2005 International Symposium on Nonlinear
Theory and its Applications (NOLTA2005)
Bruges, Belgium, October 18-21, 2005

Feasibility of a dispersion-managed electrical transmission line

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Abstract—We examine whether the dispersion management technique, which appeared for the first time in 1995 in the field of optical communications is applicable to discrete electrical transmission lines. We present numerical simulations showing that an appropriate concatenation of two types of electrical lines leads to interesting properties of pulse stability, which suggest the possibility of achieving highly stable pulse propagation in a discrete dispersion-managed electrical transmission line.

1. Introduction

During the last decade, information transmission via fiber optics has achieved spectacular developments in terms of transmission capacities. The breakthrough of fiber optics is due on one hand to the utilization of ultra-short light pulses (solitons) as information-coding elements, and on the other hand, to innovating concepts in the engineering of soliton-based transmission systems. In particular, dispersion management [1, 2] is, without doubt, the most innovating technique that has appeared in the last decade in the area of optical telecommunications with a view to overcome the detrimental effects of chromatic dispersion on the pulse propagation. Schematically, a dispersion-managed optical transmission line is composed of a succession of fibers with alternately positive and negative dispersions. The basic idea in such an arrangement is to locally impose a high dispersion while keeping the average dispersion within very small levels. Moreover, recent studies in optical transmission systems, have shown that dispersion management reinforces substantially the pulse stability against detrimental effects such as the chromatic dispersion, the amplifier noise, or four wave mixing [2].

Knowing these remarkable properties, one can ask the following question: Is the dispersion-management technique applicable to all wave guides? In particular, is it possible to substantially reinforce the pulse stability in an electrical transmission line by applying the technique of dispersion management.

In the present study, we present some results of numerical simulations, that we have carried out by using discrete electrical transmission lines as a typical example of pulse-bearing wave-guide, from which one can formulate a fair answer to the above question. The advantage of this waveguide is that it permits experimental measurements of the pulse parameters at any site of the electrical lattice whereas in optical fibers, such measurements are possible only at the two ends of the optical fiber. The second advantage of electrical transmission lines lies in its low cost and its simplicity.

We first present separately the different discrete electrical lines that we have used to build our dispersion managed transmission line. Then we perform numerical simulations of the pulse propagation in our system, firstly in the linear regime then in the non linear regime.

2. Low-pass electrical line in the linear regime

The low-pass electrical line is one of the basic elements that can be used in the construction of a dispersion-managed transmission line. The standard low-pass line is composed of a network of elementary cells each cell being composed of an inductance and a voltage-dependant capacitor $C_l(V)$ (reversed-biased diode BB112 [4]) as schematically represented in fig 1 where the resistance r accounts for the dissipation in the line.

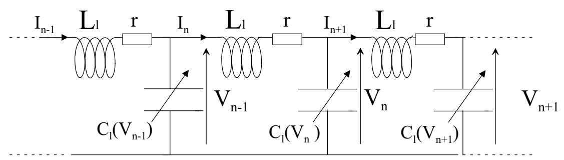

Figure 1: Non-linear and dissipative low-pass electrical line.

In the first part of the present study, for sake of simplicity, we will not consider dissipation and non linearity, we will concentrate solely on dispersion effects. Under these conditions, we can replace the voltage-dependent capacitor by a simple capacitor C_l and set r to 0. Kirchoff's laws lead to the following set of propagation equations

$$\frac{d^2V_n}{dt^2} = \frac{1}{L_lC_l}\left(V_{n+1} + V_{n-1} - 2V_n\right) \qquad n = 1, 2..N \tag{1}$$

and the corresponding dispersion relationship

$$\omega = \frac{2}{\sqrt{L_l C_l}} sin\left(\frac{k}{2}\right) \qquad (2)$$

where ω is the angular frequency and k the wave number. The dispersion coefficients are given by

$$\beta_i = \frac{1}{i!}\frac{d^i\omega}{dk^i} \quad . \qquad (3)$$

Figure 2(a) represents the dispersion law (2)

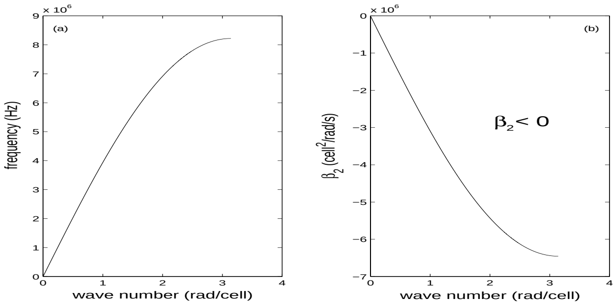

Figure 2: (a)Dispersion law of the low-pass electrical line, (b)Second order dispersion of the low-pass line for L_l=15μH et C_l =100pF.

