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Spatio-temporal Phase Patterns in Coupled Chaotic Maps with Parameter Deviations

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Abstract

In this study, spatio-temporal chaotic behavior in coupled chaotic maps with parameter deviations is shown. The chaotic map which has been governed by a fifth-power polynomial function is properly selected as a chaotic cell. We consider a simple model which each cell has parameters with slight error margin are connected to four neighbors by arbitrary coupling strength. Analytical bifurcation diagram and Lyapunov exponent of the chaotic map are investigated rigorously. Several phase patterns of spatio-temporal chaos are shown.

1. Introduction

It is interested in various pattern formations how to create in the natural world very much in a broad sense. Coupled chaotic systems attract many researchers' attention as a good model which can realize the complicated phenomena in the natural world, and further its dynamics can yield a wide variety of complex and strange phenomena. The coupled systems existing in nature exhibit great variety of phenomena such as complex mechanisms for all of the systems in the universe. These phenomena can be found in a metabolic network, a human society, the process of a life, self organization of neuron, a biological system, an ecological system and so many nonlinear systems. Among the studies on such coupled systems, many interesting researches relevant to the spatio-temporal chaos phenomena on the coupled chaotic systems have been studied until now, e.g. mathematical model in one- or two-dimensional network investigated earnestly by Kaneko[1]-[4], and found in physical circuit model[5]. Moreover, research of complicated phenomena and emergent property in the coupled cubic maps on two-dimensional network system has been also reported[6]. The studies of coupled map lattice(CML), globally coupled maps(GCM) and so many studies concerned with such complex systems provided us tremendous interesting phenomena. We had reported the research on spatio-temporal phase patterns in coupled maps, using a fifth-power function[7], in which it has been carried out in the unique case which parameters of the chaotic map and coupling strength are all the same settings. However many coupled chaotic systems have wide variety of features and moreover its dynamics is also expected to be applied much engineering applications, there are many problems which should be solved in large scale coupled network systems by their complexity.

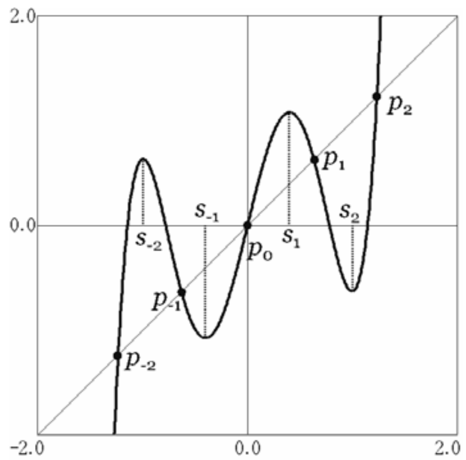

Figure 1: Chaotic map model by a fifth-power polynomial function for *a*=5.2, *b*=10.0 and *c*=4.1.

In this study, spatio-temporal chaotic behavior in coupled chaotic maps with parameter deviations is investigated from the point of view in more faithful natural world. The chaotic map which has been governed by a fifth-power polynomial function is properly selected as a chaotic cell. We consider the model which chaotic cells are mutually connected to some neighbors for spreading on the two-dimensional space by arbitrary coupling strength. Then, we show some phenomena which spatio-temporal chaos, complex behavior and several phase patterns can be found in the proposed coupled systems.

2. Model Description

Chaotic maps are generally used for several approaches to investigate chaotic phenomena on coupled chaotic systems. Especially, the logistic map and the other types of chaotic maps such as a cut map, a circle map, a tent map, a cubic map are well known and popular. Let us consider a fifth-power polynomial function as a chaotic subsystem in each cell written as follows.

$$f(x) = ax^5 - bx^3 + cx, \qquad a, b, c > 0 \tag{1}$$

where *a*, *b* and *c* are parameters which can determine for their chaotic feature. We can easily confirm that it generates chaos in this function. The function (1) is shown

