delayed feedback control is illustrated in Fig. 1. The dotted line indicates the delayed feedback controller. E is the voltage of a DC power source. r, L, and C represent the equivalent resistance, the equivalent inductance, and the equivalent capacitance, respectively. $v_P(t)$ and $i_L(t)$ denote the bus line voltage and the current through L. The CPL consumes the time-varying power $P(t)$ which satisfies

$$i_P(t) = \frac{P(t)}{v_P(t)}, \ \forall t \geq 0, \tag{1}$$

where $i_P(t)$ is the current through the CPL. The delayed feedback controller measures $v_P(t)$, and then outputs the control current,

$$i_u(t) = \frac{1}{r_k} \{v_P(t - \Gamma) - v_P(t)\}. \tag{2}$$

Here $i_u(t)$ is proportional to the difference between $v_P(t)$ and $v_P(t - \Gamma)$, where $\Gamma \geq 0$ is the delay time.

The dynamics of the DC bus system can be reduced to a dimensionless form,

$$\begin{cases} \dfrac{dx}{d\tau} = -\dfrac{a(\tau)}{x} + by + u \\ \dfrac{dy}{d\tau} = -x - by + 1 \end{cases}, \tag{3}$$

where u is the control signal,

$$u = k(x_T - x). \tag{4}$$

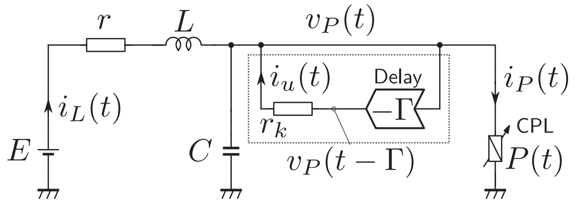


Figure 1: Simplified DC bus system with delayed feedback control.

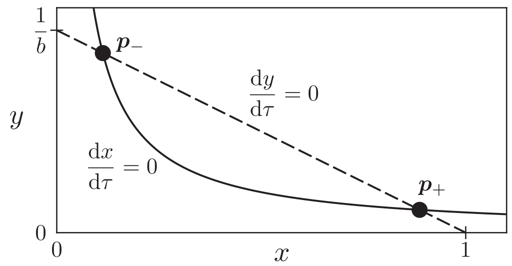


Figure 2: Nullclines and equilibrium points.

k is the feedback gain, and T is the delay time. The state variables and time are transformed as

$$x := \frac{v_P}{E},\ x_T := \frac{v_P(t-\Gamma)}{E},\ y := \frac{Li_L}{rCE},\ u := \frac{ri_u}{E}, \tag{5}$$

$$\tau := \frac{t}{rC},\ T := \frac{\Gamma}{rC}. \tag{6}$$

The system parameters are transformed as

$$a(\tau) := \frac{rP(rC\tau)}{E^2},\ b := \frac{r^2C}{L},\ k := \frac{r}{r_k}. \tag{7}$$

Remark that the time-varying parameter $a(\tau)$ is proportional to the time-varying power $P(t)$.

3. Response for time varying $a(\tau)$

Let us consider the dynamics of DC bus system (3) without control ($u \equiv 0$). Figure 2 shows the nullclines (i.e., sets of $\mathrm{d}x/\mathrm{d}\tau = 0$ and $\mathrm{d}y/\mathrm{d}\tau = 0$) and the two equilibrium points. The locations of these equilibrium points, which depend on the time varying parameter a,

$$\boldsymbol{p}_+(a) := [x_+^*(a), y_+^*(a)]^{\mathrm{T}},\ \boldsymbol{p}_-(a) := [x_-^*(a), y_-^*(a)]^{\mathrm{T}},$$
$$x_\pm^*(a) := \frac{1}{2}\left(1 \pm \sqrt{1-4a}\right),\ y_\pm^*(a) := \frac{-x_\pm^*(a)+1}{b}, \tag{8}$$

do not move even if controller (4) is added to the system. However, controller (4) can change the local stability of $\boldsymbol{p}_\pm(a)$. The instability of $\boldsymbol{p}_-(a)$ is demonstrated in our previous study [15]; thus, the present paper focuses only on the stability of $\boldsymbol{p}_+(a)$.

