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Phase Pattern Switching and its Control in Pulse-Driven Star-Coupled LC Oscillators

Seiichiro Moro†, Keisuke Hamamoto†, and Tadashi Matsumoto‡

†Department of Electrical and Electronics Engineering, University of Fukui
3-9-1 Bunkyo, Fukui 910-8507 Japan
‡Department of Electrical and Electronics Engineering, Fukui University of Technology
3-6-1 Gakuen, Fukui 910-8505, Japan,
Email: moro@ppc8100.fuee.fukui-u.ac.jp, matsu@ccmails.fukui-ut.ac.jp

Abstract—In the system of the LC van der Pol oscillators star-coupled by one resistor, N-phase oscillations can be stably excited when N is prime number and the strength of nonlinearity of the system is sufficiently large. In this case, the system exhibits $(N-1)!$ different stable states, and it is considered that it can be used as some kinds of neural networks and associative memories. If we use the system as neural networks, the control of the phase pattern should be an important problem. To achieve this, we have proposed the oscillators systems with pulse driving units. In this paper, we show that we can derive all the possible phase patterns with at least $N-2$ pulse stimulation units.

1. Introduction

There have been many investigations of mutual synchronization and multimode oscillation in coupled oscillators [1]–[4]. In particular, we have reported synchronization phenomena observed from N oscillators with the same natural frequency mutually coupled by one resistor [3, 4]. In LC oscillators systems, we have confirmed that N-phase oscillation can be stably excited when each oscillator has strong nonlinearity and N is a prime number [3]. In this case, there exist $(N-1)!$ stable phase states according to the initial states. Moreover, we have investigated the coupled system with RC Wien-bridge oscillators. This system is suitable for VLSI implementation because the system does not include any inductors. They also exhibits the "phase-shift synchronization" and we can get 3^{N-1} different stable phase patterns [4]. Because these "star-coupled" oscillators exhibit a large number of different steady states, they will be used as a structural element of large scale memories and neural networks.

When we use such coupled oscillators systems as neural networks and large scale memories, it should be an important problem how to control the systems to get the appropriate phase patterns. To achieve the phase pattern control, we have proposed the star-coupled system of Wien-bridge oscillators driven by the periodic pulse train and confirmed that the stimulation of the pulse train can cause the phase pattern switching [5]. In this system, however, only the phase of the oscillator where the pulse train is directly added switches [6]. On the other hand, in LC oscillators systems, it is predicted that the effect of the pulse stimulation propagates to the whole system because each oscillator has to take a different phase each other. In this paper, we propose the star-coupled LC oscillators with pulse stimulation units and we show the phase pattern switching phenomena in the proposed system. Moreover, due to "the phase shifting rule" in the proposed system, we suggest that at least $N-2$ switching units are needed to derive all the possible phase patterns.

2. Circuit Models

The circuit models are shown in Fig. 1 (a) and (b). In these circuits, five identical LC van der Pol oscillators are coupled by one linear resistor r. The construction of the nonlinear negative conductor included in each oscillator is shown in Fig. 1 (c). In this study, we propose the following two models.

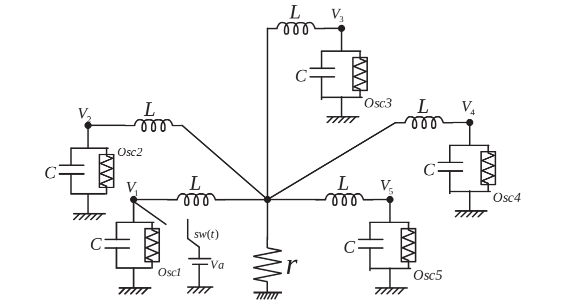
Model 1 The single switch unit is connected to *Osc* 1.

Model 2 The multiple switch units are connected to the oscillators.

