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Modeling of Competitive Networks using Coupled Chaotic Circuits

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Abstract—Coupled oscillatory circuits provide simple models for high-dimensional nonlinear real-world phenomena. Synchronization, in particular, is one of the most important features that can be described and explored with the help of oscillators, because, upon their coupling, strongly correlated rhythms emerge among the oscillators, called synchronized states. Here, we present results obtained for a new paradigm of coupled oscillatory networks, called competitive interaction networks.

1. Introduction

Coupled oscillatory circuits provide simple models for describing high-dimensional nonlinear phenomena occurring in our everyday world. Synchronization, in particular, is one of the most important features that can be described and explored with the help of oscillators, because, upon their coupling, strongly correlated rhythms among the oscillators emerge, called synchronized states. Therefore, many researchers have proposed different coupled oscillatory networks and have discovered many interesting synchronization phenomena [1], [2].

Recently, synchronization in complex networks with different types of interactions has been extensively investigated for understanding important role played by the interactions. This is because interactions in networks leads to the emergence of key synchronization phenomena, especially competitive coupling can be observed in real world networks. In our research group, we have investigated synchronization phenomena in complex networks by using analog electrical oscillators [3]-[6]. Giron et al. investigated synchronization phenomena in plant communities where usually both facilitation and competition coexist, playing a key role in the structure and organization of these communities. They showed that synchronization provides an efficient way to unveil how species sharing facilitative interactions group of the plant community [7].

However, a node in a complex network is expressed by a mathematical model in most studies of synchronization of complex networks with competitive interactions. Although, it is very important to use mathematical model for complex networks in order to understand the synchronization states by approaching theoretical methods, we also need to consider physical models for future engineering applications.

In this study, we focus on synchronization state observed in two networks of chaotic circuits which are coupled in one direction hierarchically. We analyze the role of synchronization by changing the competitive coupling strategies. A schematic competitive network is shown in Fig. 1.

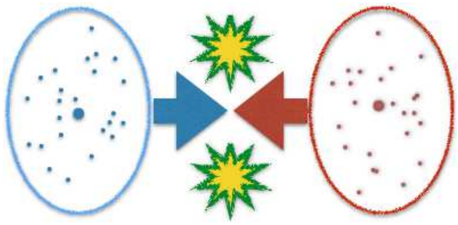


Figure 1: A schematic competitive network.

2. Network Model

Figure 2 shows the proposed network model of this study. There are two hierarchical networks (Group A and Group B) including the directed and undirected connections. Each network consists of 11 chaotic circuits and the adjacent chaotic circuits are coupled by resistors. The coupling strength in the group is defined as δ_{in} and the coupling strength between the groups is defined as δ_{out}. The coupling methods of the directed and undirected connections are shown in Fig. 3.

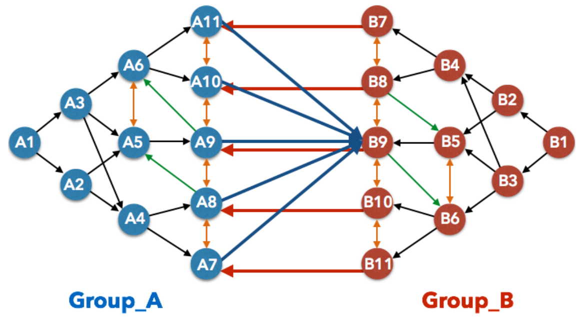


Figure 2: Network model.

We define two types of competitive coupling between two hierarchical networks. One is the concentrated attack which the lowest five chaotic circuits (A7 - A11) in Group A attack to only one chaotic circuit (B9) in Group B. Another one is the distributed attack which the lowest five chaotic circuits (A7 - A11) in Group B attack to in front of the chaotic circuits (B7 - B11) one by one in Group B.

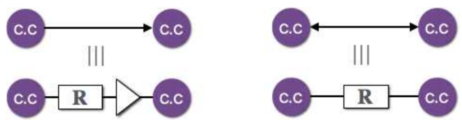


Figure 3: Coupling method.

