2017 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2017, Cancun, Mexico, December 4-7, 2017

Particle Swarm Optimization for Calculating Local Bifurcation Point in One-dimensional Discrete Dynamical Systems

Yusho Tomimura†, Hiroaki Kurokawa††, Haruna Matsushita††† and Takuji Kousaka†

†Faculty of Science and Technology, Oita University
700 Dannoharu, Oita shi, Oita 870–1192, Japan
††School of Engineering, Tokyo University of Technology
1404–1 Katakura–machi, Hachioji, Tokyo 192–0982, Japan
†††Faculty of Engineering, Kagawa University
2217–20 Hayashi–cho, Takamatsu, Kagawa 761–0396, Japan
E-mail: tomimura@bifurcation.jp, takuji@oita-u.ac.jp

Abstract—In this paper, we propose the local bifurcation point derivation method of the one-dimensional discrete dynamical systems using the particle swarm optimization (PSO). First, we define the equation of the one-dimensional discrete dynamical system and show the local bifurcation point derivation method in that system. Next, we apply the proposed method to the Circle map which is a one-dimensional discrete map. Finally, we consider the effectiveness of the proposed method using the application result.

1. Introduction

Bifurcation phenomena occur in dynamical systems such as electrical systems and mechanical systems [1]. Derivation of the bifurcation set in the parameter space in the elucidation of the qualitative nature of these systems is standard. In mind, the engineering applications, various periodic points and the bifurcation point derivation methods have been proposed. However, most conventional techniques use of Newton's method. Therefore, the derivation of the differential information necessary for the calculation is complex, and the computer implementation is complicated. As an approach not using Newton's method, Refs. [2-5] derive the unstable periodic orbits in a chaos using the PSO [6]. In Ref. [7], Matsushita et al. derived the intersection point of the maximum power line and the period doubling bifurcation curve in the circuit model of the boost converter having the solar cell using the PSO. However, the method of Ref. [7] derives a fixed point by the manual calculation, so its versatility is low. Therefore, we previously proposed a period doubling bifurcation point(PD) derivation of the one-dimensional discrete dynamical system using the PSO [8]. Then in the Ref. [9], we showed problems and the solution method when we applied the method of Ref. [8] to the saddle-node bifurcation(SN). In this paper, we improve the method of Refs. [8][9]. And we show an algorithm deriving a local bifurcation point irrespective of the type.

2. Particle swarm optimization

2.1. Optimization problem

The optimization problem is a problem of search for $\boldsymbol{x}$ that maximizes or minimizes a particular objective function F. Various optimization methods have been proposed so far

$$F(\boldsymbol{x}) \leq C, \quad \boldsymbol{x} = (x_1, \cdots, x_D) \in \boldsymbol{R}^D. \tag{1}$$

Equation (1) is a problem of search for a suboptimal solution $\boldsymbol{x}$ that minimizes the objective function F, C is a threshold value, and $\boldsymbol{R}^D$ is a search space.

2.2. PSO

The PSO is an optimization method modeling movement of fish school and bird flock with swarm intelligence. The PSO has multiple solution candidates called particles, and each particle has position and velocity information. Each particle of the particle swarm has the property of search for an optimal solution while exchanging position information of each particle. Since the PSO has features such as not requiring gradient information by differentiation and allowing change to a warping direction of the solution, the PSO is good at problems including local solutions. Moreover, the PSO is attracting attention because it has advantages such as the speed of convergence, the simplicity of algorithm.