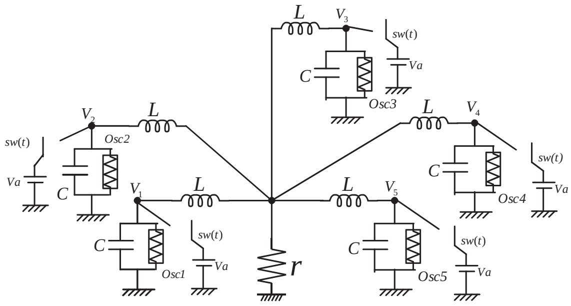
The control signal of the switch is shown in Fig. 2. In this case, because the switch closes Δt seconds in every T seconds, the periodic pulse stimulation with period T is added to the system. T should be sufficiently large to achieve the synchronization within one switching period. Without the pulse stimulation units, the system exhibits 5-phase oscillations because the system tends to minimize the current through the coupling resistor. As a results, we can derive (5-1)!=24 different stable phase patterns considering the combination and permutation of the oscillators' phases [3].

3. Simulation Results

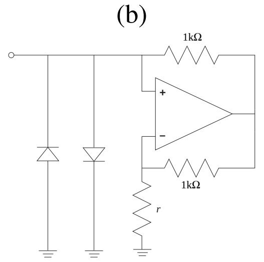
In the following subsections, we show the simulation results using circuit simulation package SPICE. In the following results, we take $r = 300[\Omega]$, $L = 10[\text{mH}]$, $C = 0.068[\mu\text{F}]$ and $r = 150[\Omega]$. The nonlinear negative conductors consist of op amps.



(a)



(b)



(c)

Figure 1: Circuit models. (a) Model 1. (b) Model 2. (c) Construction of the nonlinear negative conductor.

3.1. Simulation Results for Model 1

Figure 3 shows an example of the phase pattern switching phenomena seen in Model 1. In this case, the phase pattern is changed form A to B which are shown in Fig. 5. The magnified figures both before and after the pulse is added are shown in Fig. 4. In the Model 1, from pattern A as an initial pattern, we can see only phase pattern A, B, C, and D as shown in Fig. 5. For example, when $\Delta t = 1[\mu\text{sec}]$ and $V_a = 30[\text{V}]$, in the bold line range shown in Fig. 6, the phase pattern switching occurs. The precise range is shown in Table 1. Therefore, we cannot derive all the possible patterns in Model 1. Thus, it is considered that the multiple switch units are needed to derive all of them.

3.2. Simulation Results for Model 2

Figure 7 shows an example of the phase pattern switching phenomena seen in Model 2. From the results, we can see not only the patterns A–D but the other patterns.

From the results shown in previous subsection, it is shown that the multiple switch units (abbr. SU) are needed

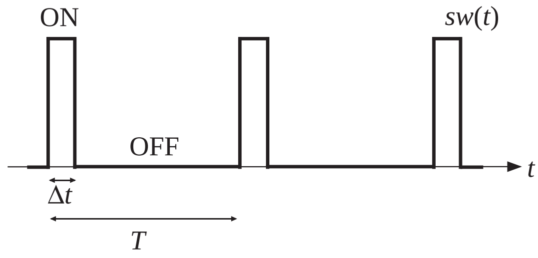


Figure 2: Control signal of the switch $sw(t)$.

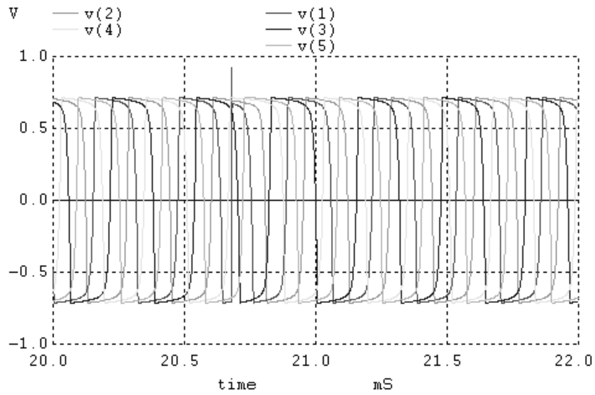


Figure 3: An example of the phase pattern switching in Model 1.

to derive all the possible phase patterns. From Figs. 5, 6 and Table 1, some rules about the phase pattern switching with single SU can be found. The rules are shown as follows:

I. When the pulse is added during the interval I in Fig. 8, the phase switches to one-delayed (72° delayed) position, e.g. A→B in Fig. 5.

