Influence of Randomly Connecting External Inputs on Associative Chaotic Neurodynamics

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Abstract—In the present paper, we investigate influence of randomly connecting external inputs on retrievals in associative chaotic neural networks. The aim of the investigation is to find out possible roles of adult neurogenesis in memory neural networks. As a first step, we numerically analysed the influence of randomly connecting external inputs on retrievals when the associative chaotic neural network retrievals the stored patterns with chaotic dynamics. As the result, the randomly connecting external inputs make less frequent retrieval of the stored patterns than the case without the inputs. However, the randomly connecting external inputs cause the network to switch the retrieval pattern more frequent than the case without the inputs.

1. Introduction

A chaotic neural network model was proposed[1][2] and it has been applied to an associative memory[3]. The network model consists of model neurons that exhibit deterministic chaos by themselves, namely, without connections to the other neurons in the network[1][2] their behaviour is chaotic. The chaotic neuron model was intended to model dynamical response to stimuli that is observed in squid giant axons and in the Hodgkin-Huxley equations[4] in simple difference equations. Not only for a single neuron, existence of chaos in natural neural network has been also discussed by Freeman[5][6]. Freeman and his colleague also proposed an artificial neural network model, which is called KIII model, exhibiting chaotic dynamics[7].

In order to study memory system in the brain, Hippocampus has been paid attention by many researchers. As model of the memory network, associative network is the most popular model neural network. The conventional associative network is composed of simple static neurons that are mutually connected. A common feature of the associative network and CA3 region in Hippocampus is that both of them can be seen as recurrent networks. Recently adult neurogenesis has been found in Dentate Gyrus (DG)[8] that has connection with CA3 in Hippocampus. A model study based on the layered network for investigating the role of the adult neurogenesis in DG is recently reported[9][10], however, the layered network is not suitable as a model of CA3 since the layered network does not have any recurrent structure. The results of the model studies are limited because they consider only static mapping between the input to the output.

In this paper, we investigate influence of randomly connecting external inputs on retrievals in associative chaotic neural networks. The associative chaotic neural network has recurrent structure and it exhibits chaotic retrieval of the stored patterns[3]. We compare the retrieval characteristics between the associative chaotic neural network and the network with randomly connecting external inputs. Here, we intend to mimic the input from the neurons caused by adult neurogenesis by the randomly connecting external inputs. At the moment, we can not find any experimental result on the distribution of the spatial connection of the neurons caused by the adult neurogenesis. Therefore, we assume that the new neurons connect randomly to the existing neurons. From the above comparison with such an assumption we may find possible role of the adult neurogenesis on the memory retrieval of the stored pattern before and after the adult neurogenesis.

2. Associative Chaotic Neural Network

The associative chaotic neural network to be investigated in the present paper consists of a chaotic neuron model[1][2] that exhibits deterministic chaos by itself. The synaptic weights of the associative chaotic neural network are determined by the conventional auto–associative matrix[12]–[13] of the stored pattern vectors.

The operation of the associative chaotic neural network model is represented by the following equations:

$$x_i(t + 1) = f\{\eta_i(t + 1) + \zeta_i(t + 1)\}, \tag{1}$$

$$\eta_i(t + 1) = k_f \eta_i(t) + \sum_{j=1}^{16} w_{ij} x_j(t), \tag{2}$$

$$\zeta_i(t + 1) = k_r \zeta_i(t) - \alpha x_i(t) + a_i \tag{3}$$

where $x_i(t)$ denotes output of the ith neuron at discrete-time t. The variables $\eta_i(t)$ and $\zeta_i(t)$ denote internal states for feedback inputs from the constituent neurons and for the refractoriness, respectively. k_f and k_r are the decay parameters for the feedback inputs and the refractoriness, respectively. α denotes refractory scaling parameter. The parameters w_{ij} and a_i denote the synaptic weights from the jth neuron to the ith neuron and the bias to the ith neuron ($a_i = a$ for every neuron in this paper), respectively. Output function of the neuron is denoted by f; in this paper, we use the logistic function represented by

