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MAP Code Acquisition Shows Superiority of Bologna Codes in a Class of Multiuser CDMA Systems

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***Abstract*— A maximum *a posteriori* probability (MAP) code acquisition scheme is proposed which utilizes all the available information by exploiting stochastic properties of spreading codes. Application of this method shows: i) code acquistion superiority of Bologna codes (Markov codes having eigenvalue of the transition probability matrix $\lambda = \sqrt{3} - 2$) over independent and identically distributed (i.i.d.) codes in chip-asynchronous multiuser systems, when acquisition time is fixed and ii) comparsion of Markov codes with different values of λ shows that code acquisition performance is best for Bologna codes.**

I. Introduction

It was reported in [1] that bit error rate (BER) performance of Bologna codes[1] was superior to i.i.d. random codes as well as Kasami and Gold codes in singleuser asynchronous (in this paper the term asynchronous implies chip-asynchronous systems) direct sequence (DS) CDMA systems. This result was supported by several following papers [3]–[5]. However, all these contributions assumed that synchronization between the transmitter and receiver has already been established. The process of synchronization comprises of code acquisition, by which time delay between received signal and locally generated signature waveform is brought within a fraction of a chip, and tracking, which performs fine-tuning and delay is further reduced. Code acquisition is of crucial importance in DS/CDMA systems. Spreading code sequences have noise-like properties so correct demodulation is not possible if synchronization is off even by one chip. Thus, performance of a DS/CDMA system depends both on BER and synchronization. Without comparing the synchronization performance it was not possible to declare whether Bologna codes were better than i.i.d. ones. Synchronization issue was discussed in [6] and [7] using conventional acquisition methods and performance was observed to be unaffected by type of code in singleuser case.

Multiuser detection (MUD) schemes can significantly improve BER performance of CDMA systems [8]. In [9], using a parallel interference canceller, BER performance of Bologna codes was shown to be better than i.i.d. codes in asynchronous environment. Superiority of Bologna codes in MUD systems, in terms of synchronization performance, was not yet established. Using data and timing estimates of already synchronized users, which are available at base station, multiple access interference (MAI) can be cancelled before code acquisition (see Fig. 1) thus improving timing accuracy [10]–[13]. In [10]–[12] coventional code acquisition is performed after MAI cancellation. We proposed a scheme based on *a posteriori* probability calculation for chip-synchronous systems in [13] and for asynchronous systems in [14]. The method proposed in [14] had a flexible acquisition time without any upper bound and the stress was on acquiring signal with a theoretically guaranteed minimum probability of correct acquisition. In certain practical applications, acquisition has to be performed after a fixed time. In such case, MAP algorithm proposed in this paper will be more suitable. Conventional acquisition schemes show performance to be independent of code type. However, we apply proposed algorithm for both Bologna and i.i.d. codes and the computer simulations' results show higher correct acquisition probability for Bologna codes. The proposed method has also been applied for acquisition of Markov codes having different values of λ (eigenvalue of transition probability matrix). Computer simulations show that performance of Markov codes having $\lambda = \sqrt{3} - 2$, i.e., Bologna codes is superior to others.

II. System Model

Consider a baseband DS/CDMA system with spreading factor N and J users. Let T_c be the chip time, $n(t)$ be additive white Gaussian noise. Data and SS code signals of j-th user be,

$$d^j(t) = \sum_{p=-\infty}^{\infty} d_p^j u_T(t - pT), \quad (1)$$

$$X^j(t) = \sum_{q=-\infty}^{\infty} X_q^j u_{T_c}(t - qT_c), \quad (2)$$

where $u_D(t) = 1$ for $0 \le t < D$, $u_D(t) = 0$ otherwise and $d_p^j, X_q^j \in \{+1, -1\}$. The received signal is given by

$$r(t) = \sum_{j=1}^{J} d^j(t - t_j) X^j(t - t_j) + n(t), \quad (3)$$

[1]This fact was pointed out by researchers from Bologna university, thus, Markov codes having $\lambda = \sqrt{3} - 2$ are referred to as Bologna codes [2].