Figure 2(b) illustrates the second order dispersion coefficient, $\beta_2$which is negative at any frequency. In other words, if a pulse is injected in such a line, it will broaden continually during its propagation. According to the basic principles of dispersion management, it should be possible to compensate this dispersion effect (i.e. pulse broadening) by propagating the pulse in a line for which the sign of the coefficient β_2 is opposite to that of the low-pass line. We show in the following section that a band-pass line offers the advantage of having a second order dispersion whose sign is opposite to that of the low-pass line.

3. Band-pass electrical line in the linear regime

A linear, non-dissipative band-pass electrical line is schematically represented in figure 3.

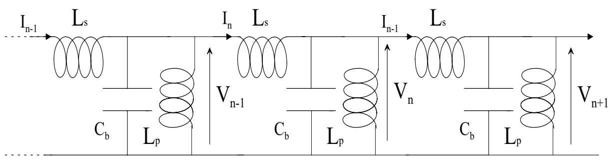

Figure 3: Linear, non-dissipative band-pass electrical line.

Applying Kirchoff's laws to this line, we obtain the following propagation equations:

$$\frac{d^2V_n}{dt^2} = \frac{1}{L_sC_b}(V_{n+1} + V_{n-1} - 2V_n) + \frac{V_n}{L_pC_b} \qquad (4)$$

and the corresponding dispersion law:

$$\omega = \sqrt{\frac{1}{L_pC_b} + \frac{4sin^2\left(\frac{k}{2}\right)}{L_sC_b}} \quad . \qquad (5)$$

Figures 4(a) and 4(b) illustrate the dispersion law and the second order dispersion β_2, for the band-pass line, for L_s=220μH, L_p=470μH and C_b=320pF. Here the most important point to be noticed is the existence of a region where the coefficient β_2 is positive This region ranges from :

$$\omega_{min} = \frac{1}{\sqrt{L_pC_b}} \qquad (6)$$

and

$$\omega_{max} = \sqrt[4]{\frac{1}{L_p^2C_b^2} + \frac{4}{L_sL_pC_b^2}} \qquad (7)$$

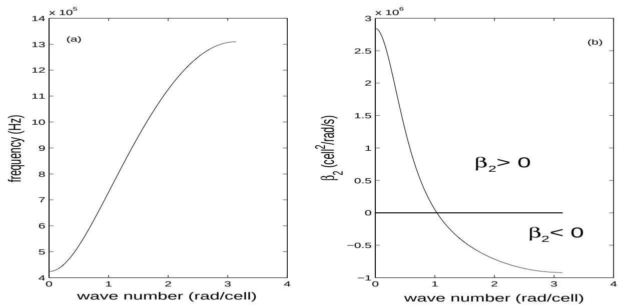

Figure 4: (a) Dispersion law of the band-pass electrical line (b) Second order dispersion of the band-pass line for L_s=220μH, L_p=470μH et C_b=320pF.

4. Linear low-pass/band-pass map

With low-pass and band-pass lines having second order dispersions of opposite signs, it is theoretically possible to build a dispersion-managed system by juxtaposing these two lines, as shown in figure 5.

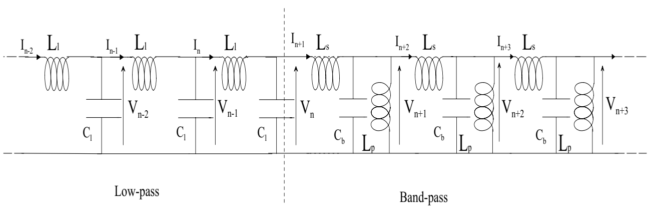

Figure 5: Linear low-pass/band-pass dispersion-managed map.

