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Reconstruction of Chaos Attractor with DT-CNN

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Abstract—We propose reconstruction of chaos attractor by Discrete Time Cellular Neural Network (DT-CNN). The feedback **A**-matrix can be obtained in form of a tri-diagonal matrix by using House Holder Transformation for an algebra equation constructed from observed time sequences. Since the **A**-matrix can be changed to satisfy the diagonal dominant based on the use of virtual capacitors, the Gauss Seidel Method (Linear DT-CNN) can be used to solve the equation. As a simulation, we reconstruct the chaos attractor of Chua's circuit. The attractor which is reconstructed from the observal time sequence based on our method is almost same as that of attractor derived from the Chua's differential equation.

1. Introduction

In this paper, we propose Discrete Time Cellular Neural Network (DT-CNN)[1][2] for the reconstruction of chaos attractor. It is said that the prediction of chaos is impossible because chaos behaves irregularly. Then, we consider the reconstruction of chaos attractor in the limited time interval.

The coefficients of the reconstructed nonlinear differential equations can be obtained by equilibrium points of the DT-CNN state equation. That is, the DT-CNN is used as a machine learning solver. The feed-forward and feed-back templates of its nonlinear state equation are expressed by using past values in observed time sequences based on linear Gray Theory[3]. The reconstructed coefficient matrices corresponding to the templates must be obtained by a new machine learning algorithm. If the **A**-matrix is not diagonal dominant, the DT-CNN state equation is changed to nonlinear algebra equation which should be solved by Newton Raphson Method. The relaxation-based method (Linear DT-CNN) is used to solve the linearized equations by the Newton Raphson Method. Diagonal dominant for the Jacobian matrix used in the Newton Raphson Method can be satisfied by inserting virtual capacitor to each node. In this process, House Holder Transformation is used to transform its dense matrix into tri-diagonal sparse matrix. The tri-diagonal matrix is corresponding to the feedback A-template of state equation of the DT-CNN.

As a simulation, we reconstruct the attractor of Chua's circuit in the short time. The attractor which is reconstructed from the observal time sequence based on our method is almost same as that of attractor derived from the Chua's differential equation.

2. Machine Leaning

Let u_i be an integral continuous variable for observed input variable du_i/dt, then the proposed model is described by using state equation as

$$\begin{aligned}\frac{du_i}{dt} &= -f(x_{ii})u_i + f(x_{i1})u_1 + \cdots \\ &\quad + f(x_{ij})u_j + \cdots + f(x_{iN})u_N \end{aligned} \quad (1)$$

where x_{ij} is a coefficient weight connected from cell j to i, N is the number of input variables and a and b_j are its nonlinearity for quantization.

We define 1-Accumulated Generating Operation (AGO) of u_i as

$$u_i^{(1)}(k) = \sum_{m=1}^{k} u_i(m). \quad (2)$$

1-Inverse Accumulated Generating Operation (IAGO) of u_i is opposite to AGO. In the case of $\Delta t = 1$, du_i/dt is IAGO of $u_i(k+1)$ and

$$u_i = \frac{\Delta u_i(k+1)}{\Delta t} \quad (3)$$

are used by this model discretization for time $k+1$.

Assuming that there is no rapid changes of u_i from discrete time k to $k+1$ in the discrete time space, the variable u_i is expressed in the mean value of $u_i(k)$ and $u_i(k+1)$,

$$u_i = \frac{1}{2}[u_i^{(1)}(k) + u_i^{(1)}(k+1)]. \quad (4)$$

Let $\mathbf{u} = [u_i(1), u_i(2), \cdots, u_i(i-1), u_i(i+1), \cdots, u_i(n)]^T$, where n is the number of the data, be a vector for the number $n-1$ of past observed samples in time series, then we can derive the following nonlinear algebra equation from Eqs.(1), (2), (3) and (4):

$$\mathbf{u} = \mathbf{G}\mathbf{f}(x) \tag{5}$$

where

$\mathbf{f}(x) = [f(x_{ii}), f(x_{i1}), f(x_{i2}), \cdots, f(x_{ij}), \cdots, f(x_{iN})]^T$

and $\mathbf{G}$ is $(n-1)\times N$ rectangular matrix given by

$$\mathbf{G} = \begin{bmatrix} -\frac{1}{2}(u_1^{(1)}(1)+u_1^{(1)}(2)) & u_2^{(1)}(2) & \cdots & u_N^{(1)}(2) \\ -\frac{1}{2}(u_1^{(1)}(2)+u_1^{(1)}(2)) & u_2^{(1)}(3) & \cdots & u_N^{(1)}(3) \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{2}(u_1^{(1)}(n-1)+u_1^{(1)}(n)) & u_2^{(1)}(n) & \cdots & u_N^{(1)}(n) \end{bmatrix}.$$