The parameter b is set to 0.33. The locations of $\boldsymbol{p}_+(a)$ are plotted as ◇, ○, and × for $a = a_{\mathrm{H}} = 0.18$, $a = a_{\mathrm{L}} = 0.13$, and $a = a_{\mathrm{L}} = 0.11$, respectively, as shown in Fig. 3. It should be emphasized that $\boldsymbol{p}_+(a)$ moves only on the nullcline $\mathrm{d}y/\mathrm{d}\tau = 0$ for any a, since the nullcline does not depend on a. Throughout this paper, the parameter a is supposed to be varied as

$$a(\tau) = \begin{cases} a_{\mathrm{L}} & (\tau < \overline{\tau}) \\ a_{\mathrm{H}} & (\tau \geq \overline{\tau}) \end{cases}, \tag{9}$$

where $\overline{\tau}$ is the time when a jumps from a_{L} to a_{H}.

We now consider the following situation: both $\boldsymbol{p}_+(a_{\mathrm{L}})$ and $\boldsymbol{p}_+(a_{\mathrm{H}})$ without control are stable. Here $a_{\mathrm{H}} = 0.18$ and $\overline{\tau} = 50$ are fixed. For $a_{\mathrm{L}} = 0.13$, the trajectory $[x, y]^{\mathrm{T}}$ after jumping, which is plotted as a solid black line in Fig. 3(a), converges on $\boldsymbol{p}_+(a_{\mathrm{H}})$. For $a_{\mathrm{L}} = 0.11$, the trajectory $[x, y]^{\mathrm{T}}$ after jumping (see the dotted black line) does not converge on $\boldsymbol{p}_+(a_{\mathrm{H}})$. This fact indicates that the load variation with large amplitude may lead to the voltage collapse. The difference between these convergence and divergence is due to the initial value (i.e., the location of $\boldsymbol{p}_+(a_{\mathrm{L}})$). This is because $\boldsymbol{p}_+(a_{\mathrm{H}})$ has its own basin of attraction. The boundary of this basin is plotted as a bold blue line, which is equivalent to the unstable periodic orbit with $a = a_{\mathrm{H}} = 0.18$. If an initial value (i.e., $\boldsymbol{p}_+(a_{\mathrm{L}})$) is within the basin, then the trajectory converges on $\boldsymbol{p}_+(a_{\mathrm{H}})$, otherwise it does not converge. Therefore, we notice that the size of basin is strongly related to robustness of DC bus system with time-varying parameter (9).

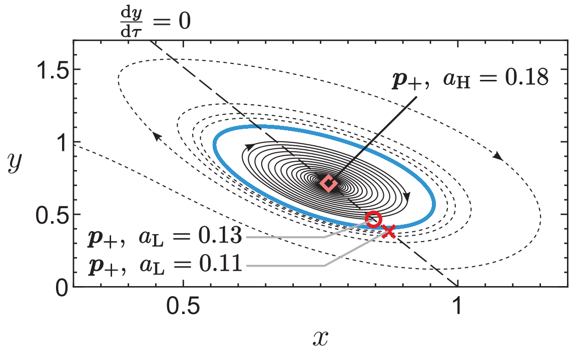
Now, controller (4) is connected to the DC bus system. The controller parameters are set to $k = 0.1$ and $T = 5$. For $a_{\mathrm{L}} = 0.11$, the trajectory converges on $\boldsymbol{p}_+(a_{\mathrm{H}})$ as shown in Fig. 3(b). This result demonstrates that controller (4) improves the transient stability of the DC bus system. In general, the basin of time-delay dynamical systems is expressed as a set of time functions (i.e. initial functions). Therefore, it is quite difficult to analytically investigate the basin of $\boldsymbol{p}_+(a_{\mathrm{H}})$ with delayed feedback control (4). The basin is numerically and experimentally investigated in the next section.

4. Size of basin

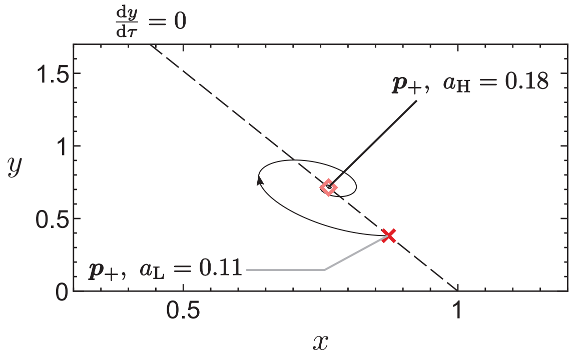
This section investigates the size of basin of the controlled equilibrium point $\boldsymbol{p}_+(a_{\mathrm{H}})$. In order to simplify our investigation, we employ the following assumptions: (A-1) $a(\tau)$ varies as Eq. (9) with $\overline{\tau} \gg T$. (A-2) both $\boldsymbol{p}_+(a_{\mathrm{L}})$ and $\boldsymbol{p}_+(a_{\mathrm{H}})$ can be stabilized by controller (4); (A-3) the controlled trajectory $[x, y]^{\mathrm{T}}$ before jumping remains on $\boldsymbol{p}_+(a_{\mathrm{L}})$ for a long time. From (A-1) and (A-3), we just have to consider the dynamics of controlled DC bus system (3) (4) with $a = a_{\mathrm{H}}$ from the initial conditions,

