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Obstacle Avoidance of 4-wheeled Vehicle Using Receding Horizon Control with iLQG Method Considering Computational Delay

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Abstract—We consider an obstacle avoidance problem of 4-wheeled vehicle. We represent the obstacle by a repulsive potential function and introduce a control cost function evaluating the state and the input. Then, we formulate the problem as a receding horizon control problem where the sum of both the repulsive function and the control cost function over the given time interval is optimized on-line. We also take into consideration a delay for computation of the control input and use an iLQG based method. By computer simulation, it is shown that the vehicle heading towards the target position along an acceptable path with avoiding the obstacle.

1. Introduction

With the development of autonomous cars, a problem of the automatic generation of an optimal path with avoiding obstacles has been paid much attention to. The method of generating the obstacle avoiding trajectories can be classified in two different points of view. One is based on the road map decomposition, which can be classified into a global and local decomposition[1]. The other is based on the avoidance manoeuvre concepts such as the potential field method, the vector histogram method, the curvature-velocity mothed, and artificial intelligence tools.The potential field method generates the obstacle avoidance trajectory under the influence of an artificial potential produced by the goal point and the obstacles, which can be described by the potential function[2, 3].

On the other hand, the on-line generation of an optimal trajectory is an important issue in robotics. The receding horizon control(RHC) is a useful approach to the on-line trajectory optimization. To reduce the computational time for solving the optimal control problem, the trajectory optimization approach is useful where differential dynamic programming (DDP) is one of the well-studied methods[4]. Recently, the iterative linear quadratic Gaussian (iLQG) method has been proposed, which is a simpler variant of DDP[5]. In this method, the first derivatives of dynamics are used so that the computation time for solving the optimal control problem is reduced with guaranteeing the precision of the optimal solution. Recently, iLQG based RHC method with computational delay has been proposed[6].

In this paper, we propose an iLQG based RHC for the optimal trajectory generation under the existence of obstacles. We represent the obstacle by a repulsive potential function and introduce a control cost function evaluating the state and the input. Then, we formulate the problem as a RHC problem where the sum of both the repulsive function and the control cost function over the given time interval is optimized on-line. We apply the iLGQ method taking into consideration the computational delay of the control input to solve the optimization problem.

2. Four-Wheeled Vehicle

In this section, we consider a four-wheeled vehicle as shown in Fig. 1, which is constrained by its position and velocity. Let (r_x, r_y) be the coordinate of the midpoint between the two rear tires, θ denotes the angle between r_x axis and the vertical direction of the vehicle representing the vehicle traveling direction, and ϕ represents a steering angle. The state variable is defined by $x = [r_x \quad r_y \quad \theta \quad \phi]^T$. Let u_v and u_w be the vehicle traveling velocity and the angular velocity of steering as the inputs for the system. Then the state equation of the four-wheeled vehicle is written by

$$\frac{d}{dt}\begin{bmatrix} r_x \\ r_y \\ \theta \\ \phi \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ \frac{1}{2W}\tan\phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_v \\ u_w \end{bmatrix}, \tag{1}$$

where $2W$ represents the distance between the front and rear tires. Taking the following coordinate and control in-

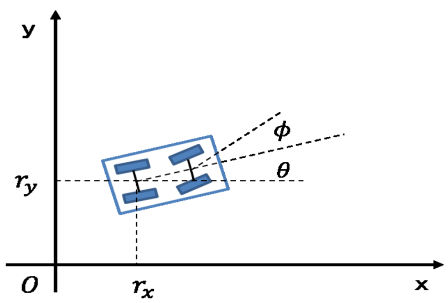

Figure 1: Four-wheeled vehicle.

put transformation,

$$\begin{cases} \xi_1 = r_x, \\ \xi_2 = \dfrac{1}{2W}\sec^3\theta\tan\phi, \\ \xi_3 = \tan\theta, \\ \xi_4 = r_y, \\ u_v = \sec\theta v_1, \\ u_w = -\dfrac{3}{2W}\sin^2\phi\tan\theta\sec\theta v_1 + 2W\cos^2\phi\cos^3\theta v_2, \end{cases}$$

we have the following time-state control form:

$$\frac{ds}{dt} = w, \tag{2a}$$

$$\frac{dz}{ds} = Az + Bu, \tag{2b}$$

where $w = v_1$, $u = v_2/v_1$, and

$$z = \begin{bmatrix} \xi_2 \\ \xi_3 \\ \xi_4 \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

3. Obstacle avoidance problem

3.1. Obstacle

The general structure of potential function E is given by

$$E = E_{att} + E_{rep}, \tag{3}$$

where E_{att} denotes the attractive potential of the goal point which pulls the car toward them, and E_{rep} represents the repulsive potential of the obstacles which pushes the car away from them. The balance between the two potentials deform the desired trajectory and transforms it into an obstacles avoidance trajectory. In other words, finding an appropriate potential function is critical. The attractive potential function generally described by two types of attractive wells: a parabolic and a conical well. The purpose of the repulsive potential function is to create a repulsive potential barrier around the obstacle region, which won't affect the motion of the car when it is sufficiently far away from the obstacles. Thus, the repulsive potential functions depend on various shapes of the obstacles.