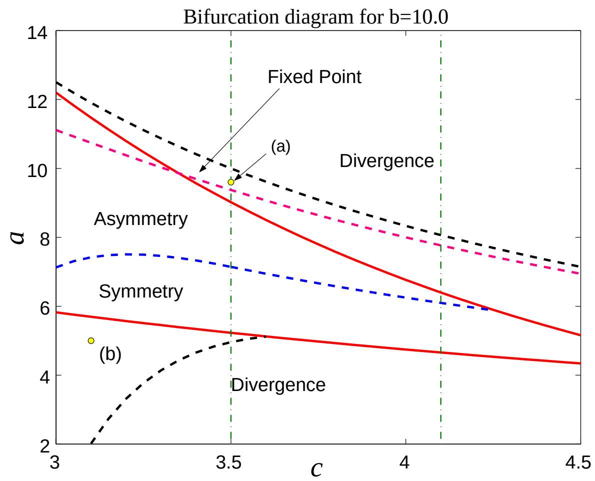

Figure 2: Bifurcation diagram for $b = 10.0$.

in Fig. 1 with some equilibrium points. In general, more plural patterns can be expected to be generated compared with [6], because many equilibrium points can be taken in a higher dimensional map.

From (1), it can be calculated rigorously several bifurcation conditions and boundary region. For instance, a necessary and sufficient condition for having fixed points of $f(x^*) = x^*$ from (1) except the origin can be calculated exactly as

$$b^2 - 4a(c-1) > 0\,. \tag{2}$$

We can only confirm that the system diverges in this setting. Since this system (1) is formed with respect to the origin, variable x moves around neither plus or minus region symmetrically by setting of the parameter. Similarly, this condition can be calculated exactly as

$$b^2 - 4ac < 0\,. \tag{3}$$

However, even if in case of the condition (3), chaotic oscillation is not symmetry with respect to the origin except in special case, e.g. $a = 8.0$, $b = 10.0$ and $c = 3.0$, if satisfied as follows.

$$f\,(f(s_1)) > 0\,. \tag{4}$$

Since this equation can not be calculated explicitly, the bifurcation curve is obtained numerically by computer calculation.

Furthermore, we can also obtain the conditions that solution emanates to infinity by calculating from the following evaluation. The values of $[f'(x) = 0, x > 0]$ at coordinate s_i while in the condition (2),

$$s_1 = \sqrt{\frac{3b - \sqrt{9b^2 - 20ac}}{10a}}\,, \qquad s_2 = \sqrt{\frac{3b + \sqrt{9b^2 - 20ac}}{10a}}\,, \tag{5}$$

