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Scaling Property of Wealth Concentration in a Model of Artificial Market

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Abstract– This paper deals with the scaling property observed in the accumulated wealth distribution known as Pareto's law in a model of artificial market. Assuming a simple model of independent agents playing minority game, we discuss the condition for the wealth distribution to satisfy the scaling property having Pareto's index in the range of empirical value.

1. Introduction

Modeling an artificial society need to satisfy various conditions. Among them, Pareto's law observed in the real society [1-5] constrains the model from the point of the income distribution. Earlier we investigated how the system stability can be maintained by adjusting price increment parameters [6] in a homogeneous-agent model of traders that exhibits two phases in the average price time series [7].

In this paper we discuss a model of artificial market made of heterogeneous agents playing minority game [8-11], which represents the behavior of traders in a simple manner.

2. Minority Game

The game is played by odd numbers of (N) independent agents, who decide which of two actions (e.g., buy or sell) to make. The agents who have chosen the minority action get rewarded by one unit of wealth, while the majority agents loose one unit each. Agents do not communicate each other and they use only the public information of which choice won. In order to reflect reality, agents are given only limited intelligence in terms of time as well as space. All the agents use the same length of memory (M) and the same fixed number (S) of strategies. Individual agent uses the same strategy given at the beginning of the repeated games and different agents are given different set of strategies.

An example of strategy set owned by an agent for the case of M=2 and S=5. Each agent chooses one action out of 0 or 1 at each match based on the best-scored strategy out of S strategies in its own bag. All the agents have 5 strategies although the content of the bag is different for a different agent. The history column shows the array of winning choices for M consecutive steps. There are $L=2^M$ possible arrays of history for the memory length M, for each of which there are two choices of action, 0 or 1. Thus the total number of possible strategies mounts to $2^L=16$ for M=2.

Table 1. Example of strategy table held by one agent for the case of M=2, S=3. The bottom row contains scores given to each strategy as a result of games. For example, when 1-0 is the winning record of the past two matches, strategy1 having the best winning score tells you to vote for the action 0 in the next match.

Winner History	Strategy 1	Strategy 2	Strategy 3
00	1	0	0
01	1	1	0
10	0	0	1
11	0	1	1
Scores →	3	1	-1

The reason to call MG as a game lies in the following elements:

(1) Each agent chooses one out of multiple choices of actions
(2) Each agent has a set of strategies and uses the best-scored one
(3) The winners of the game are rewarded and the losers pay penalties/commissions

3. Accumulated Distribution of Income

Pareto's law is stated that the accumulated wealth distribution density P(x) follows the power law as a function of wealth x

$$P(x) = Ax^{-a} \quad (1)$$

where x represent income and P(x) the rate proportional to the number of agents who earns more than x, A is a positive constant, and the index α is a positive number called Pareto's index. The wealth is more evenly distributed among agents for smaller value of a and the other way around for larger value of a. Montroll and Shlesinger [12] showed that this rule indeed holds for very rich side of the society by using the U.S. statistics, while income distribution of the working-class people follows the normal distribution.

Computing P(x) in the artificial society of agents playing MG defined in the past two chapters, we see that the Pareto's index a is close to 48, much larger than the real value of 1-2 [3-5]. This implies the wealth of the total society is evenly distributed over the whole society and very few agents have considerably richer than others. To remedy this situation, we need a mechanism to create some rich agents

Toda and Nakamura [12] considered a modified version of minority game in which the agents invest a fixed percentage Y out of the current wealth and receive the profit in proportion to the invested amount only when it wins. In doing so they obtained much smaller value of a and argued that the real society can be simulated by this kind of modification. However, the accumulated distribution of wealth does not have wide enough range of power law behavior to see the fractal property.

In order to promote incentive to win, we have adopted a biased rewarding assignment by giving +2 for each winner and -1 for each looser. Combining the investment activity and the biased rewarding system, we have obtained a power-law result. The case of Y=0.001 is shown in Fig.1. We can see that Pareto's index is close to one in the region of the straight line.

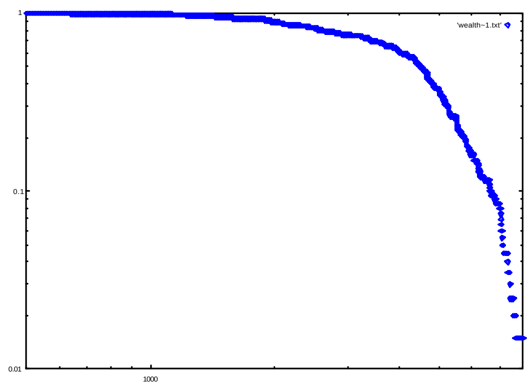

Fig. 1. Accumulated distribution of wealth obtained in our model for investment rate Y=0.001. The value of Pareto's index obtained from the straight line is about one which is close to the empirical value.

4. Gini Coefficient

Next we examine the distribution of wealth by means of Lorenz curve and Gini's coefficient of concentration. Lorenz curve is a plot of accumulated population rate (in %) on Y-axis earning less than X, as a function of X. The diagonal line corresponds to the case of perfectly even distribution. On the other hand, large deviation from the diagonal line shows that the distribution is uneven.[6]

Gini's coefficient is an index to quantify the degree of unevenness. It is define as the ratio of the area surrounded by the Lorenz curve and the diagonal line over the maximum area (most unevenly distributed case, i.e. the total wealth concentrated to one agent.)