$$f(y) = \frac{1}{1 + \exp(-y/\varepsilon)} \qquad (4)$$

where ε is a parameter for the steepness of the function[1][2]. We examine on the associative chaotic neural network with 16 chaotic neurons. The stored patterns for the network are three 16-dimensional binary patterns that are orthogonal with each other and with average firing rate of each pattern is set to be equal to 0.5. Therefore, the synaptic weights are determined by the following equation[12][13].

$$w_{ij} = \frac{4}{3} \sum_{p=1}^{3} (x_i^{(p)} - \overline{x})(x_j^{(p)} - \overline{x}) \qquad (5)$$

with $w_{ii} = 0$ where $x_i^{(p)}$ is the ith component of the pth stored pattern. $\overline{x}$ denotes spatially averaged value of the stored patterns.

In the following numerical experiment, we use the three orthogonal stored patterns as shown in Fig.1.
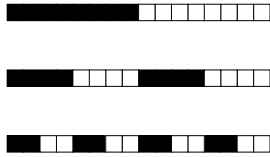

Figure 1: Stored patterns of the associative chaotic neural network with 16 neurons. Each row is a stored pattern vector. The filled and open squares in the rows represent 1 and 0 that correspond to the neuronal outputs, respectively.

It has been reported that the network exhibits chaotic sequential patterns that include the stored patterns when the parameters of the network are set to certain values[2]. It has also been reported that the network, as a whole, in such chaotic retrieval of the stored patterns shows orbital instability which implies deterministic chaos[3]. Figure 2 shows the time evolutions of direction cosine $dc^{(p)}$ between the output pattern of the original network with 16 neurons and the three stored patterns of Fig.1. Such chaotic associative dynamics is treated as control case that is going to be compared with the case with the randomly connecting external inputs in the present paper. The direction cosine is

computed by

$$dc^{(p)}(t) = \frac{1}{N} \sum_{i=1}^{N} x_i(t) x_i^{(p)}, \qquad (6)$$

for the pth pattern, where the pth patterns for $p = 1, 2, 3$ represent the patterns of Fig.1 from the top to the bottom row, respectively. When the network retrieves the pth exact stored pattern and its reverse one, $dc^{(p)}$ becomes 1 and -1, respectively.
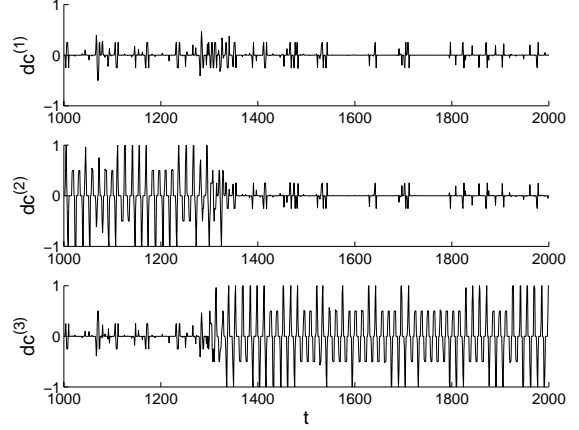

Figure 2: Time evolutions of the direction cosine between the output pattern of the original network with 16 neurons and the three stored patterns.

3. Associative Chaotic Neural Network with Randomly Connecting External Inputs

We consider the retrieval characteristics of the associative chaotic neural network with randomly connecting external inputs. Here, we intend to mimic the input from the neurons caused by adult neurogenesis by the randomly connecting external inputs. For the network with randomly connecting external inputs, the updating equation (3) of the internal state ζ is replaced by the following equation,

$$\zeta_i(t + 1) = k_r \zeta_i(t) - \alpha x_i(t) + a_i + e_i(t), \qquad (7)$$

where $e_i(t)$ denotes randomly connecting external input to the ith neuron at t. Here, the value of external input e_i depends on the index i. The indexes are grouped into the following two sets, one is the set of active input indexes $I^{(a)}$ and the other is for the set of inactive ones $I^{(i)}$. The elements of $I^{(a)}$ and $I^{(i)}$ are randomly determined for every trial. In the following numerical experiments, we examined for the two external inpus as follows.

