

# Temporal Sequences of Patterns with an Inverse Function Delayed Neural Network

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**Abstract**—A network based on the Inverse Function Delayed (ID) model, which can recall a temporal sequence of patterns, is proposed. The classical problem, that the network is forced to make long distance jumps due to strong attractors that have to be isolated from each other, is solved by the introduction of the ID neuron. The ID neuron has negative resistance in its dynamics, which makes a gradual change from one attractor to another possible. Also a second version of the model with paired conventional and ID neurons is presented.

## 1. Introduction

For autocorrelation associative memory models, the synaptic connections are chosen in such a way that equilibrium states of the network coincide with states that represent stored static patterns. The synaptic connections are preferably symmetric, which makes the network relax to a state which is a local minimum of a global energy function. However, such a model does not allow temporal sequences. For temporal sequences of patterns, there is no equilibrium state, since the network retrieves different patterns sequentially.

The sequence of  $m$  patterns is typically stored in the connection weights of a fully connected network. The patterns form a cross correlation matrix,  $\mathbf{W}$

$$\mathbf{W} = \frac{1}{n} \sum_{\mu=1}^m \vec{\xi}^{\mu+1} \vec{\xi}^{\mu T} \quad (1)$$

where  $n$  is the total number of neurons and the pattern vector  $\vec{\xi}^{\mu}$  is pattern  $\mu$ , where  $\vec{\xi}^{\mu} = (\xi_1^{\mu}, \dots, \xi_n^{\mu})^T \in \{-1, 1\}^n$ . Further, let the sequence be cyclic in a way that the first pattern follows the last,  $\vec{\xi}^{m+1} = \vec{\xi}^1$ .

In 1972, Amari used a simple model for temporal sequences of patterns with discrete time and two-state neurons [1]. Each individual state is updated synchronously according to

$$x_i(t+1) = \text{sgn} \left( \sum_{j=1}^n w_{ij} x_j(t) \right) \quad (2)$$

where  $x_i$  is the output for the  $i$ th neuron and  $w_{ij}$  is the connection strength from the  $j$ th neuron to the  $i$ th one. The

performance is considered to be good. After recall of a part of the sequence, the network is resting in that very embedded pattern, from which the flow to the next pattern, in accordance with how the patterns are stored, is as big as possible. Due to discrete time property of the model and by using the sign function as a noise reduction tool, the network is then able to jump the distance to the next pattern in one update.

10 years later, Hopfield's paper [2] described an auto-correlation associative model with continuous time, but the paper also contained some statements about sequential patterns. Continuous time leads to that the network will not reach the next pattern in the sequence after just one update, it will instead gradually approach the pattern. However, some special mechanism is needed for the network to reach the next pattern, since the flow varies its direction when the state of the network starts to change. The network is typically attracted by other close patterns and the sequence will soon be lost. Thus, the question of how the sequence successfully was to be retrieved was left unsolved.

In 1986, Sompolinsky and Kanter [3] and Kleinfeld [4] each gave their solution to the problem. They suggested that sequence generation depends on the interplay between two sets of synaptic connections; one that stabilises the network and one that makes the network move on to the next pattern. A time delay was introduced which stabilises the system in each state before it makes the transition.

Other authors discussed solutions where synaptic connections changed their connection strength in time [5] and where sparse coding and noise were introduced [6]. However, what Morita [7] did in 1996 was to change the output function from the conventional monotonic sigmoid function to a non-monotonic one. With the new dynamics the network showed a rather high performance. However, in order to be able to recall the sequence, stabilizing patterns had to be interpolated between the target patterns.

## 2. The ID model

The Inverse Function Delayed (ID) model was proposed by Nakajima and Hayakawa [8] in 2002 and further studied by Li *et al.* [9] for autocorrelation associative memory. It is a time continuous model that considers the output *not* to be

instantaneous, but instead introduces a time delay. Further, it allows the output function to be of an S-shape, hence it can have hysteresis characteristics.