The transformation to non-singular equation is applied to minimize the norm $\|\mathbf{G}\mathbf{f}(x)-\mathbf{u}\|$ as

$$\mathbf{G}^T\mathbf{G}\mathbf{f}(x) - \mathbf{G}^T\mathbf{u} = 0 \tag{6}$$

where $\mathbf{G}^T$ is the transposed matrix of $\mathbf{G}$.

Clearly, the Eq.(6) is a nonlinear algebra equation in which the variable x must be solved to satisfy it.

3. Reconstruction by DT-CNN

Since the matrix $\mathbf{G}^T\mathbf{G}$ is non-singular, the Eq.(6) can be solved by the Newton Raphson Method directly if the initial value is in the neighbor of the solution. However, it is very difficult to find the initial value of the Newton Raphson Method. Therefore, we think that we solve the Eq.(6) by using the state equation of DT-CNN. That is, the nonlinear coefficient vector $\mathbf{f}(\mathbf{x})$ can be obtained by

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{f}(\mathbf{x}(k)) + \mathbf{B}\mathbf{u} + \mathbf{T} \tag{7}$$

where

$$\begin{cases} \mathbf{A} = -\mathbf{G}^T\mathbf{G} \\ \mathbf{B} = \mathbf{G}^T \\ \mathbf{T} = 0 \end{cases}.$$

For the correspondence to the state equation of DT-CNN, $\mathbf{x}$ is a state variable vector, $\mathbf{u}$ is an input variable vector, $\mathbf{f}(\mathbf{x}(k))$ is an output variable vector, $\mathbf{A}$ and $\mathbf{B}$ are feed back and feed-forward weight matrices corresponding to **A**- and **B**- templates respectively. Then, we consider the House Holder Transform in order that it is easy to change the dense matrix to the sparse matrix.

The matrix $\mathbf{A}$ transform into tri-diagonal matrix by the House Holder matrix $\mathbf{P}$. Tri-diagonal matrix $\mathbf{A}_{3d}$ is expressed as

$$\begin{aligned} \mathbf{A}_{3d} &= \mathbf{P^{-1}AP} \\ &= \begin{bmatrix} \alpha_1 & \beta_1 & & & \\ \beta_1 & \alpha_2 & \beta_2 & & 0 \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \beta_{n-1} \\ & 0 & & \beta_{n-1} & \alpha_n \end{bmatrix}. \end{aligned} \tag{8}$$

Since $\mathbf{x}(k) = \mathbf{P}^{-1}\mathbf{y}(k)$ and $\mathbf{P}\mathbf{P}^{-1} = 1$, the Eq.(7) can be rewritten as

$$\mathbf{y}(k+1) = \mathbf{A}_{3d}\mathbf{f}(\mathbf{y}(k)) + \mathbf{PBu}. \tag{9}$$

Because the **A**-matrix is transformed into tri-diagonal sparse matrix $\mathbf{A}_{3d}$ in Eq.(9), the Eq.(9) can be rewritten by using the state equation of DT-CNN for the constant vector $\mathbf{PBu}$. It makes it possible to reduce the computation drastically. Basically, the coefficients of the reconstructed nonlinear differential equations(1) can be obtained by equilibrium points of the DT-CNN state equation(9). We can prove the convergence of DT-CNN state equation when $\mathbf{A}_{3d}$ is diagonal dominant.

The guarantee of the convergence is in that the matrix $\mathbf{A}_{3d}$ must be changed to the diagonal dominant matrix. That is, we must select the process whether each row of the tri-diagonal matrix $\mathbf{A}_{3d}$ is diagonal dominant to use the nonlinear DT-CNN or otherwise to use Newton Raphson Method including linear DT-CNN.