where t_j is the the time delay of j-th user. We have assumed a coherent receiver and all users are transmitting with equal powers. Let T denotes the time period of one data symbol such that $T/T_c = N$ and $T_c = 1$. Purpose of code acquisition process is to find the code chip corresponding to time delay t_j where $0 \le t_j < T$. Components of data sequence are assumed to be equi-probable, i.i.d. binary random variables i.e., Prob. $(d_p^j = +1)$ = Prob. $(d_p^j = -1) = 1/2$. Code sequences are generated by mutually independent Markov chains whose components are equi-probable. If λ be the eigenvalue, other than 1, of transition probability matrix, for $1 \le i, j \le J$ and $n, m \ge 0$

$$E_{X^i}[X_n^i] = 0,$$
$$E_{X^i X^j}[X_n^i X_m^j] = \begin{cases} \lambda^{|m-n|} & \text{if } i = j, \\ 0 & \text{otherwise} \end{cases}, \tag{4}$$

where $E_Z[.]$ denotes expectation with respect to the distribution of Z. It is clear that i.i.d. codes may be regarded as a special case of Markov codes when $\lambda = 0$. Suppose Z_n^i models the i-th correlator's output (i-th user is target user) at time instant n. Then, ignoring channel noise

$$Z_n^i = \int_{nT_c}^{nT_c+T} r(t) X^i(t - nT_c) dt = S_n^i + \sum_{j=1, j\neq i}^{J} I_n^{i,j}. \tag{5}$$

The correlator output is composed of desired signal plus self-interference S_n^i of i-th user and MAI $I_n^{i,j}$ due to the j-th user.

III. Previous results

Let J be the total number of users in the system out of which, K are already synchronized and L are new users yet to be synchronized. Using SS codes X^j, data estimates $\hat{d}^j$ and timing estimates $\hat{t}_j$ of already synchronized users $1 \le j \le K$, their signals are subtracted from total received signal and the residual signal is

$$s(t) = r(t) - \sum_{j=1}^{K} \hat{d}^j (t - \hat{t}_j) X^j (t - \hat{t}_j). \tag{6}$$

Assume $\hat{t}_j = t_j$ i.e., perfect timing estimates. Any error in data estimation of already synchronized users will increase variance of correlator output.

It has been shown in [3] that in asynchronous system, variance of normalized MAI per user is

$$E_D[Var_X[I_n^{i,j}/\sqrt{N}]] = \sigma_I^2 = \frac{2}{3} . \frac{1 + \lambda + \lambda^2}{1 - \lambda^2}, \tag{7}$$

where E_D denotes expectation with respect to data. Explicit expression for variance of normalized self-interference is given in [16]. However, if we average that variance over time, for non-synchronization chip, it can be regarded as one more user contributing to MAI. Thus,

$$E_D[Var_X[S_n^i/\sqrt{N}]] = \sigma_S^2 = \frac{2}{3} . \frac{1 + \lambda + \lambda^2}{1 - \lambda^2}. \tag{8}$$

Let $p = \lfloor (n - t_j)/N \rfloor$ where n means n-th time instant, $1 \le j \le K$, $j \ne i$ and

$$M_1 \triangleq \{j | \hat{d}_p^j \ne d_p^j, 1 \le j \le K\}, \tag{9}$$
$$M_2 \triangleq \{j | \hat{d}_{p+1}^j \ne d_{p+1}^j, 1 \le j \le K\}. \tag{10}$$

If $\sigma_{H_1}^2$ denotes subtracted correlator output variance corresponding to synchornization chip and $\sigma_{H_0}^2$ denotes the variance otherwise, it has been shown in [14] that

$$\sigma_{H_1}^2 = \sigma_I^2 (L - 1 + 2m), \tag{11}$$
$$\sigma_{H_0}^2 = \sigma_{H_1}^2 + \sigma_S^2. \tag{12}$$

where $m = |M_1| + |M_2|$ ($|A|$ means cardinality of set A) and $0 \le m \le 2K$ from independence of data.

