Effects of synaptic plasticity
on the neural network topology

Akira SAKUMA   Osamu ARAKI

Institute of Pure and Applied Physics, Graduate School of Science, Tokyo University of Science
1-3 Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan
Email:j1205627@ed.kagu.tus.ac.jp, o-araki@rs.kagu.tus.ac.jp

Abstract—The purpose of this research is to clarify how the topological properties of a neural network change by a rule of synaptic plasticity. To evaluate the topology, we use three variables: (1) average path length (L) that denotes an average of minimum distance between two neurons, (2) clustering coefficient (γ) that denotes an average fraction of actual number of connections over total possible edges in the neighborhood of a neuron, and (3) group coefficient (G) that denotes an average number of connections between any groups of neurons, which is an extension of γ from a neighborhood to any groups of a neuron. We simulated one dimensional circulated neurons of integrate & fire models with Hebbian or STDP synaptic plasticity alternatively. When the initial topology is "regular" (with large γ, G, and L), L decreases in spite of maintained large values of γ and G as the learning proceeds. When the initial topology is "random" (with small γ, G, and L), G increases while L is kept small. These results suggest the synaptic plasticity changes the network topology so that each neuron transfers information more efficiently.

1. Introduction

Learning methods of neural networks by synaptic plasticity have been studied so far. Hebbian rule is one of the most known rules of synaptic plasticity [1]. On the other hand, recently, spike-timing-dependent plasticity (STDP) has been discovered, in which synaptic weights change depending on the difference between pre- and post-synaptic spike timings [2]. And some studies reported that STDP have relationship to the synchronous firing [3][4].

On the other hand, Watts showed that in real networks the average path length L is relatively small and the clustering coefficient γ is relatively large, which are explained by the "Small-World" model. The "Small-World" networks include Film actor relationship, the power grid, neural network of C. *elegans*, for example [6]. In a neural network model with small L and large γ such as a Small-World, synchronous spikes occur more frequently than that with large L and large γ [5]. This implies that the network topology affects the spike patterns and that these variables represent the essential property of the topology.

From these, we assume that L and γ represent essential property of network topology and examine how the two variables L and γ are changed in neural networks by the Hebbian rule or STDP. In this paper, we simulated one dimensional circulated network of integrate & fire neuron models with Hebbian or STDP synaptic plasticity alternatively. We evaluate the network topology by L, γ, and G. In this paper, we propose a new measure G because γ cannot evaluate clustering among spatially separated but strongly connected neurons. G is an extended measure of γ from neighborhood to any groups of a neuron. When the initial topology is "regular" (with large γ, G, and L), L decreases in spite of maintained large values of γ and G as the learning proceeds. When the initial topology is "random" (with small γ, G, and L), G increases while L is kept small. These results suggest the synaptic plasticity changes the network topology so that each neuron transfers information more efficiently.

2. Method

2.1. Model

In this paper, we used a neural network model composed of leaky integrate & fire neurons. This model obeys the following equations:

$$x_i(t+1) = \sum_j^N s_j w_{ij} y_j(t)$$
$$+ A e_i(t) + B x_i(t) \quad (1)$$

$$y_i(t) = \begin{cases} 1 & \text{if } x_i(t) > \theta \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

where $x_i(t)$ is an internal activity of i-th neuron at time t, $y_j(t)$ is an output value of j-th neuron at time t, N is the total number of the neurons, s_j is the parameter of whether j-th neuron is excitatory or inhibitory, w_{ij} is the synaptic weight from j-th neuron to i-th neuron, A is the size of an external input, and $e_i(t)$ is the on-off signal of external input for i-th neuron, B is the decay parameter at every step ($0<B<1$), and θ is the threshold for firing. And, each neuron has absolute refractory period that has a constant interval (3 steps). Each cell has an excitatory or inhibitory property alternatively, which determines the value of s_j. In the simulations, about 16% of all cells are inhibitory, and the rest (84%) are

excitatory. For simplicity, time is assumed to be discrete.

