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Asymptotic Stabilization of Nonholonomic Four-Wheeled Vehicle with Hysteresis Mechanism

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Abstract—In this paper, we propose a new asymptotically stabilizing control law for a four-wheeled vehicle with a steering limitation. We improve a previously proposed control law by authors [1], which uses a locally semiconcave control Lyapunov function (LS-CLF) and includes a saturation function and a signum function. The signum function makes the vehicle velocity nonzero except at the origin so that the angular velocity can be manipulated within the input constraint. However, the signum function may cause a chattering phenomenon at some state far from the origin. Thus, we integrate a hysteresis mechanism on the vehicle velocity with the control law. The mechanism makes a sign of the vehicle velocity maintain if the input value decreases the LS-CLF. We confirm the effectiveness of the mechanism via an experiment.

1. Introduction

In this paper, we propose an asymptotically-stabilizing control law under a steering-angle limitation for nonholonomic four-wheeled vehicles. We have previously studied on this problem [1], but the previous method causes a chattering phenomenon, which sometimes makes it hard for the vehicle to arrive at the origin. Therefore, we improve our previous method by adopting a a hysteresis mechanism so that the four-wheeled vehicle with the input constraint reaches the desired point and attitude from any initial state.

A lot of researchers studies the stabilization problem of the nonholonomic mobile robot [2, 3]. It is well known that the chained system and the Brockett integrator are essentially equivalent to the nonholonomic mobile robot [4, 5, 6], which allows pivot turns. Recently, the control Lyapunov function approach is applied to this problem. Kimura et al. [7] proposed a locally semiconcave control Lyapunov function (LS-CLF) and a control law for a chained system. The control law cannot be applied to the four-wheeled vehicle system because no pivot turn is allowed for the vehicles front tyres of which steers. In our previous work [1], we converted Kimura's LS-CLF into a vehicle system by using a coordinate transformation without any singular point, and added a saturation function and a term including a signum function to the Jurdjevic-Quinn type controller to make the car velocity nonzero except at the origin. Since the car velocity is not zero, no pivot turn occurs and nonzero angular velocity can be generated except at the origin. However, the control law causes a chattering phenomenon for the vehicle velocity at some state far from the origin, and in experiments, the vehicle is sometimes stuck at the point.

In this paper, we reveal that such a bad behavior is caused by the signum function included in the previous control law. Thus, we improve the control law by using a hysteresis mechanism, which is similar to Nonaka et al. [8]. The mechanism decreases the number of times of the switching by choosing the sign of the vehicle speed appropriately, and prevents the chattering. Even when the hysteresis mechanism is introduced, the time derivative of the control Lyapunov function is negative except at the origin, and the origin of the system is globally asymptotically stable and locally exponentially stable. The control law makes the vehicle arrive at the destination without causing the chattering from any initial points. Finally, we confirm the effectiveness of the new control law via an experiment.

2. Four-Wheeled Vehicle System

In this paper, we consider the asymptotic-stabilization problem of a nonholonomic four-wheeled vehicle. We define $[X, Y]^\top \in \mathbb{R}^2$ as the center of the rear wheel on the Cartesian coordinate, $\theta \in \mathbb{S}$ as the angle between the heading direction and X-axis, v as the vehicle velocity, and δ as the steering angle. If the vehicle causes no sideslip, the four-wheeled vehicle system is described by

$$\dot{x} = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ v\dfrac{\tan\delta}{L} \end{bmatrix}, \tag{1}$$

where $x = [X, Y, \theta]^\top \in \mathbb{R}^2 \times \mathbb{S}$ is the state vector, $[v, \delta]^\top$ is the input vector, and L denotes the length between the axle center of the front wheels and the axle center of the rear wheels. We propose a method of the asymptotical stabilization of the vehicle system (1) at the origin. As with the real vehicle, we consider the limit on the steering angle as $-\delta_{\max} \le \delta \le \delta_{\max}$, where maximum steering angle $\delta_{\max}$ is a positive constant less than $\pi/2$.

By using an input transformation

$$\omega = \frac{v}{L}\tan\delta, \tag{2}$$

we can express the four-wheeled vehicle system to the state equation

$$\dot{x} = B(x)\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v \\ \omega \end{bmatrix}, \tag{3}$$

where the vehicle's rotation speed ω is a new input. The state equation (3) is equivalent to one for two-wheeled vehicles, which allows a pivot turn. However, the original four-wheeled vehicle system (1) can perform no pivot turn. Thus, in this paper, we focus on the asymptotical-stabilization problem under the steering restriction.

3. LS-CLF and Previous Control Law

In this section, we explain the result of our previous study [1]. In the previous study, we proposed a control Lyapunov function (CLF) and a control law for the four-wheeled vehicle system with the steering-angle limitation.