Then, we describe the Newton Raphson Method including linear DT-CNN. Since the Eq.(7) is corresponding to a nonlinear algebra equation for $\mathbf{f}(\mathbf{x})$, the Newton Raphson Method can be used to the Eq.(7). Let $\mathbf{J}$ be a Jacobian matrix given by

$$\mathbf{J} = \left.\frac{\partial \mathbf{F}(\mathbf{y})}{\partial \mathbf{y}}\right|_{\mathbf{y}=\mathbf{y}_m} \tag{10}$$

where

$$\mathbf{F}(\mathbf{y}) = \mathbf{A}_{3d}\mathbf{f}(\mathbf{y}(k)) + \mathbf{PBu} - \mathbf{y}(k+1). \tag{11}$$

Then the Newton Raphson Method for $\mathbf{F}=0$ is

$$\mathbf{J}\cdot\tilde{\mathbf{y}} = \mathbf{F}(\mathbf{y}_m) \tag{12}$$

where $\mathbf{y}_m - \mathbf{y}_{m+1} = \tilde{\mathbf{y}}$.The convergence criterion for solving the linearized equation(12) by linear relaxation method must depend on whether $\mathbf{J}$ is diagonal dominant or not.

The linearized equation(12) is assumed to describe a linear virtual circuit as a nodal equation. In order to analyze the virtual circuit by using the relaxation method, each adaptive virtual capacitor is inserted from less diagonal dominant node to the reference node. It is unnecessary for each virtual capacitor to be inserted to the node having diagonal dominant to avoid redundant process as inserting the capacitor. In other words, the value of the adaptive virtual capacitors for the diagonal dominant node is zero. The linearized nodal equations are mapped by inserting adaptive virtual capacitors to the state equation as

$$\mathbf{C}_{virtual}\frac{d\tilde{\mathbf{y}}}{dt_{virtual}} = -\mathbf{J}\cdot\tilde{\mathbf{y}} + \mathbf{F}(\mathbf{y}_m) \tag{13}$$

where the nodal capacitance matrix $\mathbf{C}_{virtual}$ is decided as making $\mathbf{J}$ more diagonal dominant. Since the matrix $\mathbf{J}$ becomes also a tri-diagonal matrix in Eq.(10), each virtual capacitor for $\mathbf{C}$ is inserted for each off-diagonal element of $\mathbf{J}$ based on

$$S_i = \begin{cases} |J_{i+1,i}| & (i=1) \\ |J_{i-1,i}| + |J_{i+1,i}| & (2 \le i < N) \\ |J_{i-1,i}| & (i=N) \end{cases} \qquad (14)$$

and

$$C_i = \begin{cases} 0 & (J_{ii} \ge L \times S_i) \\ L \times S_i - J_{ii} & (J_{ii} < L \times S_i) \end{cases} \qquad (15)$$

where the parameter L is experimentally selected by 1.25. Equation(13) is integrated by Backward Euler Method in each virtual time step h as

$$\mathbf{C}(\tilde{\mathbf{y}}_{n+1} - \tilde{\mathbf{y}}_n) = -h\mathbf{J}\tilde{\mathbf{y}}_{n+1} + h\mathbf{F}(\mathbf{y}_m), \qquad (16)$$

$$\mathbf{H}\tilde{\mathbf{y}}_{n+1} = \mathbf{C}\tilde{\mathbf{y}}_n + h\mathbf{F}(\mathbf{y}_m) \qquad (17)$$

where $\mathbf{H} = \mathbf{C} + h\mathbf{J}$. It is possible that this Eq.(17) is solved by the Gauss Seidel Method (Linear DT-CNN) because the matrix $\mathbf{H}$ is diagonal dominant by Eqs.(14) and (15). The Gauss Seidel Method (Linear DT-CNN) is given by

$$(diag\mathbf{H})\tilde{\mathbf{y}}^{k+1}_{n+1} = -\tilde{\mathbf{H}}\tilde{\mathbf{y}}^{k}_{n+1} + \mathbf{C}\tilde{\mathbf{y}}_n + h\mathbf{F}(\mathbf{y}_m) \qquad (18)$$