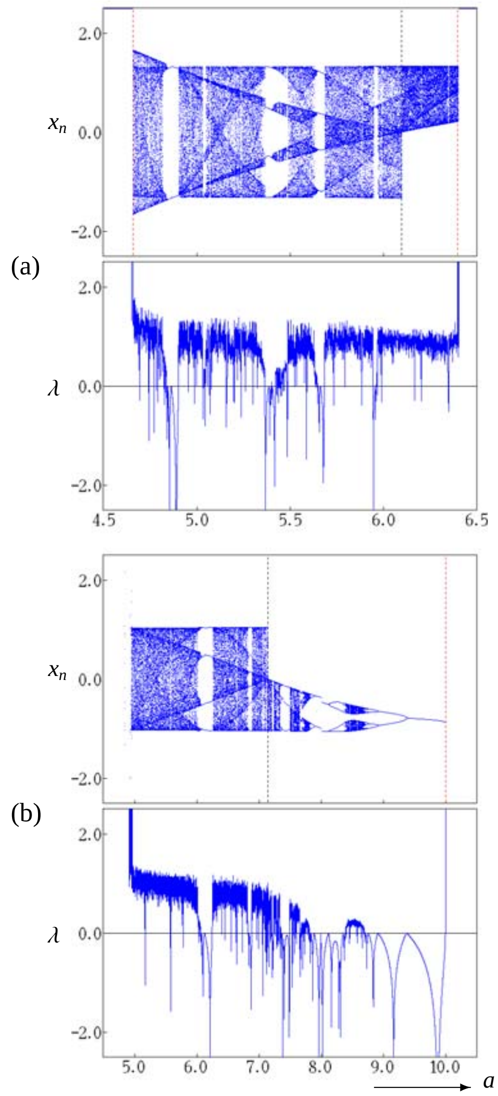

Figure 3: Bifurcation diagram and Lyapunov exponents by changing a for fixed $b = 10.0$. (a) $c = 4.1$, (b) $c = 3.5$.

then, solve $f(s_1) > p_1$ and $f(s_2) < p_2$ the following

$$f(s_1) > \sqrt{\frac{b + \sqrt{b^2 - 4a(c-1)}}{2a}}\,, \qquad f(s_2) < -\sqrt{\frac{b + \sqrt{b^2 - 4a(c-1)}}{2a}}\,, \tag{6}$$

we can obtain the parameter of divergence point numerically. For example, by calculating for the settings $b = 10.0$ and $c = 4.1$ as $a = 6.39953$ and 4.65828, respectively. Bifurcation diagram is partially illustrated in Fig. 2 by changing both a and c for fixed $b = 10.0$. Moreover, some analytical bifurcation curve can also be obtained such as tangent bifurcation, fixed point and so on.

In order to evaluate the function (1), Lyapunov exponent can be calculated as follows.

$$\lambda = \lim_{N\to\infty} \sum_{k=1}^{N} \log \left| \frac{d\,f(x_k)}{dt} \right|\,. \tag{7}$$

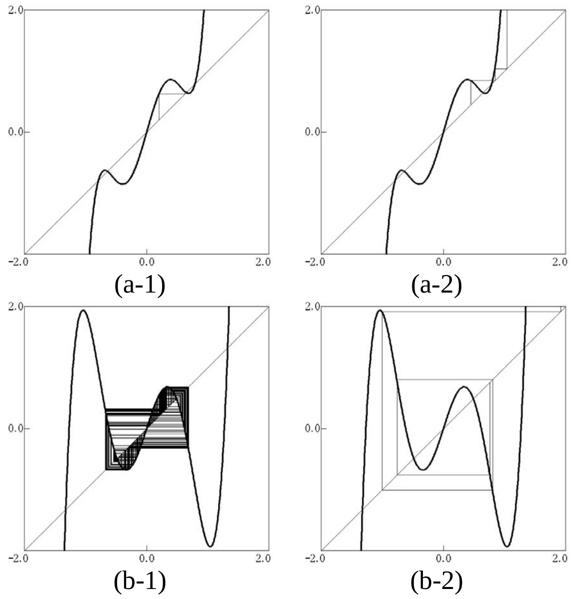

Figure 4: Snapshot of chaotic map with attractor: (a) coexist with fixed point and divergence for $a = 9.6$, $b = 10.0$ and $c = 3.5$, (b) coexist with chaotic attractor and divergence for $a = 5.0$, $b = 10.0$ and $c = 3.1$.

Lyapunov exponents with bifurcation diagram by increasing a gradually for fixed $b = 10.0$ and $c = 4.1$ are shown in Fig. 3 (a). In case of this parameter setting, we can easily calculate parameter in which symmetry collapses is as $a > 250/41$. We can confirm that solution diverges for $a > 6.39953$. Figure 3 (b) also shows bifurcation diagram and Lyapunov exponents for $c = 3.5$ across the parameter a as shown in Fig. 2. Some snapshots of attractor obtained for special cases of coexistence from (1) are shown in Fig. 4; converge to equilibrium point and divergence for (a) $a = 9.6$, $b = 10.0$ and $c = 3.5$, attract to chaotic orbit and divergence for (b) $a = 5.0$, $b = 10.0$ and $c = 3.1$. It can be confirmed that solution coexists with an equilibrium point and divergence to infinity, with chaos and divergence, respectively. As we can see, several modes of chaos, limit cycle and periodic window can be seen by changing the parameters.