3.1. Constant external inputs

In this case, the neuron whose index belongs to $I^{(a)}$ receives constant input with magnitude of E for every itera-

tion. It is represented by the following equation.

$$e_i(t) = \begin{cases} E & (i \in I^{(a)}) \\ 0 & (i \in I^{(i)}) \end{cases} . \tag{8}$$

3.2. Gaussian external inputs

In this case, the neuron whose index belongs to $I^{(a)}$ receives random values with Gaussian distribution of mean and deviation are both equal to E. It is represented by the following equation.

$$e_i(t) = \begin{cases} N(E, E) & (i \in I^{(a)}) \\ 0 & (i \in I^{(i)}) \end{cases} . \tag{9}$$

where $N(E, E)$ denotes Gaussian distribution of mean and deviation are both equal to E. In this case each neuron belongs to $I^{(a)}$ receives random values generated by a quasi-random value generator for every iteration.

4. Results of Numerical Experiments

In this paper, we examined for one to eight of randomly connecting external input(s) with magnitude of $E = 0.5$ and 1. We attempt to see the difference between the retrieval characteristics of the network with the constant and Gaussian inputs. For each case, 20 trials for random connection of the external inputs are examined and the retrieval characteristics are evaluated by averaging the 20 trials.

The results of the experiments are summarized in Figs.3 and 4. Figure 3 shows the ratio of the averaged retrieval frequency of the network with external input to that of the original network. Figure 4 shows the switching frequency among the three stored patterns. The switching frequency is important index to evaluate associative dynamics since we intend to realize not static association by the conventional associative network but dynamic association by the chaotic neural network.

From Fig.3 we find that the retrieval frequency of the network with Gaussian external input does not depend on both the number of the external inputs and their magnitude, however, for the constant inputs, it shows non–monotonic change for the number of the external inputs. For larger magnitude of the constant external inputs and many inputs, there is a tendency that the network retrieves only one of the stored patterns that is not good for the dynamical association. Such example is shown in Fig.5. Figure 5 shows an example of time evolutions of the direction cosine between the output pattern of the original network and the three stored patterns with eight constant external inputs of $E = 0.5$. In this case the retrieval frequency is high but the switching frequency among stored patterns is low. Such retrieval is not what we are intended to realize by the associative network with chaotic neurons. Therefore, not only the retrieval frequency but also the switching frequency among the stored patterns is important. From Fig.4 the switching frequency of the network with the constant inputs is not monotonic to the number of the inputs while the

frequency of the network with the Gaussian inputs shows nearly monotonic characteristics to the number of inputs. For the Gaussian inputs, the larger the magnitude of the input is, the higher the switching frequency becomes.
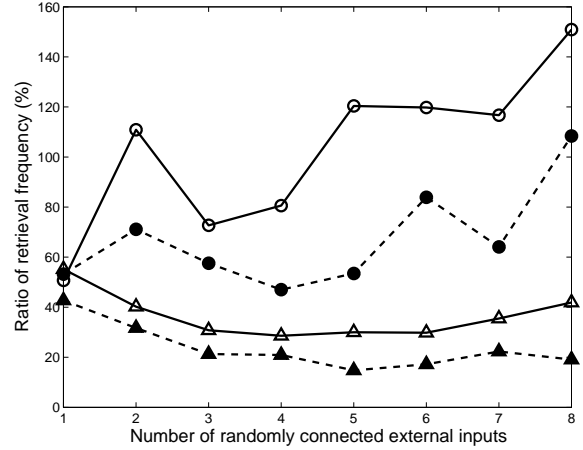

Figure 3: The ratio of retrieval frequencies of the network with constant and Gaussian inputs with magnitude of $E = 0.5$ and 1 to the network without inputs by changing the number of the external inputs. The open and filled circles denote the case with the constant inputs with $E = 0.5$ and 1, respectively. The open and filled triangles denote the case with the Gaussian inputs with $E = 0.5$ and 1, respectively.