$$\left[x(\tau),\quad y(\tau)\right]^{\mathrm{T}} = \boldsymbol{p}_+(a_{\mathrm{L}}),\ \forall\tau \in [\overline{\tau} - T,\ \overline{\tau}]. \tag{10}$$

We will numerically and experimentally find the set of initial points $\boldsymbol{p}_+(a_{\mathrm{L}})$ in condition (10) such that the controlled trajectory $[x, y]^{\mathrm{T}}$ converges on $\boldsymbol{p}_+(a_{\mathrm{H}})$. As $\boldsymbol{p}_+(a_{\mathrm{L}})$ moves only on the nullcline $\mathrm{d}y/\mathrm{d}\tau = 0$ for any a_{L}, the set can be



(a) Without control



(b) With control ($k = 0.1,\ T = 5$)

Figure 3: Trajectories of the DC bus system with time-varying parameter (9): (a) trajectories without control just before and after jumping for $a_L = 0.13$ and $a_L = 0.11$, (b) trajectories with control for $a_L = 0.11$.

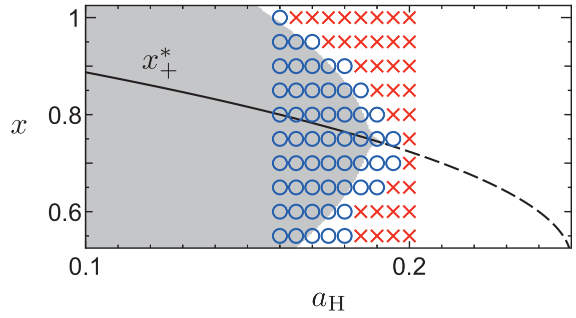
described by the range of x component of $\boldsymbol{p}_+(a_L)$. It is obvious that the range of x_0 corresponds to the size of basin. In the following subsections, we will use the initial point,

$$\boldsymbol{p}_+(a_L) = \begin{bmatrix} x_0, & (-x_0 + 1)/b \end{bmatrix}^T, \tag{11}$$

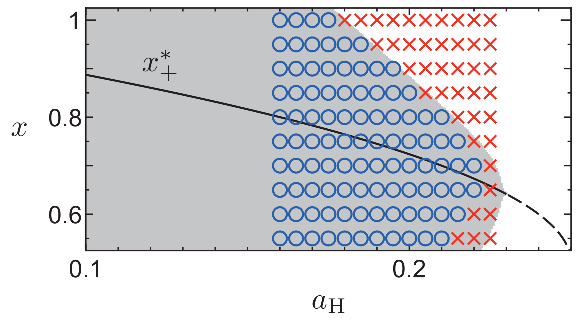
where $x_0 \in \mathbb{R}$ is the x component of $\boldsymbol{p}_+(a_L)$.

4.1. Numerical simulations

The size of basin is numerically estimated with the parameters $a_H \in [0.1, 0.25]$ and $x_0 \in [0.525, 1.025]$. The estimated range without control is shown in Fig. 4(a). The x component of $\boldsymbol{p}_+(a_H)$, that is x_+^*, is plotted: solid line (stable) and broken line (unstable). The gray area presents a set of parameters (a_H, x_0) where the trajectories converge on $\boldsymbol{p}_+(a_H)$. We also estimate the size with delayed feedback control as shown in Fig. 4(b). We can see that controller (4) stabilizes the unstable $\boldsymbol{p}_+(a_H)$ for $a_H \in [0.193,\ 0.230]$. The controller also makes the size of range wider for $a_H < 0.193$. These facts suggest that the DC bus system with delayed feedback control is robust for the step type variation of consumption.