We used two kinds of external input patterns. First, a spatiotemporal pattern is repeatedly input (pattern A). Pattern A consists of periodic Poisson trains. Pattern B is a temporally correlational pattern superimposed with uncorrelated signals which consist of aperiodic Poisson trains (Fig.1).
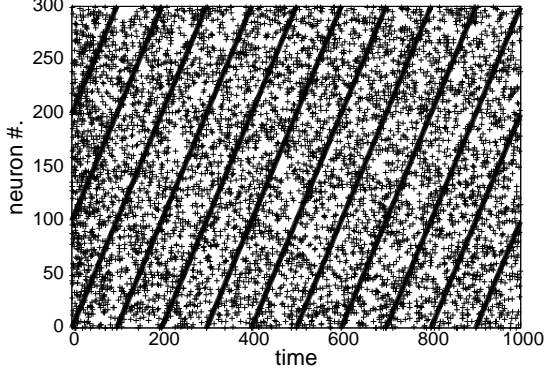

Figure 1: Input signals with spatiotemporal correlation (pattern B)

2.2. Learning rules of synaptic plasticity

In this simulation, we used two rules of synaptic plasticity which are Hebbian rule and STDP. And the rules are applied to excitatory neurons only, for simplicity.

Hebbian rule is a learning rule that the synaptic weight w_{ij} is potentiated if both of j-th and i-th neuron's firing rates are high. To prevent the network from bursting, we use the following covariance rule:

$$w_{ij}(t) = A_h(\bar{y}_j - B_h)(\bar{y}_i - B_h) + C_h w_{ij}(t-1), \quad (3)$$

where $\bar{y}_i$ and $\bar{y}_j$ are the firing rates of i-th and j-th neuron, A_h is the learning rate, B_h is the spontaneous discharge rate, and C_h is the decay parameter at every step. We assume that the firing rates are calculated from the spike trains for 10 steps before the current time.

STDP is a learning rule that synaptic weights are changed by the timing between pre- and post-neuron's spikes. In this simulation, we assume that the window function has a shape of an inverse proportion. The STDP rule of potentiation or depression obeys the following equation with using different parameters:

$$w_{ij}(t) = y_j(t) \sum_{n=-B_s, n\neq 0}^{B_s} \frac{A_s}{n} y_i(t+n) + C_s w_{ij}(t-1), \quad (4)$$

where n is the temporal difference between j-th neuron and i-th neuron. Synaptic weights are potentiated by $n > 0$ and depressed by $n < 0$. A_s is strength of potentiation or depression, B_s is the ranges of the time window of STDP, and C_s is the decay parameter at every step ($0 < C_s \leq 1$). In STDP, C_s is 1 (no decay factor).

2.3. The essential connections

To compare the topology, we should assume that the total connections on which the parameters are calculated are constant. However, every connection should have a continuous value of synaptic weight for learning. Thus, we regard the fixed number of connections as essential connections. We defined lc_{ij} as the essential connection from j-th neuron to i-th neuron. $lc_{ij}=1$ denotes connected connection, and $lc_{ij}=0$ denotes unconnected connection. To keep the number of essential connections constant, we decide the value of lc_{ij} depending on w_{ij}. First, we sort synaptic weights (w_{ij}) in order of the strength. Second, we select 10% (average number of connections in initial state) of synaptic weights in descending order. Finally we set $lc_{ij}=1$ for the pair of i and j of the selected synaptic weights w_{ij}, and the other connections are $lc_{ij}=0$.

2.4. Three parameters for network topology

2.4.1. Average path length

Assuming that lv_{ij} denotes the shortest distance from j-th neuron to i-th neuron, average path length L can be computed as follows:

$$L = \frac{1}{N(N-1)} \sum_{i=0}^{N} \sum_{j=0, j\neq i}^{N} lv_{ij}. \quad (5)$$

When the shortest distance from j-th neuron to i-th neuron passes k-th neuron, lv_{ij} denotes the sum of lc_{kj} and lc_{ik}. The larger the path length L is, the more the cost of information transformation in the network has.