The previously proposed CLF [1] for the system (3), which is locally semiconcave except at the origin, is

$$V_m(x) = \min_{k\in\mathbb{Z}} V_{m,\mathrm{pre}}(X, Y, \theta + 2\pi k) \tag{4}$$

$$V_{m,\mathrm{pre}}(x) = \left(\theta^4 + (-X\cos\theta - Y\sin\theta)^4 + \frac{|A|^3}{(\sqrt{\theta^2 + (-X\cos\theta - Y\sin\theta)^2} + \sqrt{|A|})^2}\right)^{\frac{1}{2}} \tag{5}$$

$$A = 2(-X\sin\theta + Y\cos\theta) - \theta(-X\cos\theta - Y\sin\theta). \tag{6}$$

The CLF (5) can be derived from the homogeneous locally semiconcave Lyapunov function (homogeneous LS-CLF) for the chained system, which is proposed by Kimura et al. [7]. Kimura's LS-CLF is converted for the system (3) by using a coordinate transformation [2], which has no singular point, and then the square-root function is applied to it to generate a CLF (5). The vehicle posture θ has not a value on the $\mathbb{R}$ but the value on circle $\mathbb{S}$. Because θ in (5) is regarded as a value on $\mathbb{R}$, we redefined the CLF (4), by using the multilayer minimum projection method proposed by Nakamura et al.[9]. Since the coordinate transformation does not preserve the homogeneity, the obtained CLF is not homogeneous, but we can consider an approximated homogeneous degree around the origin with the dilation [1, 2, 1]. Due to the square root function, the CLF (4) is locally semiconcave except at the origin, but for simplicity, we just call V_m an LS-CLF in this paper. The LS-CLF (4) is also adopted in this paper.

The time-derivative of V_m is represented as

$$\dot{V}_m = W_1(x)v + W_2(x)\omega = \frac{\partial V_m}{\partial x}B(x)\begin{bmatrix} v \\ \omega \end{bmatrix}. \tag{7}$$

The sgn$(\cdot)$ is a signum function defined as

$$\mathrm{sgn}(x) := \begin{cases} 1 & (x \geq 0) \\ -1 & (x < 0) \end{cases}. \tag{8}$$

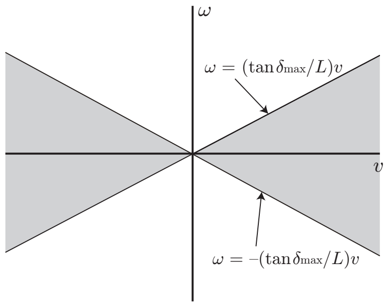

Figure 1: Input constraint set.

A Jurdjevic-Quinn type controller

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = -\begin{bmatrix} k_1 W_1(x) \\ k_2 W_2(x) \end{bmatrix} \tag{9}$$

can stabilize the vehicle system (3) if the steering-angle limitation is ignorable. However, this controller violates the input limitation. At a point satisfying $W_1(x) = 0$ except the origin, $W_2(x) \neq 0$ holds by the definition of control Lyapunov functions. The input (9) at such a point becomes $v = 0$ with $\omega \neq 0$, which makes the vehicle perform a pivot turn. Thus, the control law (9) is not applicable to the four-wheeled vehicle system (1). In addition to the prohibition of the pivot turn, a four-wheeled vehicle system has the steering-angle restriction.

Due to the steering-angle restriction, $[v, \omega]^\top$ is subjected to a set

$$\begin{bmatrix} v \\ \omega \end{bmatrix} \in \mathfrak{U} = \left\{\begin{bmatrix} v \\ \omega \end{bmatrix} \middle| |\omega| \leq \frac{|v|\tan\delta_{\max}}{L}\right\} \tag{10}$$

as illustrated in Fig. 1. For the case of $W_1(x) \neq 0$, we can make $\dot{V}_m$ negative by the input $[v, \omega]^\top = [-\mathrm{sgn}(W_1(x)), 0]^\top$, which is included in $\mathfrak{U}$. In contrast, at the point where $W_1(x) = 0$ and $W_2(x) \neq 0$, the value of v does not affect $\dot{V}_m$. Therefore, we can choose a nonzero v even when $W_1(x) = 0$, which gives a degree of freedom for determining ω within the input constraint. Consequently, for the four-wheeled vehicle system, we proposed a control law

$$\begin{aligned} v &= v_{der}(x) = -\left(k_{v_1}\sqrt{V_m} + k_{v_2}|W_1(x)|\right)\mathrm{sgn}(W_1(x)) \\ \omega &= \omega_{der}(x) = \mathrm{sat}_{|v|\frac{\tan\delta_{\max}}{L}}(-k_w W_2(x)) \end{aligned} \tag{11}$$

which makes $\dot{V}_m$ negative definite [1], where k_{v_1}, k_{v_2} and k_w are positive parameters, and the function sat_y is defined for positive y as

$$\mathrm{sat}_y(x) := \begin{cases} x & (|x| \leq y) \\ y\,\mathrm{sgn}(x) & (\text{otherwise}). \end{cases} \tag{12}$$

The control law (11) locally exponentially stabilizes the four-wheeled system, which can be proven by the approximated homogeneous degree.