From [14], expectation of normalized MAI with respect to codes is $E_X[I_n^{i,j}/\sqrt{N}] = 0$. Regarding t_i to be uniformly distributed between $-1/2 \le t_i \le 1/2$, expectation of normalized self-interference corresponding to synchronization chip is

$$E_{X,T_i}[S_n^i/\sqrt{N}] = \frac{3 + \lambda}{4} \sqrt{N}, \tag{13}$$

and corresponding to the non-synchronization chip is

$$E_{X,T_i}[S_n^i/\sqrt{N}] = 0 \quad \text{as} \quad N \to \infty \tag{14}$$

where T_i is a random variable for t_i. Normalized correlator output $Y_n = Z_n^i/\sqrt{N}$ (superscript i has been ignored for simplicity) follows density functions $f_{H_1}(y; \sigma_{H_1}^2)$ when n is the synchronization chip and $f_{H_0}(y; \sigma_{H_0}^2)$ otherwise, given by

$$f_{H_1}(y; \sigma_{H_1}^2) = \frac{1}{2} \text{ nor } (y; \mu, \sigma_{H_1}^2) + \frac{1}{2} \text{ nor } (y; -\mu, \sigma_{H_1}^2), \tag{15}$$
$$f_{H_0}(y; \sigma_{H_0}^2) = \text{nor } (y; 0, \sigma_{H_0}^2), \tag{16}$$

where, y is a realization of Y, nor $(y; u, \sigma^2) = 1/\sqrt{2\pi\sigma^2}$ exp $\left(-(y-u)^2/2\sigma^2\right)$, $\sigma_{H_1}^2$ is given by (11), $\sigma_{H_0}^2$ by (12) and μ by (13).

IV. Acquisition method

Code acquisition is a hypothesis testing problem where correct hypothesis, i.e., *synchronization* cell is denoted as H_1 while others, i.e., *non-synchronization* cells as H_0. Correlation is performed between the incoming and local signals for different chip delays and a function, known as detector, declares hypothesis H_1. Then code tracking loop is started to achieve fine tuning. In one type of conventional acquisition methods, correlation values for all chip delays are stored in memory and the chip corresponding to highest value is declared as H_1. In the other type, an energy detector compares each value of the correlation function with a threshold; if found greater, acquisition is declared otherwise, next correlator output is examined [15]. In both cases, only the information of current chip is used. Other chips which have already been examined and found to be H_0, if their information is exploited, acquisition performance can be improved.

Let $b(n, x, p)$ denotes a binomial distribution defined as $b(n, x, p) = \begin{pmatrix} n \\ x \end{pmatrix} (1 - p)^{n-x} p^x$. Then, at a particular chip interval, probability of m synchronized users having data estimation errors is $b(2K, m, p)$ (from independence of data), where p is the BER of synchronized users. When acquisition starts, synchronization chip is uniformly distributed with probability 1/*N*. After observing correlator output at time instant n, when k-th chip is the synchronization chip, conditional probability of corresponding chip being H_1 is given by the following joint density function

$$\varphi(\mathbf{y}_n \mid k) = \prod_{\substack{t=0 \\ t \neq k \bmod N}}^{n} A_{H_0}(y_t \mid m) \times \prod_{\substack{t=0 \\ t = k \bmod N}}^{n} A_{H_1}(y_t \mid m) \tag{17}$$

where

$$\mathbf{y}_n = y_0, y_1, ..., y_{n-1}$$

$$A_{H_1}(y \mid m) = \sum_{m=0}^{2K} b(2K, m, p) f_{H_1}(y; \sigma^2_{H_1}) \tag{18}$$

$$A_{H_0}(y \mid m) = \sum_{m=0}^{2K} b(2K, m, p) f_{H_0}(y; \sigma^2_{H_0}). \tag{19}$$

Strictly speaking (17) requires independence, which is true for the chip-synchronous case, but in asynchronous system, correlator outputs will not be independent. However, for sufficiently large integer NJ (N is codelength and J is the number of users), covariance is very small which is neglected and (17) is applied for both i.i.d. and Markov codes. Furthermore, it was observed that in asynchronous case, sampling once per chip, sometimes it was not possible to detect the synchronization chip. Sampling more than once per chip would introduce correlation and evaluation by (17) would be inaccurate. Thus, two parallel correlators operating independently with a time difference of $T_c/2$ (see Fig. (2)) are used. After a fixed time, (17) is evaluated for correlator 1 and correlator 2, for each possible delay and MAP algorithm will declare the chip having maximum value as H_1.