Consider the PSO algorithm for finding $\boldsymbol{x}$ satisfying the Eq. (1). The N particles move around in the D-dimensional search space R^D and search for the solution that minimizes the objective function F. Each particle is defined as position $\boldsymbol{a}^i = (a^i_1, a^i_2, \cdots, a^i_j, \cdots, a^i_D)$ and velocity $\boldsymbol{v}^i = (v^i_1, v^i_2, \cdots, v^i_j, \cdots, v^i_D)$, where i is the particle index, $1 \leq i \leq N$. Each particle starts its operation at the initial position and initial velocity given by random number in $\boldsymbol{R}^D$ and 0, respectively. The update equations of the solution are given by the following equations

$$\begin{aligned} v^i_j(k+1) &= wv^i_j(k) + \rho_1(p^i_j - a^i_j(k)) + \rho_2(g_j - a^i_j(k)), \\ a^i_j(k+1) &= a^i_j(k) + v^i_j(k+1). \end{aligned} \tag{2}$$

We define Pbest $\boldsymbol{p}^i = (p_1^i, p_2^i, \cdots, p_D^i)$ as the best position searched by the particle i. Also, we define Gbest $\boldsymbol{g} = (g_1, g_2, \cdots, g_D)$ as the best position among all the particles. In other words, Gbest is the provisional optimum solution at a time k. w is a parameter called inertia weight, ρ_1 and ρ_2 are random numbers in an appropriate range, and k is a discrete time corresponding to the number of iterations. The particle swarm continues the search until it satisfies $F(\boldsymbol{g}) < C$ or until it reaches the iteration limit. Also, Pbest and Gbest are updated with iteration.

3. Local bifurcation point derivation algorithm

Consider the next one-dimensional map

$$x_{n+1} = f(x_n, \boldsymbol{\lambda}). \tag{3}$$

$f \in \boldsymbol{R}$ is endomorphism, $x_n \in \boldsymbol{R}$ is the state variables, $\boldsymbol{\lambda} = \left(\lambda_1, \lambda_2, ..., \lambda_j, ..., \lambda_r\right) \in \boldsymbol{R}^r$ is the parameters. To derive a bifurcation point, information on the periodic point and the bifurcation parameter is required. In the proposed method, the PSO that derives the periodic point is called PSO_{pp}, and the PSO that derives the bifurcation parameter is called PSO_{bif}. We calculate PSO_{pp} and PSO_{bif} several times to derive local bifurcation points.

3.1. Search of periodic points using PSO : PSO_{pp}

Think about the search of p-periodic point. The periodic point condition is described as Eq. (4).

$$x_{n+p} = x_n = f^p(x_n, \boldsymbol{\lambda}). \tag{4}$$

If the objective function F_{pp} of PSO_{pp} defined as Eq. (5), we obtain p-periodic point regardless stability

$$F_{\text{pp}}(x_n) = |f^p(x_n, \boldsymbol{\lambda}) - x_n| < C_{\text{pp}}. \tag{5}$$

C_{pp} is the termination condition of PSO_{pp}.

3.2. Search of bifurcation parameter using PSO:PSO_{bif}

Consider the local bifurcation that occurs at the p-periodic point. In the method of Refs. [8][9], it was possible to search only the target bifurcation parameter. In the proposed method, when searching a bifurcation parameter, we calculate F_{bif} by doubling the target period p. As a result, the bifurcation parameters can be searched regardless of the type of local bifurcation. The objective function F_{bif} can be described by the following equation

$$F_{\text{bif}}(\boldsymbol{\lambda}) = \left| \frac{df^{2p}}{dx_n}(x_n, \boldsymbol{\lambda}) - 1.0 \right| < C_{\text{bif}}. \tag{6}$$

C_{bif} is the termination condition of PSO_{bif}. For the p-periodic point, the value derived by the method of Sec. 3.1 is used.

4. Application Example

To confirm the effectiveness of the previously proposed method, we apply the proposed method to the circle map of the Eq. (7)

$$\theta_{n+1} = \left(\theta_n + a - \frac{b}{2\pi}\sin 2\pi\theta_n\right) \bmod 1.0\,. \tag{7}$$

The objective functions F_{pp} and F_{bif} can be written as Eqs. (8) and (9) from Eqs. (5) and (6)

$$F_{\text{pp}}(\theta_n) = \left|\theta_{n+p} - \theta_n\right| < C_{\text{pp}}, \tag{8}$$

$$F_{\text{bif}}(a, b) = \left| \frac{\mathrm{d}\theta_{n+2p}}{\mathrm{d}\theta_n} - 1 \right| < C_{\text{bif}}. \tag{9}$$