By using the same kind of weightmatrix model as Amari did, the dynamics of the ID model is expressed by the following differential equations

$$\tau_u \frac{du_i}{dt} = \sum_j w_{ij} x_j - u_i, \quad (3)$$

$$\tau_x \frac{dx_i}{dt} = u_i - g(x_i), \quad (4)$$

where  $u_i$  is the inner potential of the  $i$ th neuron,  $\tau_u$  and  $\tau_x$  are time constants ( $\tau_x \ll \tau_u$ ) and  $g = f^{-1}$ . Note that the dynamics of the ID model are equal to those for the conventional dynamics of the Hopfield model if  $\tau_x \rightarrow 0$  in eqn. (4).

If the self connection input,  $w_{ii}$ , is separated from the sum in eqn. (3), the equation can be rewritten as  $\tau_u \frac{du_i}{dt} = w_{ii} x_i + \theta_i - u_i$ , where  $\theta_i = \sum_{j \neq i} w_{ij} x_j$ . Together with eqn. (4) the following equations can be derived

$$\tau_x \frac{d^2 x_i}{dt^2} + \eta_i \frac{dx_i}{dt} = -\frac{\partial U_i}{\partial x_i}, \quad (5)$$

$$\eta_i = \frac{dg(x_i)}{dx_i} + \frac{\tau_x}{\tau_u}, \quad (6)$$

$$\frac{\partial U_i}{\partial x_i} = \frac{1}{\tau_u} (g(x_i) - w_{ii} x_i - \theta_i). \quad (7)$$

Eqn. (5) resembles of a particle travelling motion in space, where the first and second term express the inertia and the friction, respectively, and where  $U_i$  is the potential for neuron  $i$ . There is however an oddity in the equation and that is due to the hysteresis characteristics of the S-shape output function,  $f(u)$ . The slope of the inverted output function,  $g(x)$ , will have the role as a friction coefficient (see eqn. (6)). Inside a certain region of  $g(x)$  the slope is negative, which corresponds to a negative friction, thus the model will accelerate the particle instead of slowing it down. What that means for temporal sequences of patterns may be explained by the potential function derived from eqn. (7)

$$U_i = \frac{1}{\tau_u} \left( \int^{x_i} g(x) dx - \frac{w_{ii}}{2} x_i^2 - \theta_i x_i \right). \quad (8)$$

If  $g(x)$  is not a monotonic increasing function, but instead has an N-shape (which means S-shape for  $f(u)$ ), the N-shape alone is due to a double well potential. With a self feedback connection, however, the potential function can also take the form of a single well, depending on the value of the feedback. An output function generally used for the conventional model is  $\tanh(Bu)$ . That as a reference, the inverted output function used in this paper is  $g(x) = \frac{1}{\beta} \operatorname{arctanh}(x) - \alpha x$ , where  $\alpha$  and  $\beta$  are constants. By changing the parameters at hand ( $\alpha$ ,  $\beta$  and  $w_{ii}$ ) the potential

function can take various kinds of shapes and also the size of the negative resistance region can be controlled.

The friction coefficient is thereafter expressed as

$$\eta = \frac{1}{\beta(1-x^2)} - (\alpha - \frac{\tau_x}{\tau_u}) \quad (9)$$

and the negative resistance region ( $\eta < 0$ )

$$-\sqrt{1 - \frac{\tau_u}{\beta(\alpha\tau_u - \tau_x)}} < x < \sqrt{1 - \frac{\tau_u}{\beta(\alpha\tau_u - \tau_x)}}. \quad (10)$$

With the potential function in mind, a state change for the individual neuron can be visualised as a particle moving from the higher side of the potential well to the lower side. This movement is caused by the total external input from other neurons, according to eqn. (8). However, when the network gradually starts to move toward the next pattern in the sequence, the influence from other patterns will change the direction of the flow. This may cause the potential well for the individual neuron to tilt in the other direction before the particle has been able to move across, hence there will be no state change. On the other hand, with the negative resistance property, the state change can still occur. If the state of the neuron, represented by a particle, manage to come as far as into the negative resistance region, the state change will take place even if it means an uphill-climb motion.