5. Conclusions and Discussions

We numerically analysed the retrieval characteristics of the associative chaotic neural network with randomly connecting external inputs. We intend to mimic the input from the neurons caused by adult neurogenesis by the randomly connecting external inputs.

As the result, when the inputs are constant, the relationship between the retrieval frequency of the stored pattens and the number of inputs does not show monotonic characteristics. Such charactericsitc is also found in the switching frequency among stored patterns for the constant input cases. When the external inputs are with Gaussian distribution, the inputs make less frequent retrieval of the stored patterns than the case without the inputs, regardless of the number of the inputs. On the other hand, the switching frequency of the network with the Gaussian inputs shows nearly increasing monotonic characteristics to the number of inputs.

From the above results, randomly connecting external inputs with magnitude of Gaussian distribution may have a role of eliminating the existing memory and may make the network ready to add new memory. Such a role might be a hypothesis on the possible role of the adult neurogenesis in Hippocampus. As the dynamical associative memory, the result that the switching frequency can be controlled by the
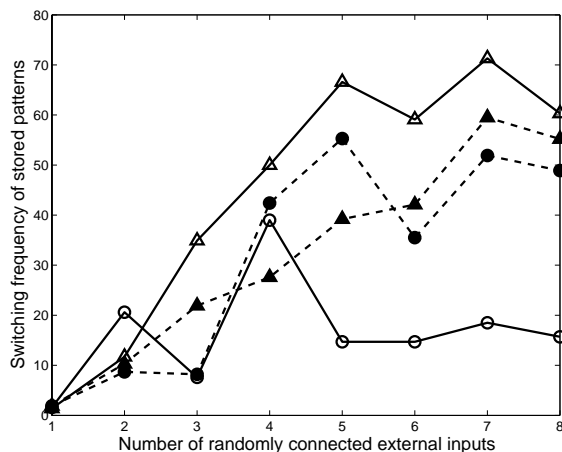
Figure 4: Switching frequencies among stored patterns of the network with constant and Gaussian inputs with magnitude of $E = 0.5$ and 1 by changing the number of the external inputs. The open and filled circles denote the case with the constant inputs with $E = 0.5$ and 1, respectively. The open and filled triangles denote the case with the Gaussian inputs with $E = 0.5$ and 1, respectively.

number of the Gaussian inputs may be useful for learning of new memory. Examining for such hypothesis including learning new memory and with biologically plausible inputs is a future problem.
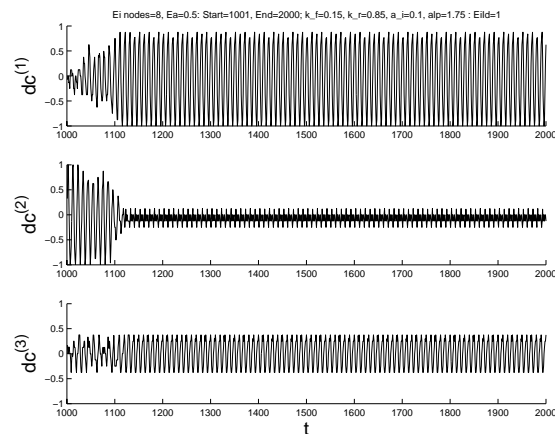
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Figure 5: Example of time evolutions of the direction cosine between the output pattern of the original network and the three stored patterns with eight constant external inputs of $E = 0.5$.

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