Below, we attempt to derive local bifurcation points of 2-period for parameters a and b. The conditions of the objective function are $p = 2$, $C_{\text{pp}} = 10^{-5}$, $C_{\text{bif}} = 10^{-3}$, and the parameters of PSO are set as follows

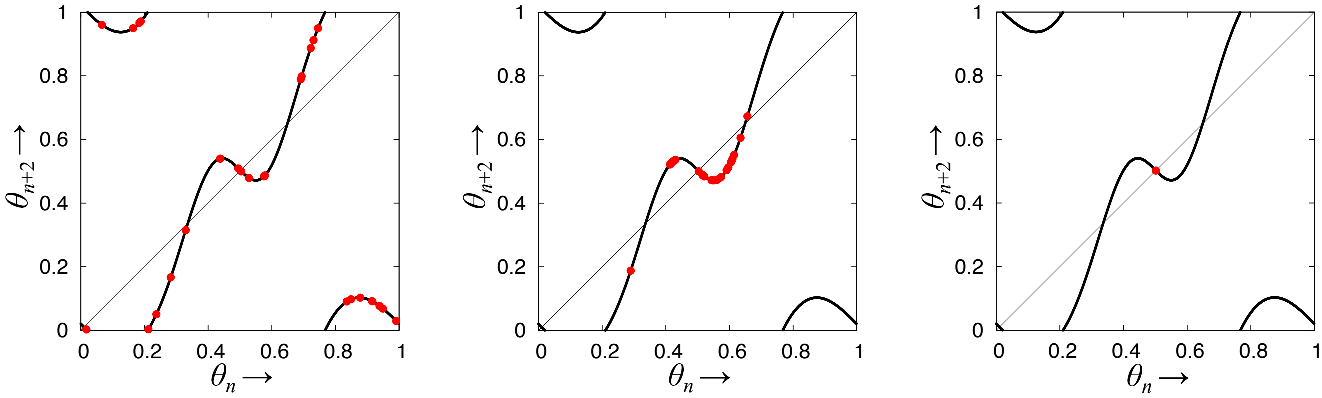
$$N = 30, \quad w = 0.729, \quad \rho_1 = \rho_2 \in \text{RND}[0, 1.414]. \tag{10}$$

Table 1 shows local bifurcation points a, b, periodic points θ_n, objective functions F_{pp} and F_{bif}, iteration count when PSO_{bif} satisfied the termination condition C_{bif}, and type of local bifurcation point derived by using the proposed method. We derived 10 local bifurcation points by using the proposed method. All F_{pp} and F_{bif} satisfied the termination condition C_{pp} and C_{bif}. In other words, we can derive the local bifurcation points using the proposed method. Also, all calculations were completed in 50 times or less. In addition, proposed method can derive local bifurcation points of either type PD or SN.

Figure 1 shows the movement of particles of F_{pp} in the parameter occurring 2-periodic period doubling bifurcation (PD2). Every time the calculation is repeated, the swarm searches for near the periodic point. Finally, a stable 2-periodic point is derived. In some cases, swarm derives unstable 2-periodic point. Figure 2 shows the movement of particles of F_{pp} in the parameter occurring 2-periodic saddle-node bifurcation (SN2). The particles search for 2-periodic point with the same movement as for PD2. Figure 3 shows the particles movement of PSO_{bif}. The lines in the figure correspond to the local bifurcation. Like PSO_{pp}, the particles of PSO_{bif} also approach the target line by repeating the calculation.