In the proposed network, each node is expressed by the chaotic circuit. Figure 4 shows the chaotic circuit which is three-dimensional autonomous circuit proposed by Shinriki et al. in Refs. [8], [9]. This circuit is composed by an inductor, a negative resistance, two condensers and dual-directional diodes. The circuit generates asymmetric attractor as shown in Fig. 5. In this simulation, the attractor of Fig. 5 (a) is used to Group-A, and the attractor of Fig. 5 (b) is used to Group-B.

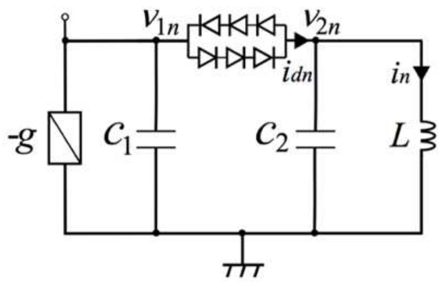
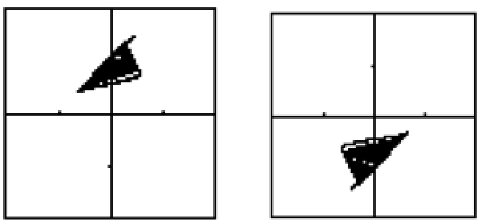


Figure 4: Chaotic circuit.



(a) Group-A. (b) Group-B.

Figure 5: Attractors ($\alpha = 0.4,\ \beta = 20.0,\ \gamma = 0.5$).

Next, we develop the expression for the circuit equations of the circuit model as shown in Fig. 4. The $i - v$ characteristics of the nonlinear resistor are approximated by the following three-segment piecewise-linear function,

$$i_{dn} = \begin{cases} G_d(v_{1n} - v_{2n} - V) & (v_{1n} - v_{2n} > V) \\ 0 & (|v_{1n} - v_{2n}| \le V) \\ G_d(v_{1n} - v_{2n} + V) & (v_{1n} - v_{2n} < -V) \end{cases} \quad (1)$$

The normalized circuit equations governing the circuit are expressed as follows.

$$\begin{cases} \dfrac{dx_n}{d\tau} = z_n \\ \dfrac{dy_n}{d\tau} = \alpha\gamma y_n - \alpha f(y_n - z_n) - \alpha\delta \displaystyle\sum_{k \in S_n} (y_n - y_k) \\ \dfrac{dz_n}{d\tau} = f(y_n - z_n) - x_n. \end{cases} \quad (2)$$

where

$$t = \sqrt{LC_2}\tau, \quad i_n = \sqrt{\frac{C_2}{L}} V x_n, \quad v_{1n} = V y_n, \quad v_{2n} = V z_n,$$

$$\alpha = \frac{C_2}{C_1}, \quad \beta = \sqrt{\frac{L}{C_2}} G_d, \quad \gamma = \sqrt{\frac{L}{C_2}} g, \quad \delta = \frac{1}{R}\sqrt{\frac{L}{C_2}}.$$

where $n = 1, 2, 3, ..., 22$ and S_n is set to nodes which are connected to chaotic circuits. The nonlinear function $f()$ corresponds to the $i - v$ characteristics of the nonlinear resistors consisting of the diodes and are described as follows:

$$f(y_n - z_n) = \begin{cases} \beta(y_n - z_n - 1) & (y_n - z_n > 1) \\ 0 & (|y_n - z_n| \le 1) \\ \beta(y_n - z_n + 1) & (y_n - z_{nn} < -1) \end{cases} \quad (3)$$

For the computer simulations, we calculate Eq. (2) using the fourth-order Runge-Kutta method with the step size $h = 0.005$.

3. Synchronization Phenomena

3.1. Attractors

Figure 6 shows the simulation results of the obtained attractors by changing the coupling strength δ_{out}. When the coupling strength δ_{out} is set to 0.1, the attractor of the chaos circuit (B9) in Group-B which is attacked from Group-B is hardly damaged. And, the attractors of the chaos circuits (A7-A11) located lowest layer in Group-A become bit small from original attractor.