By setting the potential function E described above as the cost function for the each step in the finite-horizon optimal control problem in the RHC method, we obtain a desired obstacle avoidance trajectory.

In this paper, we set a single rectangular obstacle with L and H denoting the lengths of the long and short sides of obstacle, respectively. We define the cost function J as the sum of attractive potential function E_{att} and the repulsive potential function E_{rep}.

In order to define the repulsive potentials around the rectangular obstacle, the n-ellipsoids is used which is first proposed by Khatib[7]. $\xi_d(x, y)$ denotes the distance from current state to the center coordinate of the rectangular obstacle. For every given constant value W, the curve $\xi_d(x, y) = W$ is taken as a repulsive equipotential.

$$\xi_d(x, y) = \left[\left(\frac{x}{a}\right)^{2d} + \left(\frac{b}{a}\right)^2\left(\frac{y}{b}\right)^{2d}\right]^{\frac{1}{2d}} - 1, \tag{4}$$

$$W(n) = \left[\left(\frac{i - i_m}{a}\right)^{2d} + \left(\frac{b}{a}\right)^2\left(\frac{\xi_4(n) - x_m}{b}\right)^{2d}\right]^{\frac{1}{2d}} - 1, \tag{5}$$

where a and b are the semi-major axis and the semi-minor axis, respectively, given by

$$a = \frac{L}{2}(2^{\frac{1}{2d}}) \quad \text{and} \quad b = \frac{H}{2}(2^{\frac{1}{2d}}), \tag{6}$$

and d is defined by

$$d = \frac{1}{1 - e^{-\beta W}}, \tag{7}$$

and β is an adjustable parameter. The equipotential tends toward a circle as $W \to \infty$ and $d \to 1$. It tends toward the boundary of the obstacle as $W \to 0$ and $d \to \infty$. Thus, we can define a repulsive potential function as follows,

$$E_{rep}(W) = \eta\frac{e^{-\alpha W}}{W}, \tag{8}$$

where the parameter α determines how rapidly the potential increases near the object.

3.2. Cost function

Consider a 4-wheeled vehicle which is described by a time-state control form (2). As taking the first input of the system as the time axis, we set the time that the obstacle appears $[i_{min}, i_{max}]$ as the length of the obstacle to describe the repulsive equipotential in (4). And since the last variable ξ_4 denotes the position of the vehicle, we set z_{min} and z_{max} represents the position of the obstacle. Then the central coordinate (i_m, z_m) in (4), the length L and the height H of the obstacle in (6) are given by

$$i_m = \frac{i_{max} + i_{min}}{2}, \qquad z_m = \frac{z_{max} + x_{min}}{2},$$

$$L = i_{max} - i_{min}, \qquad H = z_{max} - z_{min}.$$

The attractive potential function is described by

$$E_{att} = z^T(N)Pz(N) + \sum_{j=0}^{N-1} z^T(j)S\,z(j) + Ru^2(j), \tag{9}$$

where $z(n)$ and $u(n)$ denote the state and input of the car, and P and S are positive definite matrices representing the weights of the state z, and R is a positive definite matrix representing the weight of the control input u.

$$J_0 = E_{att} + E_{rep}(W), \tag{10}$$

3.3. iLQG method for Obstacle Avoidance Problem

To apply the proposed RHC method to the control of the 4-wheeled vehicle, we discretize (2b) by the Euler's discretization as follows.

$$z(n+1) = z(n) + h(Az(n) + Bu(n)), \tag{11}$$

where h represent the step size. By applying the cost function (10), the iLQG method to solve the finite-time horizon optimal control problem is performed by the following steps.

a. Derivatives: Compute the derivatives of $L = z^T S z + Ru^2 + E_{rep}$ and $f = z + h(Az + Bu)$:

$$\begin{cases} L_z = (S + S^T)z + \begin{bmatrix} 0 & 0 & NN \cdot M \end{bmatrix}^T, \\ L_{zz} = S + S^T + \begin{bmatrix} 0 & 0 \\ 0 & M^2 \cdot NNK + NN \cdot MK \end{bmatrix} \end{cases}$$

$$\begin{cases} V_z = (P + P^T)z + \begin{bmatrix} 0 & 0 & NN \cdot M \end{bmatrix}^T, \\ V_{zz} = P + P^T + \begin{bmatrix} 0 & 0 \\ 0 & M^2 \cdot NNK + NN \cdot MK \end{bmatrix}, \end{cases}$$