3. Spatio-temporal Chaos in Coupled Chaotic Maps

It can be considered easily that coupled chaotic systems have wide variety of phase patterns. The term "spatio-temporal" is extensively used for irregular dynamical behavior observed from large scale complex systems of the relevant to both time and space. In this study, in order to confirm spatio-temporal chaos or several phase patterns in the faithful natural world, we consider a network model of the chaotic maps placed spatially on two-dimensional space are connected to neighbors by arbitrary coupling strength ε. Generally it is able to be considered a coupled model which each cell is connected r-neighbor cells $\Xi_r(ij)$ by coupling strength ε. In order to investigate simply relevant to time and spatial transition, we define and consider an equation for a description model of the entire system that each cell is connected to 1-neighbor $\Xi_1(ij)$ with 4 cells by cyclic rule as follows.

$$x_{ij}(t+1) = (1-\varepsilon) f\Big(x_{ij}(t)\Big) + \frac{\varepsilon}{4} \sum_{kl \in \Xi_1} f\Big(x_{kl}(t)\Big) \qquad (8)$$

where t is a number of iteration in each cell, $\{i, j\}$ is an index number of cell.

Some numerical simulation results on such model (8) as size of 50×50 cells on two-dimensional space are shown in Fig. 5 for the parameter fixed as $a = 5.45$, $b = 10.0$ and $c = 4.1$. The initial condition for each cell is given at random between -1.0 and 1.0. The figure indicates a grade of synchronization state for phase difference with a nearby average, which black ■ and white □ colors correspond to synchronous and asynchronous states, respectively. Hereby the synchronous states are displayed with 100 steps gray scale colors. Figure 5 shows some snapshots of phase patterns as time increasing for $a = 5.45$, $b = 10.0$, $c = 4.1$ and $\varepsilon = 0.25$. The pattern at time $t = 1$ means initial state, which black region almost occupied visibly. However, as time passed, the entire state becomes asynchronously. Finally, some part only remains synchronous state at $t = 1000$. In this case, the phase pattern behaves almost asynchronous well, however some parts produce unique patterns while keeping synchronous state as a cluster.

On the other hand, we now consider the situation that some chaotic cells have slight error margin as compared with the other cells in order to investigate from the point of view in more faithful natural world. Figure 6 shows some numerical simulation results at time $t = 1000$ for giving the maximum parameter deviations with 0.1%, 0.5%, 1% and 10% to only a, respectively. The initial conditions are putting as all the same values in each result. Hence it has just different parameters a with slight error margin. We can confirm that some results could be obtained as different phenomena. In order to investigate the influence on synchronization depending on the parameters, a synchronous ratio was calculated. Results of the ratio of synchronous states with 5% maximum parameter deviations of a are shown in Fig. 7. We can confirm that a synchronous ratio is a high average according to the area of asymmetric region as shown in Fig. 2. Similar results can be confirmed when there is no error margin of the parameters. These results are some typical examples obtained by numerical simulation. From some numerical results, it has been confirmed that a slight error margin does not exert or influence on the entire system too much. However, the phenomena such the mutation can be also confirmed with parameter deviations in some situations. Therefore, some phenomena of spatio-temporal chaos or complex behavior of several phase patterns can be found in these systems.