Simulation Results

Computer simulations are performed with spreading factor N = 63, new users L =1 and already synchronized users K =10, 20,..., 40. Code acquisition is declared after two data periods in the favour of chip having maximum value given by (17). The channel is assumed to have a single path. Knowledge of system BER is required which is given by the Q-function of SINR (signal-to-interference and noise ratio) where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(\frac{-u^2}{2}\right) du$. i.i.d. codes are generated by one dimensional chaotic maps proposed in [17].

Fig. 3 shows that correct acquisition probability of Bologna codes is always higher than i.i.d. codes and the performance difference increases steadily with the number of already synchronized users. We have also performed simulations for Markov codes having different values of λ and results are presented in Figs. 4, 5, 6 and 7. It can be seen that in all cases correct acquisition probability is highest for $\lambda = \sqrt{3} - 2$ showing the code acquisition superiority of Bologna codes.

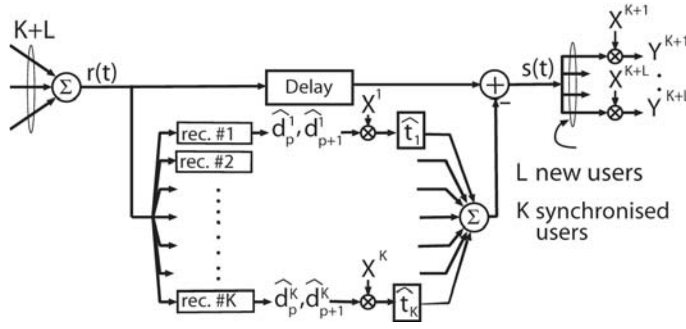

Fig. 1. Base station model using MAI cancellation

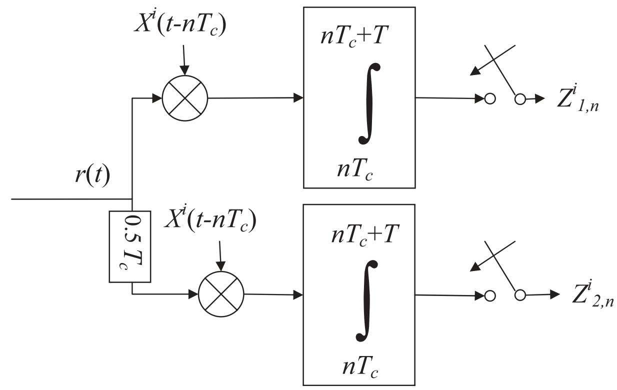

Fig. 2. Two correlators operating independently with time difference of $0.5T_c$

V. Conclusions

MAP code acquisition in asynchronous MUD system shows superiority of Bologna codes over i.i.d. codes when acquisition time is fixed. Furthermore, performance of Bologna codes is found to be superior to other Markov codes having different values of λ. We have considered a simple channel model consisting of a single path. The more practical case of multi path channels will be considered in future research.

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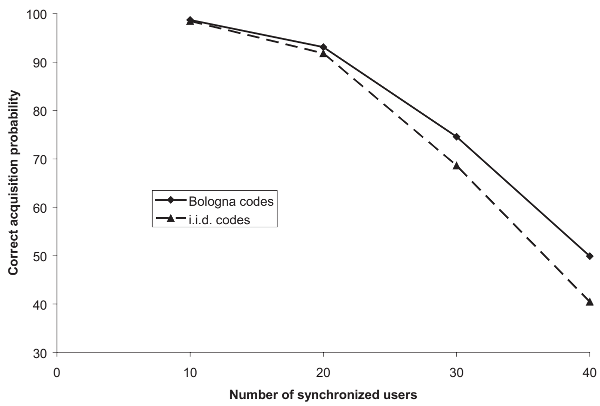