However, to obtain a highly stable propagation in such a system, the two lines must have similar characteristic impedances so as to avoid large energy losses (due to reflections) at the junction point of the two lines. The characteristic impedances of the two lines in the long wavelength approximation are respectively given by [3]

$$Z_{cl} = \sqrt{\frac{L_l}{C_l}} \tag{8}$$

for the low-pass line and

$$Z_{cb} = \sqrt{\frac{L_s L_p \omega^2}{L_p C_b \omega^2 - 1}} \tag{9}$$

for the band-pass line. Minimisation of the reflection is subject to the condition that $Z_{cl} = Z_{cb}$. This leads to the following condition on the carrier frequency:

$$\omega = \sqrt{\frac{L_l}{L_l L_p C_b - L_s L_p C_l}} \tag{10}$$

To evaluate the ability of the pulse to propagate in this type of system, we have carried out numerical simulations of pulse propagation in the system for the following set of parameters: $L_l = 470\mu H$, $C_l = 320pF$ for the low-pass line and $L_s = 220\mu H$, $L_p = 470\mu H$, $C_b = 320pF$ for the band-pass line, using a fourth order Runge-Kutta algorithm with time step $dt = \frac{1}{100 f_{max}}$, f_{max} being the maximum frequency supported simultaneously by the two lines. In this configuration, the frequency defined by (10) lies within the band defined by (6) and (7) and also within the band defined by the low-pass line (2).

From the non linear Schrödinger equation for the two lines, we have found that the broadening due to second order dispersion effects of a modulated gaussian pulse is governed by the following parameter:

$$\frac{\beta_2 N}{v_g^3} \tag{11}$$

where β_2 is the second order dispersion coefficient , N the number of elementary cells constituting the line and v_g the group velocity of the line.

The condition of dispersion compensation is thus given by :

$$\frac{\beta_{2l} N_l}{v_{gl}^3} + \frac{\beta_{2b} N_b}{v_{gb}^3} = 0 \tag{12}$$

where subscript l stands for the low-pass and subscript b for the band-pass line.

We have injected a modulated gaussian pulse with central frequency (given by (10)) $f = 562kHz$ and initial temporal full width at half maximum (FWHM) of about $12\mu S$. The losses at the junction point of the two sub-systems (low-pass and band-pass lines) are kept to a minimum level($< 10\%$ of the pulse energy is reflected). It should be noted that it is impossible to completely cancel out these losses since our pulse has a certain spectral width and the impedance of the band-pass line is frequency dependent as shown in equation (9). This reflection is qualitatively different from the coupling losses occurring in fiber systems. Indeed, in the electric line, the reflected energy propagates in the backwards direction of the incoming pulses in the line. As a consequence, in the case of the transmission of a coded sequence, the reflected pulses will inevitably interact with incoming pulses.

At this frequency, according to (12), the propagation lengths in the low-pass and band-pass must be in the ratio of 0.77.

In figure 6 we present the results of our numerical simulations of a single-pulse propagation in a system which consists of a low-pass electrical network ranging over 250 cells and a band-pass network ranging over the last 350 cells.

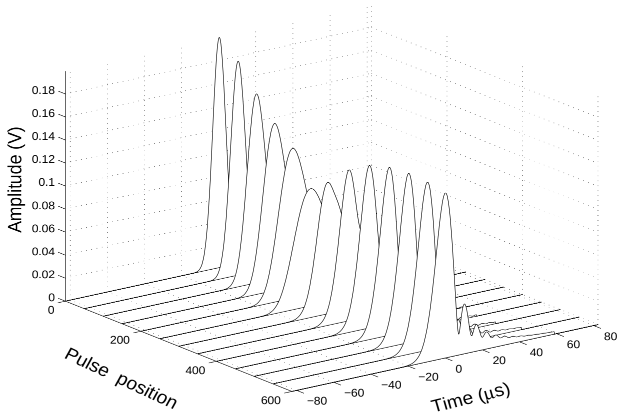

Figure 6: Evolution of an initially unchirpped gaussian pulse in a linear low-pass/band-pass dispersion-managed electrical line.