### 3. Temporal sequence of patterns

For the computer simulations, the model worked with 100-400 dimensional patterns with random elements of ones and minus ones.

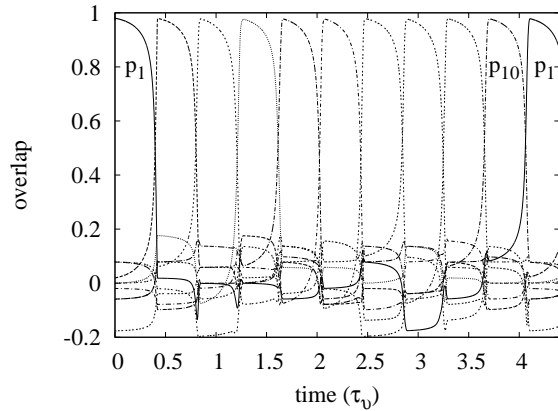


Figure 1: Behaviour of the ID model. The starting state of the network was chosen very close to one of the embedded 10 patterns. 100 dimensional patterns with random elements of ones and minus ones were used and recalled in sequence with a period of about  $4\tau_u$ .

Trying to store the longest sequence of patterns as possible, and still be able to completely recall the sequence, the

overlap was defined to visualise the progress

$$p_\mu = \frac{1}{n} \sum_{i=1}^n x_i \xi_i^\mu, \quad (11)$$

where  $p_\mu$  is the overlap for every pattern  $\mu$ . The starting state of the network is chosen very close to one of the embedded patterns. Figure 1 shows 10 patterns being recalled, in sequence, in a network of 100 neurons. After the last pattern in the sequence has been retrieved, the network starts all over again, with a period of about  $4\tau_u$  and where  $\tau_u = 100\tau_x$ . Further  $\alpha = 1$ ,  $\beta = 1.8$  and  $w_{ii}(i = 0, \dots, n) = -0.71$ , which gives a negative resistance region of about  $-0.66 < x < 0.66$  according to eqn. (10). However, even though the direction of the flow for each neuron is correct, the neurons switch states at slightly different times, due to attraction from other patterns than the next in sequence, and soon the sequence is lost. In order to solve this problem the sign function was implemented, making the state switches for the neurons synchronised. Adding the sign function, however, will make each unit consist of two elements instead of just one; firstly the sign function element and secondly the previous element with the  $g$ -function. With a future hardware implementation in mind that works for analog circuits it is realistic to add a delay to the sign function element. This was done by realising it with a conventional neuron, creating a model with units of a conventional neuron and an ID neuron connected pairwise in this way (see also Figure 2):

$$\tau_c \frac{du_i^c}{dt} = \sum_{j \neq i} w_{ij} x_j - u_i^c, \quad (12)$$

$$x_i^c = \tanh(B \cdot u_i^c), \quad (13)$$

$$\tau_u \frac{du_i}{dt} = w_{ii} x_i + x_i^c - u_i, \quad (14)$$

$$\tau_x \frac{dx_i}{dt} = u_i - g(x_i), \quad (15)$$

where  $u_i^c$  is the inner potential for the conventional neuron  $i$ ,  $x_i^c$  its output and  $B$  is a constant,  $B \gg 1$ .

- The input to each pair is collected by the conventional neuron.
- The output from the conventional neuron is collected by the ID neuron of the pair.
- The ID neuron has a self feedback connection.
- The output of the ID neuron is sent to each other pair.