(a) Without control



(b) With control ($k = 0.1,\ T = 5$)

Figure 4: Numerical and experimental estimation of the size of range with $b = 0.33$. Gray area (simulations) and circles (experiments): trajectories converge on $\boldsymbol{p}_+(a_H)$. White area (simulations) and crosses (experiments): trajectories do not converge on $\boldsymbol{p}_+(a_H)$.

4.2. Circuit experiments

This subsection demonstrates the above-mentioned numerical results by circuit experiments.

Figure 5 shows our experimental diagram of DC bus system. The DC source voltage and the passive devices are set to $E = 18.0$ V, $r = 22.2\ \Omega$, $L = 22.8$ mH, and $C = 15.4\ \mu$F, which correspond to $b = 0.33$ (see Eq. (7)). The CPL is implemented as illustrated in Fig. 6. This circuit consists of the Zener diode (1N5357BRLG, ON Semiconductor)[1], the switching regulator (LM2675-5.0EVAL, Texas Instruments)[2], the load resistance $R_L = 6\ \Omega$, the amplifier, and the voltage source $e(t)$. The regulator maintains the output voltage at 5 V regardless of the input voltage $v_p(t)$. The amplifier operates as a voltage buffer. Thus, the consumption of the CPL is given by

$$P(t) = v_P(t) i_P(t) = \frac{5}{\eta R_L} \{5 - 0.5 e(t)\}, \tag{12}$$

under condition $e(t) < 10$, where $\eta = 0.9$ is the efficiency of power conversion of the regulator. Equation (12) implies that $P(t)$ can be controlled by adjusting $e(t)$.

[1] The bus line voltage is restricted by the Zener voltage 20V to avoid high voltage.

[2] The regulator's capacitor for filtering input signals is replaced by 5μF capacitors (204N3502 105K4, MATSUO ELECTRIC).

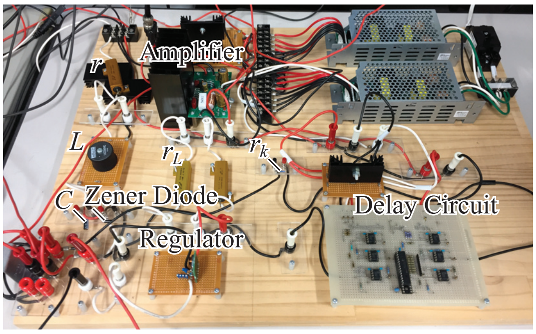


Figure 5: Experimental diagram of DC bus system.

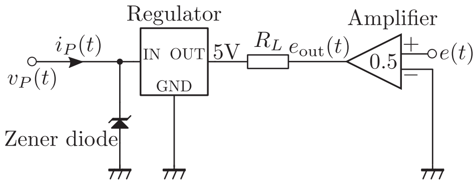


Figure 6: CPL circuit.

The delay unit of controller in Fig. 1 is implemented by using a PIC (PIC18F2550-I/SP, Microchip Technology) and op-amps. The PIC, which has an analog-to-digital converter, measures the voltage $v_P(t)$ at sampling intervals of 25μs. The measured voltage is stored in the memory of PIC as 8-bit digital data. The data is delayed via the algorithm of first-in-first-out queue. The exported data is transformed to an analog voltage by a digital-to-analog converter. The size of queue is proportional to the length of delay time. The parameters of the implemented controller are set to $r_k = 220\ \Omega$ ($\leftrightarrow k = 0.1$) and $\Gamma = 1.7$ ms ($\leftrightarrow T = 5$).

Experimental results are plotted in Fig. 4. These results are obtained by the following procedure; a is set to a_{H} by tuning P; x_0 is fixed by connecting an external voltage source to the bus line; the source is disconnected; the trajectory $[v_P, i_L]^{\mathrm{T}}$ is observed; the above procedure is repeated with different a_{H} and x_0. The circles (crosses) represent the point (a_{H}, x_0) where the trajectory converges (does not converge) on $\boldsymbol{p}_+(a_{\mathrm{H}})$. From Figs. 4(a) and 4(b), we can see that the number of circles increases owing to delayed feedback control. In addition, the experimental results almost agree with the numerical simulations.

5. Conclusion

We have shown that the transient stability of a DC bus system for a step type variation of DC load depends on the size of basin of its equilibrium point. In addition, it has been demonstrated that the delayed feedback controller makes the size larger. These results have been verified numerically and experimentally.

Acknowledgments

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