2.4.2. Clustering coefficient

The clustering coefficient γ denotes an average degree of clustering of each neuron's at neighborhood. Assuming that D denotes the radius of a neuron's neighborhood, γ is calculated as follows:

$$\gamma = \frac{1}{N} \sum_{i=0}^{N} \frac{R_i}{I_i} = \frac{1}{N} \sum_{i=0}^{N} \frac{\sum_{j=i-D}^{i+D} lc_{ij}}{2D}, \quad (6)$$

where R_i is the number of real connections around i-th neuron's neighborhood, and I_i is the maximum number of connections around i-th neuron's neighborhood. The larger value of γ means that the more local connections (more clustering) exist.

2.4.3. Group coefficient

We propose the group coefficient G which Frage degree of clustering at each group of neurons.

$$G = \frac{1}{N} \sum_{i=0}^{N} \frac{R_{gi}}{I_{gi}} = \frac{1}{N} \sum_{i=0}^{N} \frac{\sum_{j=i}^{i+D}(\sum_{k=i}^{i+D} lc_{jk} + \sum_{m\in N}^{m+D} lc_{jm})}{2D^2}, \quad (7)$$

where R_{gi} is the number of real connections among around i-th neuron's neighborhood (group A) and another m-th neuron's neighborhood (group B). m is selected so that R_{gi} has the maximum value. I_{gi} is the maximum number of connections among group A and B. In this simulation, we used $D = 5$ at calculating γ and G. We consider that G can evaluate clustering among spatially separated but strongly connected neurons.
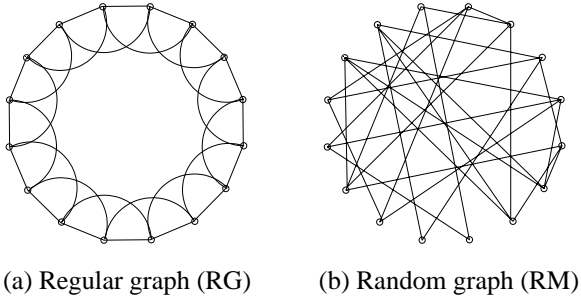

(a) Regular graph (RG)   (b) Random graph (RM)

Figure 2: Graphs of one dimensional circulated neurons

2.5. Initial condition

In this simulation, we assume two initial conditions that are one-dimensional circulated neurons. One is a regular graph (RG) which connects each neuron's neighborhood (refer to Fig.2(a), N=16,D=2). Another one is a random graph (RM) which connects two neurons randomly (refer to Fig.2(b), N=16). The number of all neurons is 300. Each neuron has random initial potentials distributed plainly from 0 to 80% of the value of θ. We assume an average number of connections is 10% in the initial state for both of RG and RM. Therefore D of RG is 15. We assume that initial synaptic weights are w_{ij}=0.45 at the initial connections. And synaptic weights can change between 0.000185 to 1.9 because of synaptic plasticity. We set 0.000185 for the minimum synaptic weight because we make a chance to be potentiated even for completely depressed synaptic weights.

A spatiotemporal pattern is repeatedly input as an external input (pattern A) on a RG. On the other hand, a temporally correlational pattern is input (pattern B) on a RM. This is because stronger correlational patterns will be needed for the topological change of RM than that of RG. The pattern A consists of periodic Poisson trains (av.ISI=50, A=5.0, period=50steps). The pattern B consists of aperiodic Poisson trains (av.ISI=50, A=5.0) plus spatiotemporal correlations (Fig.1). Each neuron's parameter is as follows: θ=12, B=0.95, s=2.73 or -4.0. Parameters of Hebbian rule are A_h=0.12, B_h=0.05, C_h=0.999, and parameters of STDP are A_s=10, B_s=5 with $n > 0$ (potentiation) and A_s=8, B_s=13 with $n < 0$ (depression).