II. When the pulse is added during the interval II in Fig. 8, the phase switches to two-delayed (144° delayed) position, e.g. A→C in Fig. 5.

III. When the pulse is added during the interval III in Fig. 8, the phase switches to three-delayed (216° delayed) position, e.g. A→D in Fig. 5.

IV. When the pulse is added during the other interval, the

Table 1: The phase pattern switching range.

t[msec]	phase pattern
20.02399–20.05460	B
20.05467–20.08621	C
20.08622–20.11838	D

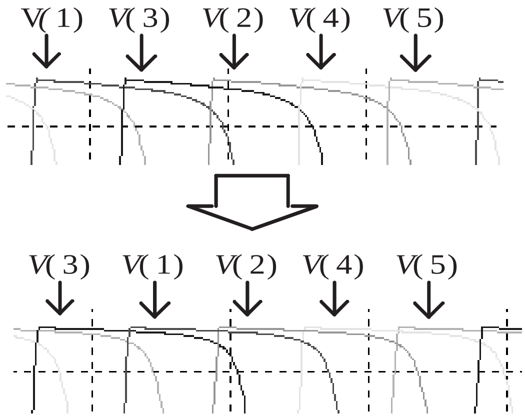


Figure 4: The magnified figures before (top) and after (bottom) the pulse is added. In this figure, the phase pattern is changed from A to B.

A	V(1)	V(3)	V(2)	V(4)	V(5)
	stimulation				
B	V(3)	~~V(1)~~	V(2)	V(4)	V(5)
C	V(3)	V(2)	~~V(1)~~	V(4)	V(5)
D	V(3)	V(2)	V(4)	V(1)	V(5)

Figure 5: Switching pattern of phases of the oscillators in Model 1.

phase does not switch.

Because these rules can be applied to each oscillator, using the permutation of the rules and the oscillators, we can derive all of the phase states. Moreover, from the rules, we can control the system to get the appropriate patterns by choosing when and to which oscillator pulses should be added. From the rules, the number of the SU needed to achieve the all the possible patterns from pattern "a" can be derived as shown in Table 2. From the single SU case, we can derive $3 = (5 - 2)$ phase patterns and to cover all the phase patterns, we need $3 = (5 - 2)$ SU. If the above rule is suitable for larger N, to derive all the possible phase patterns, it is considered that we need $N - 2$ SU.

From the phase switching rules, we can control the phase patterns from any pattern to pattern. Moreover, from Fig. 8, because the regions I–III are relatively large, the tolerance of the stimulation timing may be large. However, these rules can be applied only to the system with the condition described in this paper, and are not universal rules for any star-coupled LC oscillators systems. To find the universal phase switching rules will be our future problems.

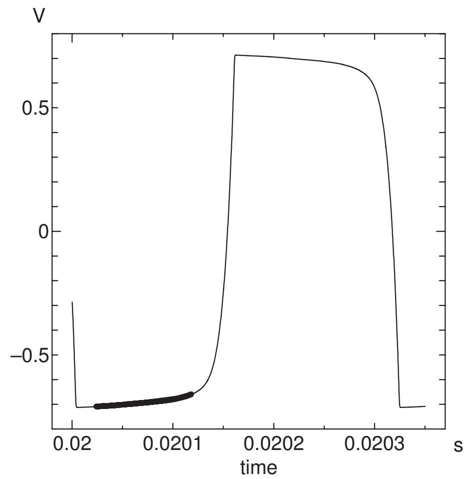


Figure 6: The phase pattern switching range.