However, when the proposed controller (11) is applied to an actual vehicle, a problem arises, and the vehicle is sometimes stuck at a point satisfying $W_1(x) = 0$. It is caused due to the signum function of the vehicle-velocity's control law (11). The vehicle velocity v becomes negative by the definition of the signum function on the points of $W_1(x) = 0$. However, when $W_1(x) < 0$ at the next moment, the vehicle advances. Thus, the velocity v changes its sign repeatedly, and the chattering occurs. The bad phenomenon happens regardless of the definition of sgn(0). The time-derivative of $W_1(x)$ is linear with respect to inputs; that is

$$\begin{aligned} \dot{W}_1 &= S_1(x)v + S_2(x)\omega \\ &= \left(\frac{\partial W_1}{\partial X}\cos\theta + \frac{\partial W_1}{\partial Y}\sin\theta\right)v + \frac{\partial W_1}{\partial \theta}\omega. \end{aligned} \tag{13}$$

When $W_1(x) = 0$, $S_1(x) > 0$, and $|S_2(x)|$ is small, the chattering will occur owing to the signum function.

4. New Control Law with Hysteresis Mechanism

In the previous section, we point out that there exists a state where the vehicle is stuck under the control law (11) due to the chattering phenomenon. This kind of chattering is caused by the signum function included in (11), and it is not induced by the nondifferentiable characteristic of the LS-CLF, while the chattering near the origin is the result of the lack of differentiability of the LS-CLF near the origin.

During the chattering, the input of vehicle velocity becomes both positive and negative repeatedly owing to the signum function. In other words, the direction of the car is too sensitive against the sign of $W_1(x)$. To remove the chattering, making the change of the direction insensitive to the change of the sign of $W_1(x)$ by using a hysteresis mechanism is effective. One may be worried about the bad effect of the mismatch between the direction and the sign of $W_1(x)$ on the stability. However, the velocity input v does not affect (7) where $W_1(x) = 0$, and thus, the first term $W_1(x)v$ in (7) is allowed to be positive around the set $\{x \mid W_1(x) = 0\}$ for locally bounded v. This fact means that the signum function $\mathrm{sgn}(W_1(x))$ in (11) can be flipped where $W_1(x) \approx 0$ and the change of the car direction can be delayed.

We investigate the negative definiteness of $\dot{V}_m$ when the signum function in (11) is flipped, i.e., when we adopt $v = -v_{der}(x)$. Under the input $v = -v_{der}(x)$ with $\omega = \omega_{der}(x)$, (7) can be written as

$$\begin{aligned} \dot{V}_m &= |W_1(x)v_{der}| + W_2(x)\omega_{der} \\ &= |W_1(x)|\left(k_{v_1}\sqrt{V_m} + k_{v_2}|W_1(x)|\right) + W_2(x)\omega_{der}. \end{aligned} \tag{14}$$

Where (14) is negative, we can choose the velocity input as $v = \pm v_{der}(x)$; that is, the sign of v can be selected arbitrarily. Nonaka et al. [8] introduced the hysteresis mechanism to reduce the switching of v. The mechanism keeps the sign of the velocity input as one of the last input, when it is possible. We adopt the same mechanism to prevent the chattering at the points $W_1(x) = 0$.

The area where the negativeness of (14) is guaranteed should be identified. Consider an inequality

$$|W_1(x)v_{der}| + W_2(x)\omega_{der} < -\kappa|W_1(x)v_{der}|, \tag{15}$$

where $\kappa > 0$. When the above inequality holds, $\dot{V}_m$ is negative. Notice that the approximated homogeneous degrees of both sides of (15) are same, and they are also equal to one of V_m. From (15), we can determine the 'switching-free' area

$$\begin{aligned} \mathfrak{B} = \{x \mid\ & (1+\kappa)(k_{v_1}\sqrt{V_m}|W_1(x)| + k_{v_2}W_1{}^2(x)) \\ & < |W_2(x)\omega_{der}(x)|\}, \end{aligned} \tag{16}$$