3. Simulation results

3.1. Spike patterns

Applying the Hebbian rule to both of RG and RM, we observe synchronous firings. Fig.3 shows an example of spike patterns at RG. Also applying STDP at RG, we usually observe synchronous firings of some neurons. But applying STDP at RM, we cannot observe synchronous firings.
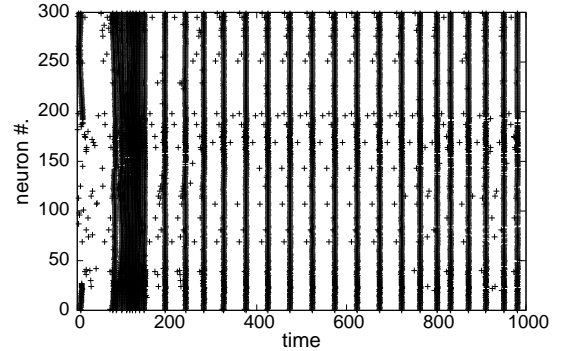

Figure 3: Spike pattern with Hebbian rule on RG

3.2. Modulation of topology

Owing to the synaptic plasticity at RG, the initial topology of regular connections changes. Fig.4(a) and (b) show the essential connections after application of Hebbian rule and STDP, respectlvely. We can see that some of essential connections are dispersed from the initial diagonal connections and are clustered.

After the synaptic plasticity at RM, we can observe the
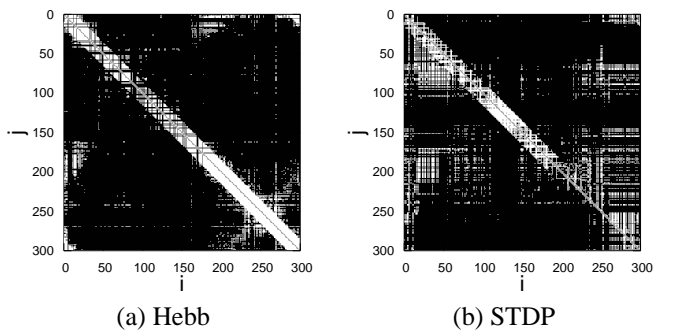

(a) Hebb   (b) STDP

Figure 4: Synaptic change (essential connections) of RG

clusters that run parallel with diagonal line (Fig.5). We consider that these parallel clusters are due to the strong spatiotemporal correlations in pattern B (Fig.1). The clusters are stronger in Hebbian rule (Fig.5(a)) than those in STDP (Fig.5(b)).

Fig.6 and Fig.7 show the average and standard deviation of L, γ, and G for 50 trials. In the case of RG, we observe that L decreases in spite of maintained large values of γ and
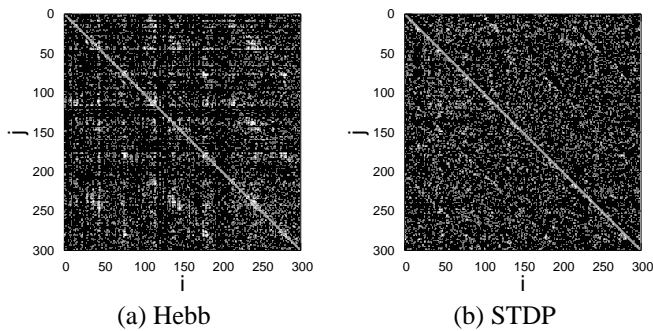
(a) Hebb   (b) STDP

Figure 5: Synaptic change (essential connections) of RM

Figure 7: Relation of *L* and *G*

Figure 8: *L* and γ of the "Small-World" graph

G. On the other hand, in the case of RM, we observe that *L* is kept small. The value of γ rarely changes. In the case of RM with Hebbian rule, we can observe that *G* increases. Fig.8 shows the relation of *L* and γ in the graph construction model of "Small-World" [6]. Comparing these figures, we can notice that the topological change by learning closely traces the *L*-γ curve of the "Small-World".