The change of the load parameter,  $m/n$ , for *absolute stability* with increasing network size for three models, is presented in Figure 3. The models are Amari's discrete model, the ID model with realised sign function and lastly the ID model with discrete sign function. If  $n$  is the number of

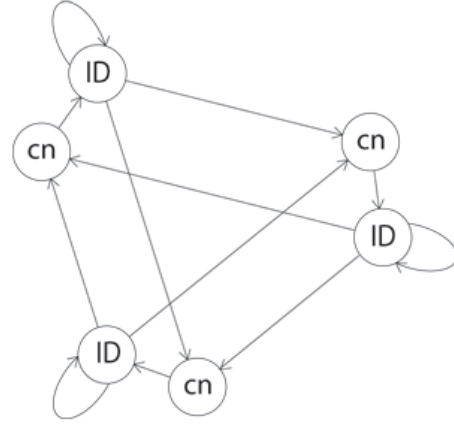


Figure 2: The ID network for temporal sequence of patterns with realised sign function. The network is constructed with pair of neurons, where every pair contains an ID neuron and a conventional neuron (cn). Every ID neuron is connected to every other conventional neuron. The conventional neuron is thereafter connected to its paired ID neuron. Further, the ID neuron is connected to itself, via a self connection.

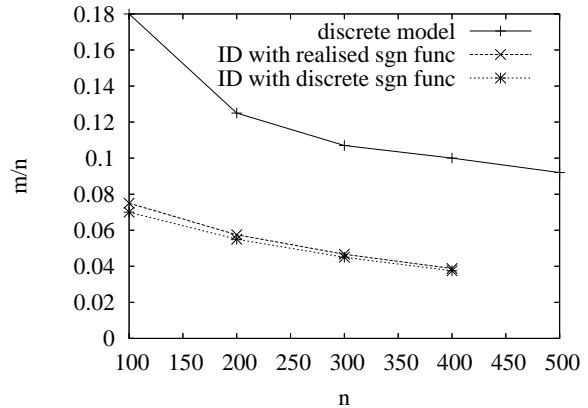


Figure 3: Amari's model gives a good measurement of the upper boundary of the load parameter for the present ID model. It shows that the *continuous* ID model has a capacity of about 38% of the *discrete* model. The network with the realised sign function performs slightly better than the ID network with discrete sign function.

units, in terms of load parameter, the ID model has a capacity of about 75% of the model Amari used, but since each unit of the ID model consists of two elements, the capacity is halved to 38%. However, a direct comparison is difficult to make. Amari's model is a discrete model, which recalls the next pattern in sequence after just one update, while the ID model is a continuous model. On the other hand, the discrete model gives a good measurement of the upper boundary of the present state of the model with ID dynamics. Interesting to see is that the version of the ID model with the realised sign function performs slightly better than the ID version with discrete sign function.

#### 4. Discussion

The memory capacity of the ID model depends on many parameters, but basically the combination of the shape of the potential well and the size of the negative resistance region are the key points in successfully making a recall of a sequence of patterns. For temporal sequence of patterns in general, the crosstalk noise for the weight matrix during recall is a big problem. An easy way to measure the memory capacity of the weight matrix is to use a discrete model, which change pattern in the sequence by every update. If focusing on units, computer simulations show that the continuous ID model has a capacity of 75% of the discrete model. However, in order to prevent individual neurons to switch their states at different time the sign function was implemented which led to two versions of the ID model; one where the sign function is realised with a conventional neuron and one where it is not. Interesting to note is that the value of the load parameter is slightly higher for the version where the sign function is realised. The reason why, is believed to originate from the fact that the conventional neuron adds a delay to the system because of its inner potential. Hence, the inner potential works as a memory and feeds the ID neuron with previous, *better* values.

#### 5. Conclusion

In this paper, a network based on the Inverse Function Delayed (ID) model, which can recall a temporal sequence of patterns has been proposed. In a sequence recalling network, strong attractors have to be isolated from each other, forcing the network state to jump a distance if it is to reach the next pattern, unless some special technique is used. A discrete model can recall the sequence by making these instant jumps. However, a more plausible way to imagine a memory working is to see the recall of a memory being done by *gradually* change the state of the network. With the negative resistance property of the suggested ID model such a gradual pattern change is made possible. Computer simulations show that the continuous ID model *can* recall temporal sequences of patterns and has a capacity of about 38% of the discrete model. As the ID model is a simple model and still being able to recall temporal sequences of

patterns, this is a promising result for future studies. The studies include developing a learning method for the ID model and to gain more memory by using a more suitable weightmatrix. The model is also expected to be able to recall plural temporal sequences.

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