4. Conclusions

In this study, a fifth-power function for using as a chaotic maps was proposed, further rigorous analysis, bifurcation

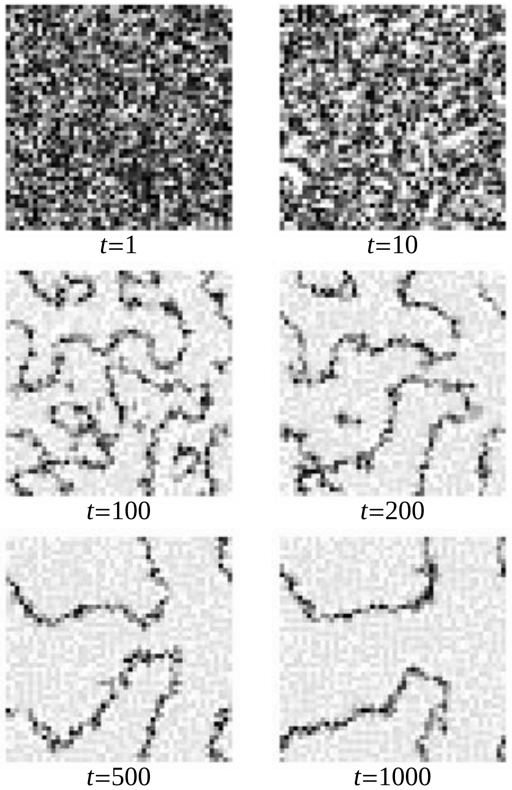

Figure 5: Some snapshots of numerical simulation results depended by time to two-dimensional space for fixed parameters a = 5.45, b = 10.0 and c = 4.1 without parameter deviations. ε = 0.25, t: number of iteration. Number of cells is 50 × 50.

diagram and Lyapunov exponents were shown. Some illustrated computer simulation results of spatio-temporal chaotic behavior and several phase patterns in coupled chaotic maps have been shown. We conclude that the supposed or similar coupled chaotic systems can be regarded as a good model for realizing complex phenomena in the universe concerned with self organization, mechanisms of pattern formation and so on. However some studies of pattern dynamics and the mechanism of clustering phenomena in such complex phenomena and many works have been left.

Acknowledgment

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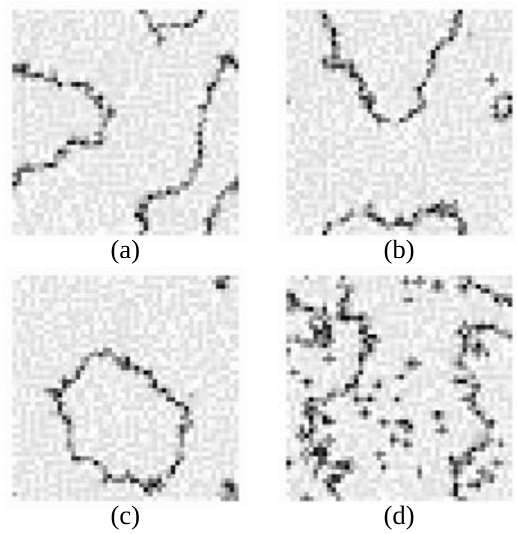

Figure 6: Some other phase patterns obtained from numerical simulation at time t = 1000 as fixed parameters b = 10.0 and c = 4.1 with maximum parameter deviations for the value of a = 5.45, (a) 0.1%, (b) 0.5%, (c) 1%, (d) 10% .

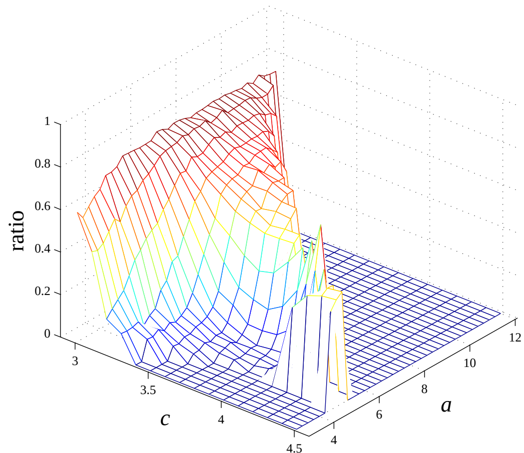

Figure 7: Ratio of synchronous states by changing the parameters a and c on coupled chaotic maps for b = 10.0 and e = 0.25 with 5% maximum parameter deviations of a at time t = 1000.

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