where $\tilde{\mathbf{H}} = \mathbf{H} - diag\mathbf{H}$. Each element of $\tilde{\mathbf{x}}_{n+1}$ for each Gauss Seidel Method's step for the tri-diagonal matrix $\tilde{\mathbf{H}}$ is expressed as

$$\left(\tilde{\mathbf{x}}^{k+1}_{n+1}\right)_i = \begin{cases} \frac{1}{H_{ii}}\{\mathbf{b}_i - \left(\tilde{\mathbf{x}}^{k+1}_{n+1}\right)_{i+1} H_{ii+1}\} & (i=1) \\ \frac{1}{H_{ii}}\{\mathbf{b}_i - \left(\tilde{\mathbf{x}}^{k+1}_{n+1}\right)_{i-1} H_{ii-1} \\ \quad - \left(\tilde{\mathbf{x}}^{k+1}_{n+1}\right)_{i+1} H_{ii+1}\} & (2 \le i < N) \\ \frac{1}{H_{ii}}\{\mathbf{b}_i - \left(\tilde{\mathbf{x}}^{k+1}_{n+1}\right)_{i-1} H_{ii-1}\} & (i=N) \end{cases} \qquad (19)$$

where $\mathbf{b}$ is a constant vector corresponding to $\mathbf{C}\tilde{\mathbf{y}}_n + h\mathbf{F}(\mathbf{y}_m)$.

4. Simulation

4.1. Chua's circuit

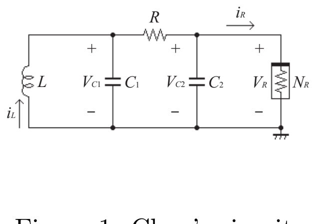

Figure 1: Chua's circuit.

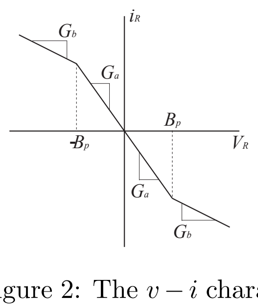

Figure 2: The $v-i$ characteristic of the nonlinear resistor.

Chua's circuit[5] is shown in Fig.1. It consists of two capacitors, one inductor, one linear resistor, and one nonlinear resistor. It is described by following set of normalized differential equations

$$\begin{aligned} \frac{dx}{dt} &= \alpha\left(y - x - f(x)\right) \\ \frac{dy}{dt} &= x - y + z \\ \frac{dz}{dt} &= -\beta y \end{aligned} \qquad (20)$$

where

$$f(x) = bx + \frac{1}{2}(a-b)\{|x+1| - |x-1|\}. \qquad (21)$$

The $v-i$ characteristic of the nonlinear resistor is shown in Fig.2.

When the nonlinear resistor behaves chaotic, Chua's Circuit makes a double scroll attractor.

4.2. Simulation result

The chaos attractor can be generated by using $\alpha = 8.99, \beta = 12.35, a = -1.09,$ and $b = -0.57$ in the given Eqs.(20) by Runge Kutta Method. If the differential equations(20) is not known, we must the chaos attractor from the observed time dataset.

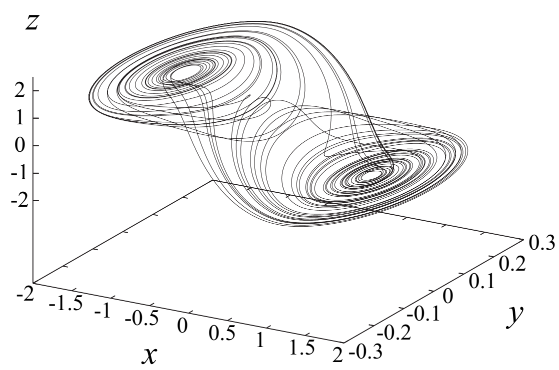

Figure 3: Chaos attractor.

The reconstructed attractor is shown in Fig.4. Comparison between original x and reconstructed x is shown in Fig.5, and comparison between original y and reconstructed y is shown in Fig.6, and comparison between original z and reconstructed z is shown in Fig.7.