Fig. 3. Correct acquisition probability comparison of Bologna and i.i.d. codes with fixed acquisition time

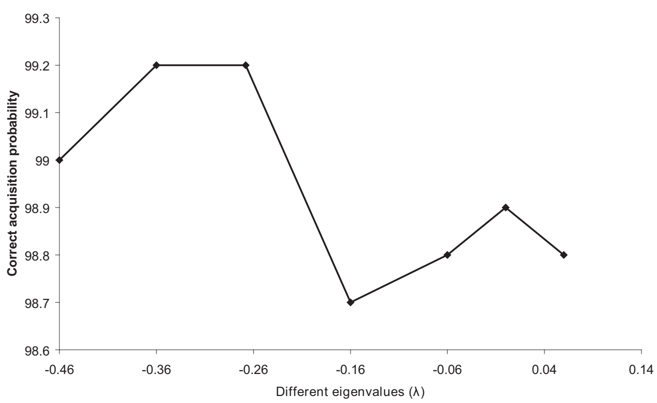

Fig. 4. Correct acquisition probability comparison for different values of λ with already synchronized users $K = 10$

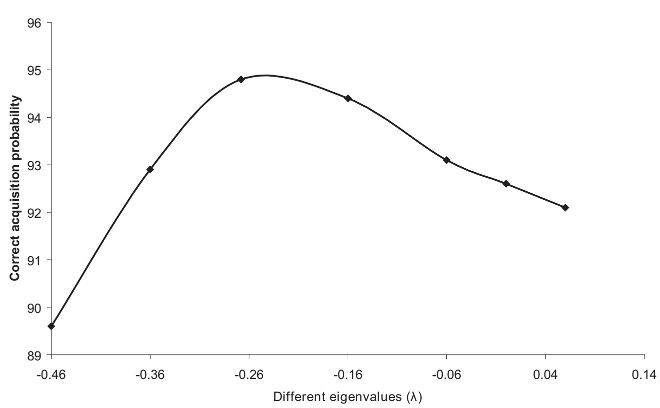

Fig. 5. Correct acquisition probability comparison for different values of λ with already synchronized users $K = 20$

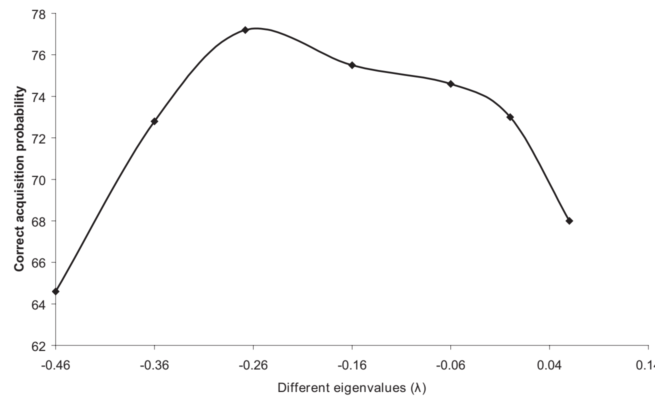

Fig. 6. Correct acquisition probability comparison for different values of λ with already synchronized users $K = 30$

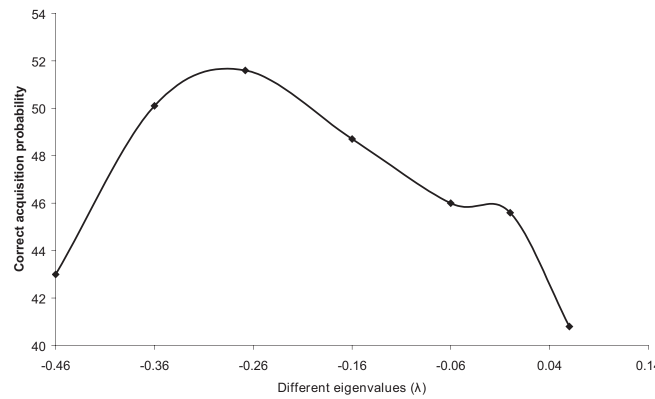

Fig. 7. Correct acquisition probability comparison for different values of λ with already synchronized users $K = 40$

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