To summarize, in the case of RG, the result suggests that the synaptic plasticity changes the network topology in which each neuron transfers information more efficiently with keeping the clustering structures. This may be the main reason why most of the output spikes become synchronous as the learning proceeds (Fig.3).
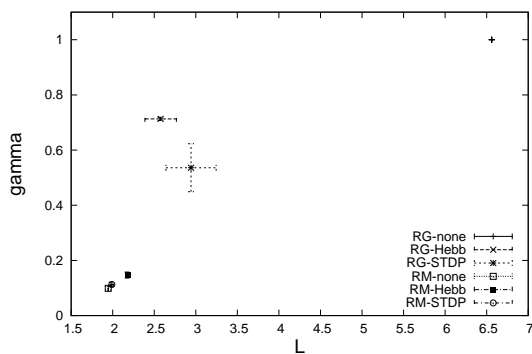

Figure 6: Relation of *L* and γ

4. Conclusion

In this paper, we study the effect of synaptic plasticity on the neural network topology by integrate & fire neuron models. In the case of RG, *L* decreases in spite of maintained large values of γ and *G* as the learning proceeds. In the case of RM with Hebbian rule, *G* increases while *L* is kept small. These results suggest the synaptic plasticity change flexibly the network topology so that each neuron transfers information more efficiently.
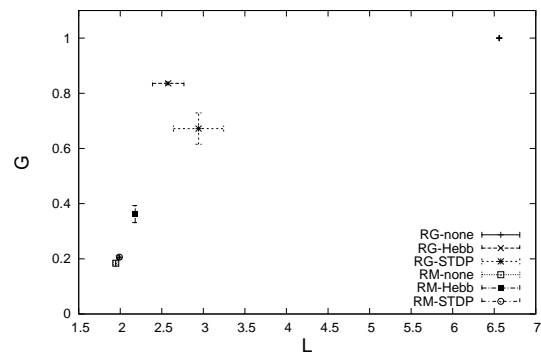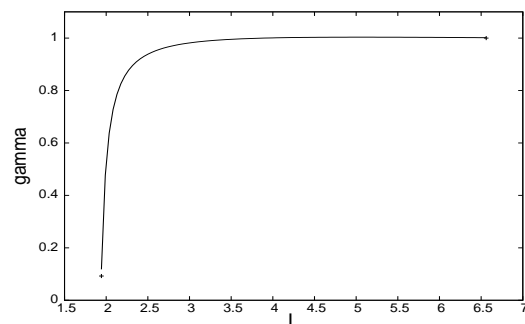
References

[1] von der Malsburg, C. "Self-organization of orientation sensitive cells in the striate cortex," *Kibernetik*, vol. 14, pp. 85-100, 1973.

[2] Markram, H., Lubke, J., "Regulation of synaptic efficacy by coincidence of postsyaptic APs and EPSPs," *Science*, vol. 275, pp. 213–215, 1997.

[3] Hosaka, R., Nakamura, H., Ikeguchi, T., Araki, O. "Information transformation from a spatiotemporal pattern to synchrony through STDP network," *Proceedings of International Joint Conference on Neural Networks*, vol. 2, pp. 1475–1480, 2004.

[4] Levy, H., Horn, D, Meilijson, I., Ruppin, E. "Distributed synchrony in a cell assembly of spiking neurons," *Neural Networks*, vol. 14, pp. 815-824, 2001.

[5] Masuda, N., Aihara, k. "Global and local synchrony of coupled neurons in small-world networks," *Biological Cybernetics*, vol. 90, No.4, pp. 302-309, 2004.

[6] Duncan J.Watts *Small Worlds*(Princeton), 1999