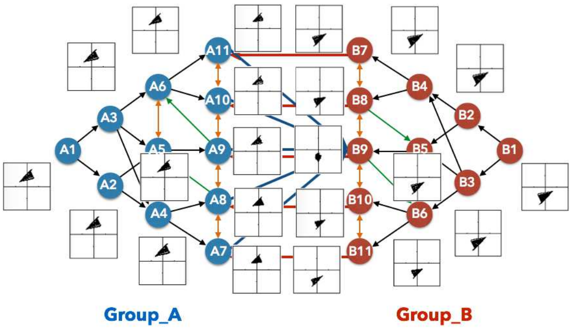
In the case of $\delta_{out} = 0.3$, the chaos circuits (A7-A11) located lowest layer in Group-A are all damaged. And, the chaos circuits (A6, B6) located in second lowest layer in Group-A and B are also damaged.

As increasing the coupling strength, the chaos circuits located in lowest and second lowest layer in Group-A and B are damaged. However, the chaos circuits (B8, B10) located in lowest layer in Group-B is recovered thier attractor.

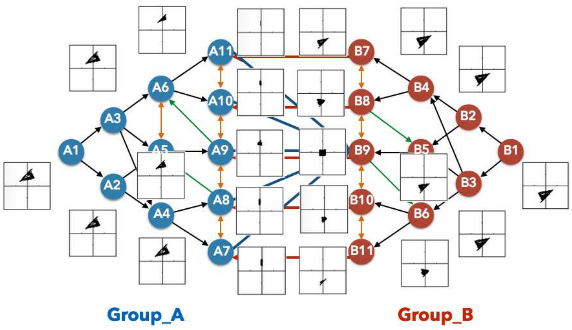
3.2. Synchronization Ratio in Each Group

In this section, we fix a certain time interval as $\tau = 100,000$ and step size $h = 0.005$. We calculate the synchronization states observed from 22 coupled chaotic circuits by using the following equation,

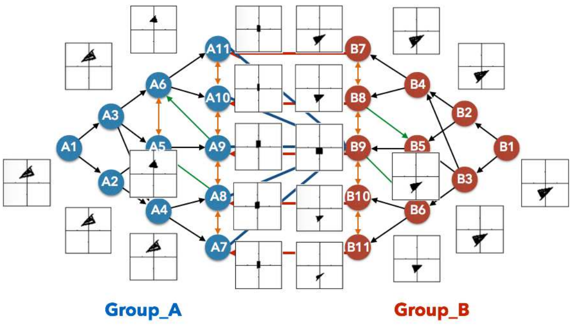
$$|y_n - y_k| = 0.01 (k \in S_n). \quad (4)$$



(a) δ=0.1.



(b) δ=0.3.



(c) δ=0.5.

Figure 6: Attractors.

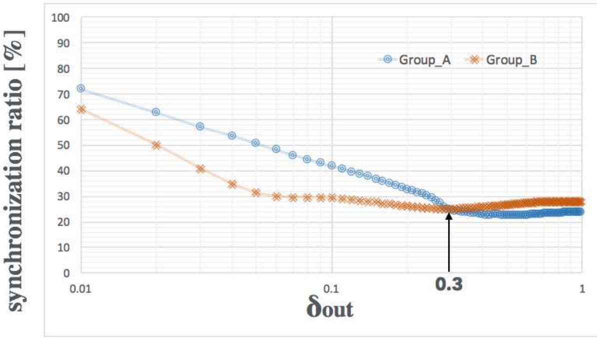


Figure 7: Synchronization ratio.

3.3. Synchronization Ratio in Each Edge

Finally, we investigate synchronization ratio in each edge. The definition of edge number is shown in Fig. 8. Figure 9 shows the simulation results with the competitive coupling strength δ_{out}. When δ_{out} is smaller than 0.1, there is big difference of synchronization ratio between Group-A and B around e12 to e19 which are located the lowest and second lowest layers.

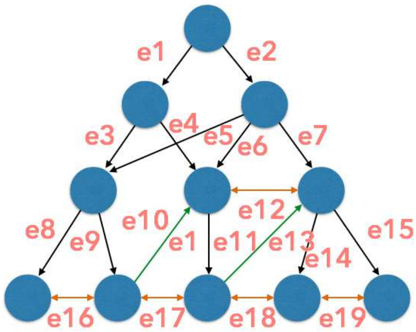


Figure 8: Definition of edge number.