$$\begin{cases} L_u = 2Ru, \\ L_{uu} = 2R, \\ L_{uz} = 0, \end{cases} \quad \begin{cases} f_z = I_3 + hA, \\ f_u = hB, \\ f_{zz} = f_{uu} = f_{uz} = f_{zu} = 0, \end{cases}$$

where

$$NN(n) = \frac{\partial E_{rep}}{\partial W} = -\eta e^{-\alpha W(n)} \left(\frac{\alpha}{W(n) + \epsilon} + \frac{1}{(W(n) + \epsilon)^2} \right),$$

$$M(n) = \frac{\partial W}{\partial z} = (\xi_4(n) - z_m)^{2d-1} \left(\frac{b}{a}\right)^2 \left(\frac{1}{b}\right)^{2d} \left[\left(\frac{b^2}{a}\right) \left(\frac{\xi_4(n) - z_m}{b}\right)^{2d} \right]^{\frac{1}{2d}-1},$$

$$NNK(n) = \frac{\partial^2 E_{rep}}{\partial W^2} = \eta e^{-\alpha W(n)} \left[\frac{\alpha^2}{W(n) + \epsilon} + \frac{2\alpha}{(W(n) + \epsilon)^2} + \frac{2}{(W(n) + \epsilon)^3} \right],$$

$$\begin{aligned} MK(n) &= \frac{\partial^2 W}{\partial z^2} \\ &= (1 - 2d)\, (\xi_4(n) - z_m)^{2(2d-1)} \left(\frac{b}{a}\right)^4 \left(\frac{1}{b}\right)^{4d} \left[\left(\frac{b^2}{a}\right) \left(\frac{\xi_4(n) - z_m}{b}\right)^{2d} \right]^{\frac{1}{2d}-2} \\ &\quad + (2d - 1)\, (\xi_4(n) - z_m)^{2d-1} \left(\frac{b}{a}\right)^2 \left(\frac{1}{b}\right)^{2d} \left[\left(\frac{b^2}{a}\right) \left(\frac{\xi_4(n) - z_m}{b}\right)^{2d} \right]^{\frac{1}{2d}-1}. \end{aligned}$$

b. Backward Pass: Iteratively calculate the following coefficient equations from $n = N - 1$:

$$\begin{aligned} Q_z(n) &= L_z(n) + f_z^T(n) V_z(n+1) \\ &= (S + S^T)z(n) + \begin{bmatrix} 0 \\ 0 \\ NN(n) \cdot M(n) \end{bmatrix} + (I_3 + hA^T) \\ &\quad \left((P + P^T)z(n+1) + \begin{bmatrix} 0 \\ 0 \\ NN(n+1) \cdot M(n+1) \end{bmatrix} \right), \end{aligned}$$

$$\begin{aligned} Q_u(n) &= L_u(n) + f_u^T(n) V_z(n+1) \\ &= 2Ru(n) + h[(P_{21} + P_{12})\xi_2(n+1) + \ldots \\ &\quad + (P_{23} + P_{32})\xi_4(n+1)], \end{aligned}$$

$$\begin{aligned} Q_{zz}(n) &= L_{zz}(n) + f_z^T(n) V_{zz}(n+1) f_z(n) \\ &\quad + V_z(n+1) f_{zz}(n) \\ &= S + S^T + \begin{bmatrix} 0 & 0 \\ 0 & G(n) \end{bmatrix} + (I_3 + hA^T) \\ &\quad \left(P + P^T + \begin{bmatrix} 0 & 0 \\ 0 & G(n+1) \end{bmatrix} \right) (I_3 + hA), \end{aligned}$$

$$\begin{aligned} Q_{uu}(n) &= L_{uu}(n) + f^T(n) V_{uu}(n+1) f_u(n) \\ &\quad + V_u(n+1) f_{uu}(n) \\ &= 2(R + h^2 P_{21}), \end{aligned}$$

$$\begin{aligned} Q_{uz}(n)T &= L_{uz}(n) + f_u^T(n) V_{zz}(n+1) f_z(n) \\ &\quad + V_z(n+1) f_{uz}(n) \\ &= hB^T \left(P + P^T + \begin{bmatrix} 0 & 0 \\ 0 & G(n+1) \end{bmatrix} \right) (I_3 + hA), \end{aligned}$$

where $G(n) = M^2(n) \cdot NNK(n) + NN(n) \cdot MK(n)$. Then, the feedback gain K and the open-loop gain k are given by

$$\begin{aligned} k(n) &= -Q_{uu}^{-1}(n) Q_u(n) \\ &= -\frac{1}{2(R + h^2 P_{21})} (2Ru(n) + hg(z(n+1))), \end{aligned} \tag{12a}$$

$$K(n) = -Q_{uu}^{-1}(n) Q_{uz}(n) = -\frac{h}{2(R + h^2 P_{21})} Y, \tag{12b}$$

where

$$\begin{aligned} g(z(n+1)) &= (P_{21} + P_{12})\xi_2(n+1) + \ldots \\ &\quad + (P_{2k} + P_{k2})\xi_k(n+1), \\ Y &= B^T \left(P + P^T + \begin{bmatrix} 0 & 0 \\ 0 & G(n+1) \end{bmatrix} \right) (I_3 + hA). \end{aligned}$$