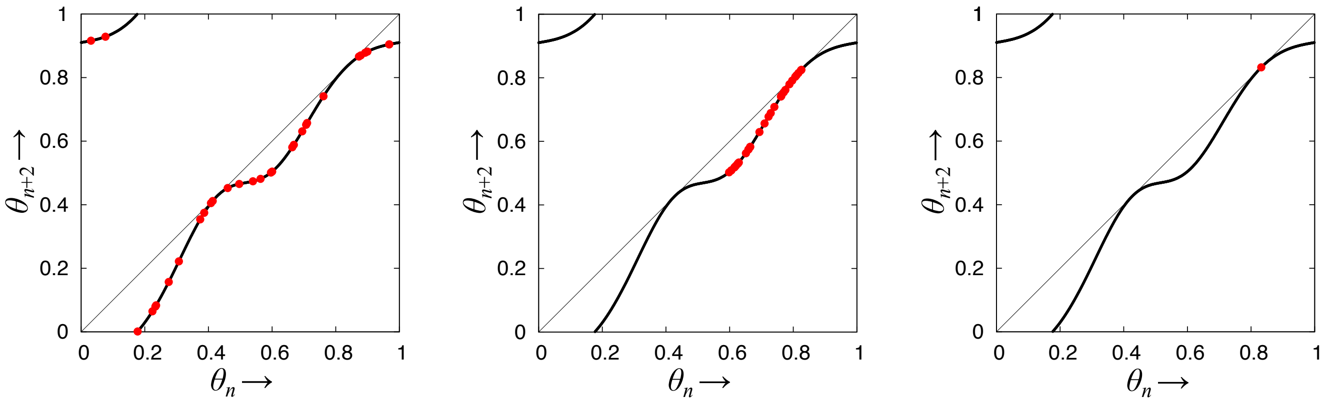
Table 1: Numerical results for deriving 2-periodic local bifurcation point.

a	b	θ_n	$F_{\mathrm{pp}}(\boldsymbol{g})$	$F_{\mathrm{bif}}(\boldsymbol{g})$	Itr.	Type
0.5059521132	1.4165127979	0.6487707922	4.8908×10^{-6}	9.6001×10^{-4}	11	PD2
0.4783598306	1.4460603428	0.4942191760	6.6715×10^{-6}	7.2322×10^{-4}	26	PD2
0.4691331608	0.9080361647	0.4270785228	5.9813×10^{-6}	4.2313×10^{-4}	13	SN2
0.5556329138	1.2474572659	0.5554151891	8.1097×10^{-6}	3.5855×10^{-4}	35	SN2
0.4974749144	1.4145193441	0.4991273891	0.4768×10^{-6}	7.8707×10^{-4}	11	PD2
0.4625336009	1.0068731515	0.4323519602	2.6732×10^{-6}	2.0096×10^{-4}	39	SN2
0.4732810807	0.8414914340	0.4234446068	1.5262×10^{-6}	2.9302×10^{-4}	12	SN2
0.5139263766	0.5992300591	0.5902006424	0.5943×10^{-6}	3.7821×10^{-4}	9	SN2
0.5307832891	1.4773922168	0.5081441749	4.9606×10^{-6}	8.1326×10^{-4}	16	PD2
0.4308177974	1.6911780513	0.9006508658	0.3309×10^{-6}	4.3864×10^{-4}	31	PD2



(a) $k = 0$ (b) $k = 10$ (c) End

Figure 1: Particles movement of $\mathrm{PSO_{pp}}$ in parameters occurring PD2.



(a) $k = 0$ (b) $k = 10$ (c) End

Figure 2: Particles movement of $\mathrm{PSO_{pp}}$ in parameters occurring SN2.