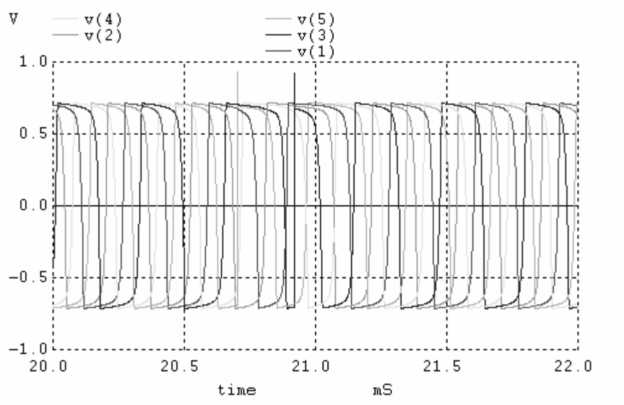


Figure 7: An example of the phase pattern switching in Model 2.

4. Conclusions

In this paper, we have proposed the star-coupled LC oscillators with the pulse stimulation units. In this system, the phase pattern switching phenomena can be seen, and by applying the phase shifting rules in single switch unit case, it is shown that all the phase patterns can be derived with at least $N - 2$ switch units. From these results, the efficient phase pattern control can be achieved in the star-coupled LC oscillators systems.

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Table 2: The number of the switch units to achieve the phase patterns from pattern “a”.

pattern	phase	# of SU	pattern	phase	# of SU
a	$V(1)V(3)V(4)V(5)V(2)$	-	m	$V(1)V(5)V(3)V(4)V(2)$	1
b	$V(1)V(3)V(4)V(2)V(5)$	1	n	$V(1)V(5)V(3)V(2)V(4)$	2
c	$V(1)V(3)V(5)V(4)V(2)$	1	o	$V(1)V(5)V(4)V(3)V(2)$	2
d	$V(1)V(3)V(5)V(2)V(4)$	1	p	$V(1)V(5)V(4)V(2)V(3)$	2
e	$V(1)V(3)V(2)V(4)V(5)$	1	q	$V(1)V(5)V(2)V(3)V(4)$	1
f	$V(1)V(3)V(2)V(5)V(4)$	2	r	$V(1)V(5)V(2)V(4)V(3)$	2
g	$V(1)V(4)V(3)V(5)V(2)$	1	s	$V(1)V(2)V(3)V(4)V(5)$	1
h	$V(1)V(4)V(3)V(2)V(5)$	2	t	$V(1)V(2)V(3)V(5)V(4)$	2
i	$V(1)V(4)V(5)V(3)V(2)$	1	u	$V(1)V(2)V(4)V(3)V(5)$	2
j	$V(1)V(4)V(5)V(2)V(3)$	1	v	$V(1)V(2)V(4)V(5)V(3)$	2
k	$V(1)V(4)V(2)V(3)V(5)$	2	w	$V(1)V(2)V(5)V(3)V(4)$	2
l	$V(1)V(4)V(2)V(5)V(3)$	2	x	$V(1)V(2)V(5)V(4)V(3)$	3

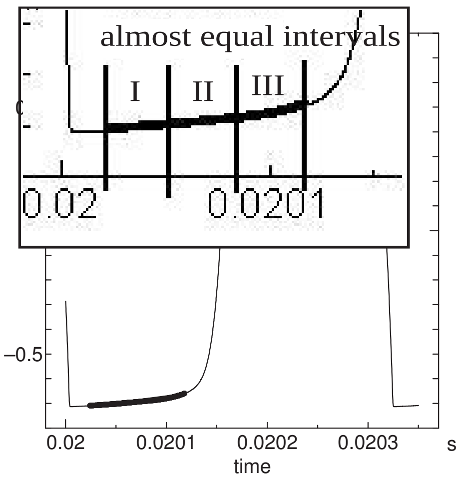

Figure 8: The phase pattern switching range.

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