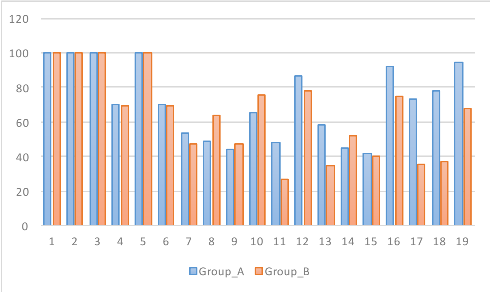
Figure 7 shows the coupling strength dependency of global synchronization in each group. The horizontal axis denotes the coupling strength δ_{out} and the vertical axis denotes the synchronization ratio. The blue line with circle mark shows the synchronization ratio of Group-A. The red line with cross mark shows the synchronization ratio of Group-B. When the competitive coupling strength is small, the synchronization ratio of Group-A is higher than Group-B. The both synchronization ratio decrease with δ_{out}, and the two graphs are crossed when δ_{out} is 0.3. After that, the synchronization ratio of Group-B is bit higher than Group-A.

From this result, we confirm that the concentrated attack is effective when the competitive coupling strength is small. While, the distributed attack is effective when the competitive coupling strength becomes strong.

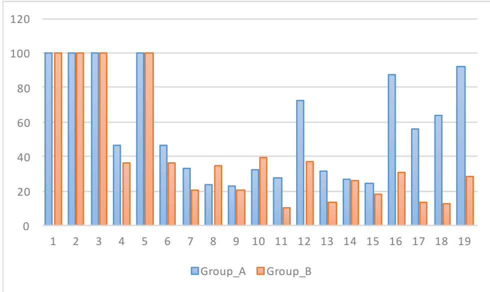
4. Conclusions

In this study, we have investigated synchronization state observed in two networks of chaotic circuits which are coupled in one direction hierarchically. We analyze the role of synchronization by changing the competitive coupling strategies. By using the computer simulations, we confirm that the concentrated attack is effective when the competitive coupling strength is small. While, the distributed attack is effective when the competitive coupling strength becomes strong.

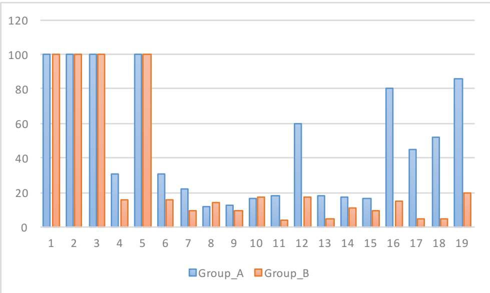
In our future works, we would like to investigate effect of competitive strategies and apply this model to more complex networks such as smart grid network and social network.



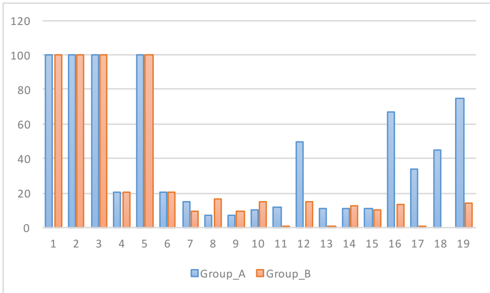
(a) δ_{out}=0.01.



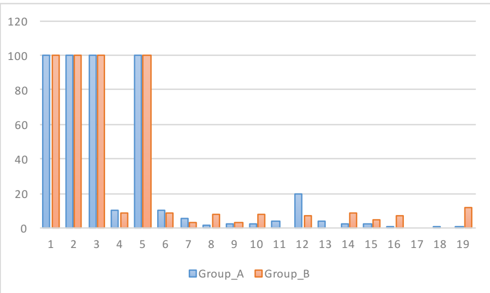
(b) δ_{out}=0.03.



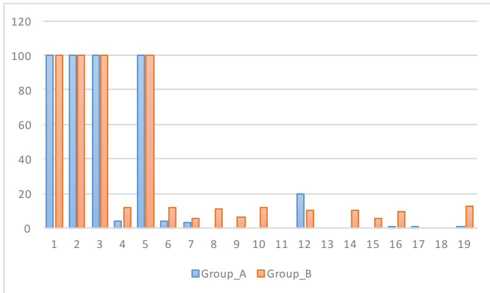
(c) δ_{out}=0.06.



(d) δ_{out}=0.10.



(e) δ_{out}=0.30.



(f) δ_{out}=0.50.

Figure 9: Synchronization ratio in each edge.

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