Update the value function as follows:

$$\begin{aligned} \Delta V(n) &= \frac{1}{2} k^T(n) Q_{uu}(n) k(n) + k^T(n) Q_u(n), \\ V_z(n) &= Q_z(n) + K^T(n) Q_{uu}(n) k(n) + K^T(n) Q_u(n) \\ &\quad + Q_{uz}^T(n) k(n), \\ V_{zz}(n) &= Q_{zz}(n) + K^T(n) Q_{uu}(n) K(n) \\ &\quad + K^T(n) Q_{uz}(n) + Q_{uz}^T(n) K(n). \end{aligned}$$

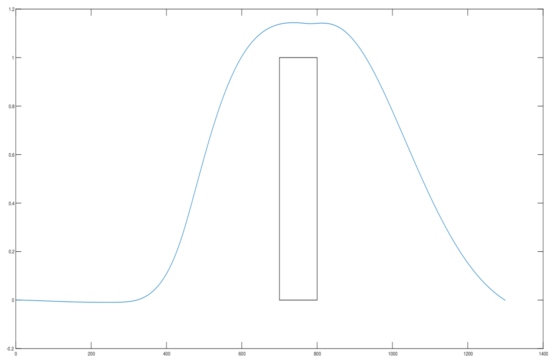

Figure 2: Obstacle avoidance simulation result ($y_0 = 0$).

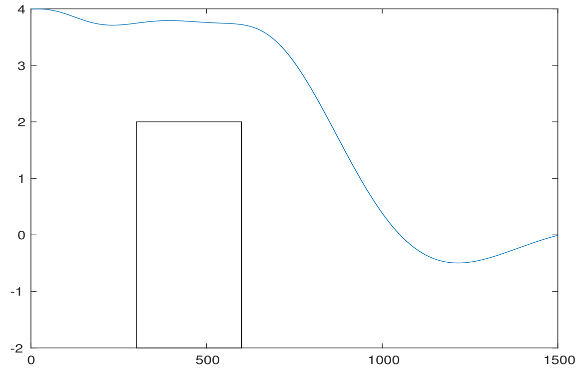

Figure 3: Obstacle avoidance simulation result ($y_0 = 4$).

Decrease $n = N - 2$. Apply the updated V back to the equations and iterate the Backward Pass step until $n = 0$
c. *Forward Pass*: Iterate the following equations forward for $n = 0, \ldots, N - 1$.

$$\begin{aligned} &\hat{z}(0) = z(0), \\ &\hat{u}(n) = u(n) + k(n) + K(n)(\hat{z}(n) - z(n)), \\ &\hat{z}(n+1) = \hat{z}(n) + h[A\hat{z}(n) + B\hat{u}(n)]. \end{aligned}$$

Repeat step **a** $\sim$ **c** until the value of the cost function converges.

4. Simulation

We set the predictive horizon of the RHC method as $N = 500$ steps, and we calculate a trajectory in the interval $[0, 1300]$. Set a single rectangular obstacle and we apply the potential function E as the cost function to the proposed RHC method where we set the weights S, P and R as follows

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \quad P = \begin{bmatrix} 5 & -5 & 5 \\ -5 & 100 & 0 \\ 5 & 0 & 100 \end{bmatrix}, \quad R = 1.$$

And W, a and b are given by (4) and (6)

Shown in Figs. 2 and 3 are the simulation results for controlling a 4-wheeled vehicle moving from an initial state to the target point with avoiding the obstacle. The curve in the figure represents the optimized driving trajectory of the vehicle. In Fig. 2, the obstacle is placed between 700 steps and 800 steps with a height $H = 1$, where the initial state is set at $y_0 = 0$. In Fig. 3, the vehicle is set to move from a initial state $y_0 = 4$ to the target point $y = 0$ while avoiding an obstacle which is placed between 300 steps and 600 steps with a height $H = 2$. The result shows that the vehicle avoids the obstacle with a smooth trajectory while heading towards to the target position.

5. Conclusion

In this paper, we considered an obstacle avoidance problem of a 4-wheeled vehicle with a computational delay. By using the proposed RHC method, we generate an acceptable trajectory towards the target position while avoiding a rectangle obstacle. It is future work to take the control limit into consideration.
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