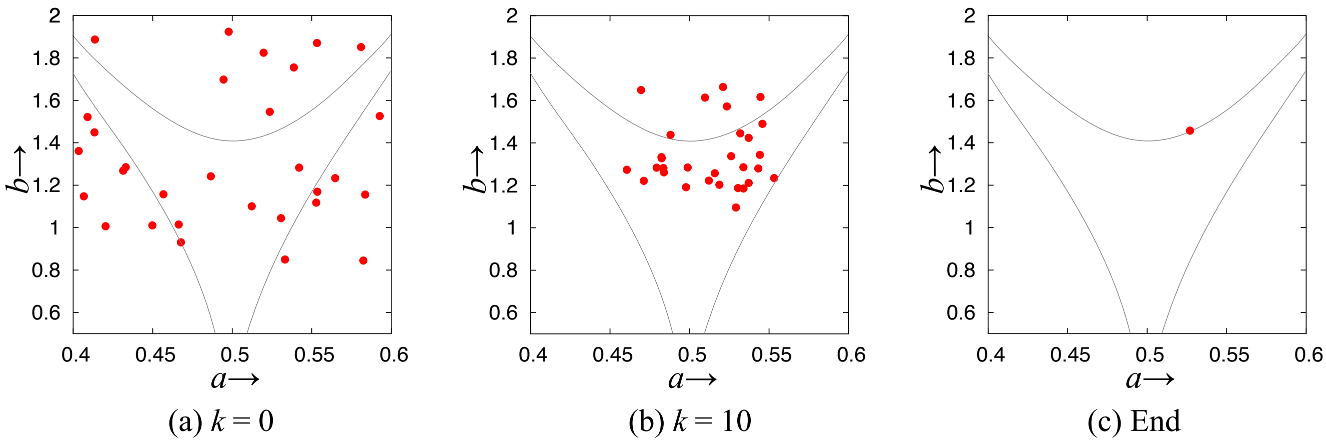


Figure 3: Particles movement of $\mathrm{PSO_{bif}}$.

5. Conclusions

In this paper, we proposed a method of deriving a local bifurcation point of a one-dimensional discrete dynamical system using the PSO. First, we explained the algorithm of PSO which is one of optimization methods. Next, we define the equation of one-dimensional discrete dynamical system and set the objective function to derive the periodic point and bifurcation parameters. We also proposed an algorithm to derive local bifurcation points by applying two PSOs repeatedly. Also, we applied the proposed method to the Circle map which is a one-dimensional discrete map. Finally, we examined the performance of $\mathrm{PSO_{pp}}$ and $\mathrm{PSO_{bif}}$ using concrete numerical examples and confirmed the effectiveness of the proposed method.

Acknowledgments

This work was supported by JSPS KAKENHI Grant Number 16K16124.

References

[1] http://www.scholarpedia.org/article/Bifurcation

[2] Yu-ping Tian, "An Optimization Approach to Locating and Stabilizing Unstable Periodic Orbits of Chaotic Systems," Internatinal Journal of Bifurcation and Chaos, Vol. 12, No. 5, pp. 1163–1172, 2002.

[3] C. O. Ourique, E. C. Biscaia and J. C. Pinto, "The Use of Particle Swarm Optimization for Dynamical Analysis in Chemical Process," Computers and Chemical Engineering, Vol. 26, pp. 1783–2793, 2002.

[4] F. Gao, H. Gao, Z. Li, H. Tong and J. J. Lee, "Detecting Unstable Periodic Orbits of Nonlinear Mappings by a Novel Quantum-behaved Particle Swarm Optimization Non-Lyapunov Way," Chaos Solitons and Fractals, Vol. 42, pp. 2450–2463, 2009.

[5] K. E. Parsopoulos and M. N. Vrahatis, "Computing Periodic Orbits of Nondifferentiable/Discontinuous Mappings Through Particle Swarm Optimization," in Proc. of IEEE SIS'03, pp. 34–41, 2003.

[6] J. Kennedy and R. Eberhart, "Particle swarm optimization," Proc. Int. Conf. on Neural. , pp.1942–1948, 1995.

[7] H. Matsushita, T. Saito, "Application of Independent-Minded Particle Swarm Optimization to Parameter Search in Switched Dynamical Systems," IEICE, Vol. 2, No. 4, pp. 458–471, 2011.

[8] H. Matsushita, Y. Tomimura, H. Kurokawa and T. Kousaka, "Period Doubling Bifurcation Point Detection Strategy with Nested Layer Particle Swarm Optimization," International Journal of Bifurcation and Chaos, Vol. 27, Issue 7, pp. 1750101-1–1750101-16, 2017.

[9] H. Matsushita, Y. Tomimura, H. Kurokawa and T. Kousaka, "Behavior of Bifurcation Point Detecteing Method by PSO on Saddle-Node Bifurcation," Proc. NOLTA Society Conf. IEICE, p. 234, 2016.