From the points on the Lorentz curve, {x(i), y(i)} (i=0,1,…,n; x(0)=y(0)=0; x(n)=y(n)=1), Gini's coefficient G is given as

$$G = 1 - \sum_{i=1}^{n} \{x(i) - x(i-1)\}\{y(i) + y(i-1)\} \tag{2}$$

It is also written as

$$G = \frac{\overline{|w_i(t) - w_j(t)|}}{2\overline{w_i(t)}} \tag{3}$$

This G takes the minimum value

$$G=0 \tag{4}$$

when complete evenness is achieved, and the maximum value

$$G=(N-1)/N \tag{5}$$

when the wealth is completely concentrated to one agent. Thus G satisfies

$$0<G<1 \tag{6}$$

and unevenness increases as G increases.

We first notice that Gini's coefficient quickly damps to invisible size in the original minority game. In the modified version having investment activity, the time series of G monotonically grows to reach one.

Considering the fact that Gini's coefficient is reported to be around G=0.5 in Japan [5], and G=0.7 in the world, we need another mechanism to keep G in this range [13]. For this purpose we apply taxing to all the winners in proportion to the gains of each winning agent. The collected tax is redistributed evenly to all the winners. Fig.2 shows the result of inventing rate r=0.01, and taxing rate Y =0.01 (top), 0.02, 0.05, 0.1 (bottom). Note that G saturates at around 0.5 for Y=0.02 and r=0.01.

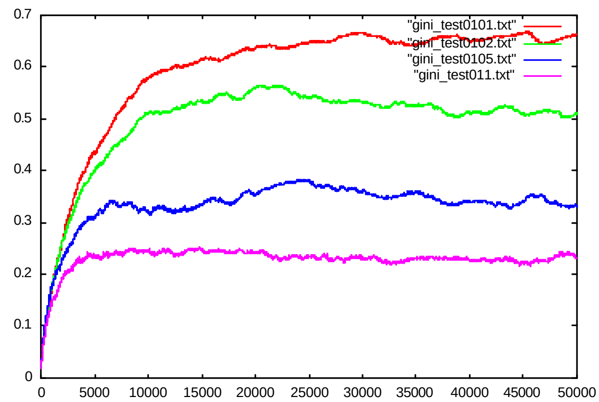

Fig. 2. Gini's index as a time series of repeated games to 50000 times, for a modifies minority game with investment rate r=0.01 and taxing rate Y=0.01 (top), 0.02 (2nd from the top), 0.05 (3rd from the top), and 0.1 (bottom). G saturates for a model with taxing and redistribution of rewards.

5. Conclusion

We considered a model of articial market which exhibit a scaling law in the wealth cncentration known as Pareto's law. In order to see the heterogenoues effect which was missing in our old model of trading society, we considered minority game as a model of trading activity. Since the wealth distribution is too flat in the original version of minority game, we incorporated investing activity and a biased rewarding system, which successfully derived a straight line in the accumulated distribution of wealth with Pareto's index close to the empirical value $\alpha \cong 1$ as shown in Fig. 1.

We further searched for a condition to have Gini's coefficient as a stable value, preperably around the empirical value of 0.5 for Japan (0.7 for world). Since G monotonically increases as a function of time in the modified version of minority game adopted in Fig.1, we apply tax in proportion to each winning agnet's gain and redistributed to all the winning agents. This version successfully derived stable time series of G as shown in Fig.2.

Acknowledgments

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References

[1] V. Pareto, "Cours d'Economique Politique" (McMillan, London 1897)

[2]. E.W. Montroll and M.F. Shlesinger, Journal of Statistical Physics (1984)

[3] H. Aoyama, W. Souma, Y. Nagahara, H.P. Okazaki, H. Takayasu, and M. Takayasu, Fractals 8 (2000) 293. W. Souma, Fractals 9 (2001) 463.

[4] A. Ishikawa, Physica A 349 (2005) 597.

[5] Ministry of Health, Labour and Welfare of Japan (2003) Report on Income Distribution in Japan, 2002,15, 5 (Nov. 2003), 795-825.

[6] Mieko Tanaka-Yamawaki and Fuminori Nakada, "Multi-agent Simulation of a Trading System", Proceedings of NOLTA 2001 (2001)

[7] Mieko Tanaka-Yamawaki, Physica A 324 (2003) 380-387

[8] Damien Challet and Yi-Cheng Zhang, Physica A 246 (1997) 407-418

[9] Ton C. Coolen, "Mathematical Theory of Minority Games" (Oxford, 2005)

[10] Damien Challet, Yi-Cheng Zhang, Matteo Marsili, "Minority Games: Interacting Agents in Financial Markets" (Oxford, 2004)

[11] Minority Game web page: http://www.unifr.ch/econophysics/minority/

[12] K. Toda and Y. Nakamura, "Wealth Dynamics in the Minority Game" (2005) to appear.

[13] Seiji Tokuoka, Bachelor Thesis submitted to Faculty of Engineering, Tottori University (March 2005)