We can reconstruct it by less than 36 datasets by using our new DT-CNN method described in section 3. When a single scroll changes toward an other scroll, error are obserbed especially. The error is being grown while the value of x is increasing. The error of y cannnot be almost obserbed at around 0. The error is grown while the value of y is decreasing or increasing from 0. However, the error of z cannot be almost obserbed constantly.

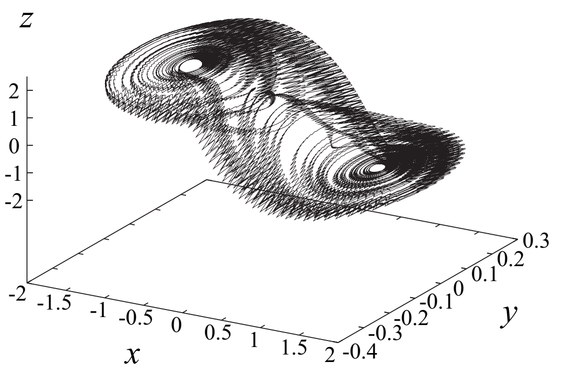

Figure 4: Reconstructed attractor.

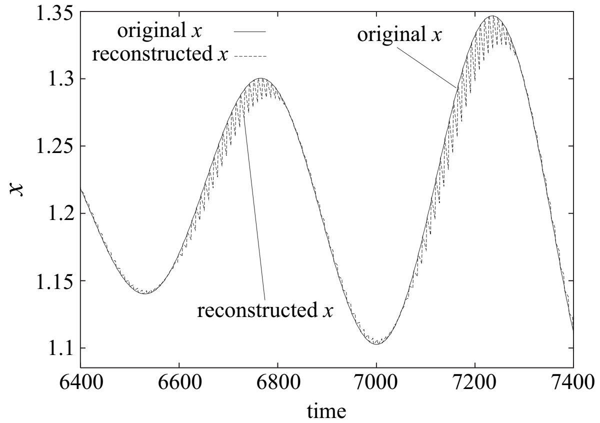

Figure 5: Comparison between original x and reconstructed x.

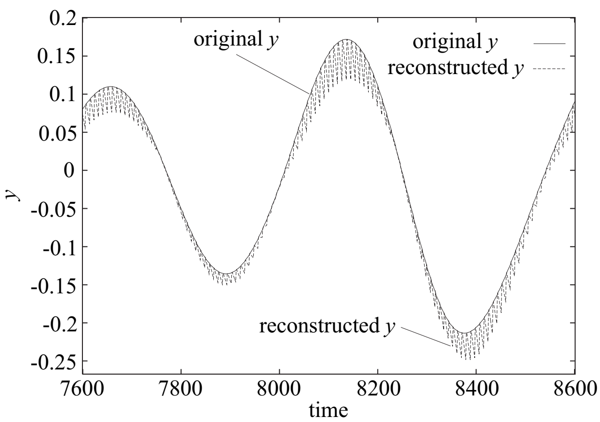

Figure 6: Comparison between original y and reconstructed y.

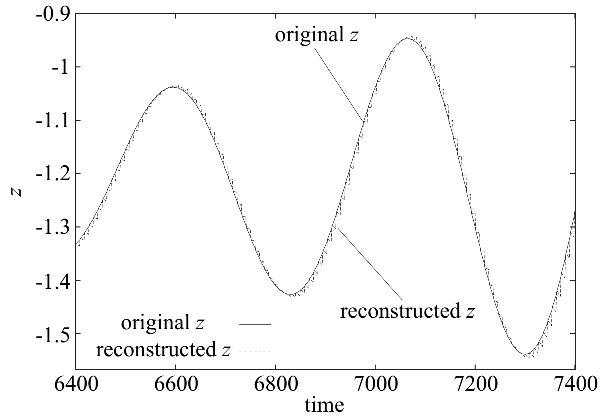

Figure 7: Comparison between original z and reconstructed z.

5. Conclusion

We could reconstruct the chaos attractor by DT-CNN which can be as machine learning solver. The attractor which is reconstructed from the observal time sequence based on our method was almost same as that of attractor derived from the differential equation of the Chua's Circuit. In the future, we will increase the number of the data and investigate the other attractor reconstruction by using CNN machine learning. It is necessary to improve the accuracy of the reconstruction.

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