One can clearly see that initially the pulse broadens until the 250^{th} cell, then progressively gets compressed. According to equation (12), the compression should take place until the 600^{th} cell but by careful inspection of Fig 6, we can observe that this compression stops at about the 500^{th} cell and then the pulse begins to broaden once more as can be seen on figure (7). This indicates that a dispersion compensation takes place. On the other hand, apart from the ”rebroadening” that takes place sooner than expected, one can also notice that the pulse progressively loses its symmetric shape as it propagates. We attribute both effects to the third order dispersion, β_3 which is not compensated with this type of map (β_3 is negative for both types of electrical lines). To avoid such a distortion of the pulse, a more

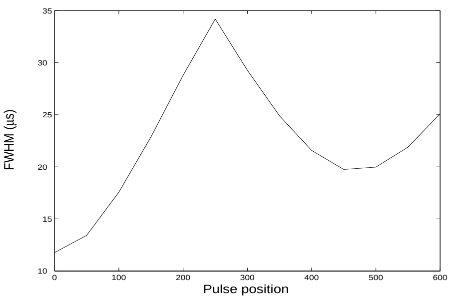

Figure 7: Evolution of the FWHM of an initially unchirpped gaussian pulse in a linear low-pass/band-pass dispersion-managed electrical line.

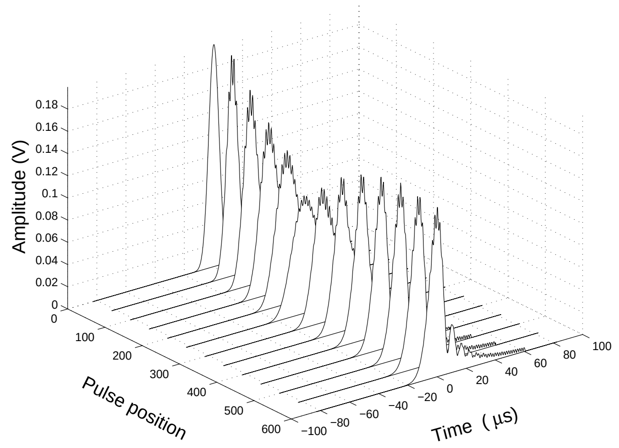

Figure 9: Evolution of an initially unchirpped gaussian pulse in a nonlinear low-pass/band-pass dispersion-managed electrical line.

refined dispersion compensation scheme should be used.

5. Nonlinear low-pass/band-pass map

Since the technique works rather well in the discrete linear lines, we have extended it to the nonlinear version of the two different electrical lines. To this end, we have replaced the capacitors C_l and C_b used respectively in the linear low-pass and linear band-pass electrical lines by a reversed-biased diode whose capacitance is voltage-dependent as shown in figure 8.

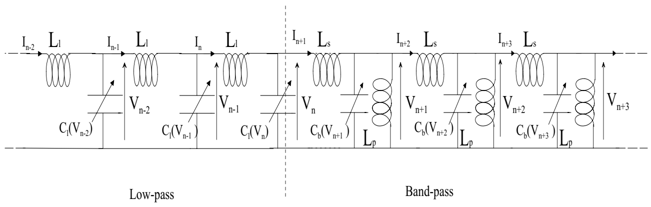

Figure 8: Nonlinear low-pass/band-pass dispersion-managed map.

At a d.c. bias of 2V, the capacitance-voltage relation can be approximately fitted by the following relation [5]:

$$C(V_0 + V_n) \quad = \quad C_0\,[1 - 2\alpha V_n]\,, \qquad (13)$$

V_0 being the d.c. bias voltage and $|V_n| \in [0, 0.5V]$; in this case, $C_0 = 320pF$ and $\alpha = 0.21V^{-1}$.

Here we have carried out numerical simulations in this nonlinear system with the same components as in the previous section except for the capacitors which have been replaced by the reversed-biased diode BB112. In figure 9 we present the results of our numerical simulations:

We can clearly see that the same breathing phenomenon takes place in the nonlinear discrete dispersion-managed line.

6. Conclusion

We have shown in this paper that it is possible to generate a breathing phenomenon of a pulse in an electrical line i.e. a complete cycle of broadening followed by a compression of the pulse. This result is fundamentally important since it is the first clear evidence that the dispersion-management technique is applicable to electrical transmission lines. However, we have also shown that important practical problems such as impedance adaptation and compensation of higher order dispersions remain to be solved to achieve highly stable pulse propagation in electrical transmission lines.

On the other hand, dissipation phenomenons should also be taken into account so as to approach experimental conditions.

References

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