where the switching of the car direction can be inhibited. Obviously, $\mathfrak{B}$ includes the set $\{x \mid W_1(x) = 0\}$ except the origin, and it excludes the set $\{x \mid W_2(x) = 0\}$. To prevent the chattering far from the origin, it is effective that the sign of v is maintained as one of the last input in the area (16), even if the sign of $W_1(x)$ varies. We redefine the control law

$$\begin{aligned} v &= \begin{cases} -v_{der}(x) & (v_{der}(x)\cdot v_t < 0 \text{ and } v \in \mathfrak{B}) \\ v_{der}(x) & (\text{otherwise}) \end{cases}, \\ \omega &= \omega_{der}(x) \end{aligned} \tag{17}$$

where v_t is the last input. Note that the control law for ω is the same as the old input $\omega_{der}(x)$, and we only modify the vehicle velocity v. The new control law can decrease the number of the switching. The vehicle reaches a desired position and attitude without a factitious chattering phenomenon.

Under the control law, the time derivative of the LS-CLF becomes

$$\begin{aligned} \dot{V}_m \le W_a(x) &= \begin{cases} |W_1(x)v_{der}(x)| + W_2(x)\omega_{der}(x) & (x \in \mathfrak{B}) \\ W_1(x)v_{der}(x) + W_2(x)\omega_{der}(x) & (x \notin \mathfrak{B}) \end{cases} \\ &< 0 \quad (x \neq 0). \end{aligned} \tag{18}$$

Note that $W_a(x)$ has the same approximated homogeneous degree as one of V_m. Hence, under the control law with the hysteresis mechanism, V_m tends to zero locally exponentially as $t \to \infty$. Therefore, the controlled system under (17) is globally asymptotically stable and locally exponentially stable.

5. Experiment of Proposed Control Law

To confirm the effectiveness of the proposed control law with the hysteresis mechanism, we perform experiments. The design parameters are chosen as $k_{v_1} = 0.1$, $k_{v_2} = 0.1$, $k_\omega = 1$, and $\kappa = 2$.

Figure 2 shows the time responses of the state variables of an experiment, Fig. 3 is the time responses of the inputs, and Fig. 4 indicates the trajectory of the vehicle. The

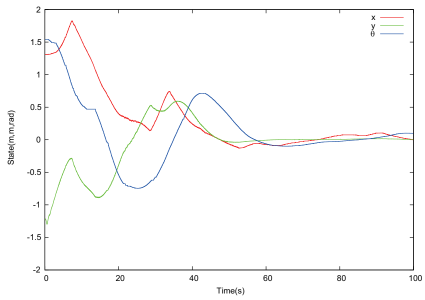

Figure 2: Time responses of the state variables for the proposed method.

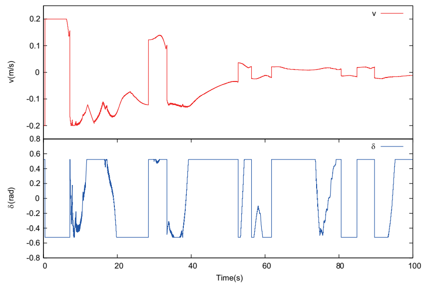

Figure 3: Time responses of the inputs for the proposed method.

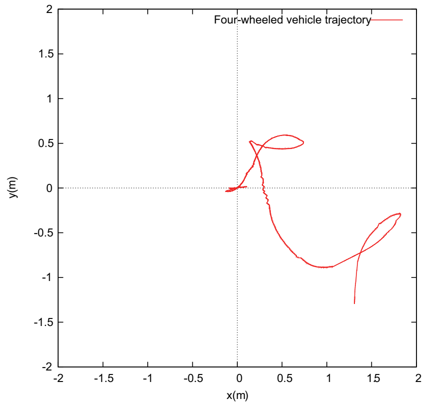

Figure 4: Trajectory of the vehicle for the proposed method.

car reaches the neighborhood of the origin, and we can see from Fig. 3 that there is no chattering phenomenon. We made several experiments for various initial states. It was confirmed that for any initial state the four-wheeled vehicle moves to the origin without causing chattering and is able to arrive in the neighborhood of the origin with no large error.

6. Conclusion

In this paper, we improve the previously proposed asymptotically stabilizing control law [1] for the nonholonomic four-wheeled vehicle with a steering-angle restriction. By adding a hysteresis mechanism to the controller, the vehicle's position and attitude converge to the origin naturally. The hysteresis mechanism prevents the chattering phenomenon except at the origin and reduces the number of the switching on the vehicle velocity. An experiment confirms the effectiveness of the proposed controller. As our future study, we will integrate an obstacle-avoidance mechanism with our controller.

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