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Delay Feedback Control in Chaotic Neural Network and Its Dynamical Properties

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Abstract—In this paper, we propose the delay feedback control method which is applicable to chaotic neural network model. In computer experiments, the method is applied to controlling dynamical associative memory in chaotic neural network model. From the results, our proposed method succeeds in stabilizing the orbits which become unstable under bifurcation processes. For instance, injecting control signal to chaotic wandering state, the network gives a certain memory pattern only.

1. Introduction

Chaotic phenomena have been observed in neuron, neural network and so on [1]-[4]. It suggests that chaos could play important roles in realizing flexible information processing and controlling actions through the processing of biological system. Related with them, in actual, several pioneer workers have investigated potentialities of chaotic dynamics in neural network model [5]-[10].

In recalling process and learning process, Skarda and Freemann presented the attractive idea based on their biological and computer experiments on the olfactory bulb [3, 4]. We focus on two results of theirs: (i) During the waiting state for input signals, the dynamical response of the olfactory system falls into a highly developed chaotic attractor. (ii) The response to a certain memorized input gives a weak chaotic attractor or a limit cycle. Thus, in recalling process, chaos could ensure rapid and unbiased access to previously trained patterns. In other words, chaos could play important roles in realizing memory search of recalling process. From the theoretical viewpoints, Nara and Davis presented interesting results in complex memory search of neural network model with multi-cyclic memory patterns [5, 6]. And also, Kuroiwa, Nara et al. gave interesting results of functional potentialities of chaotic dynamics in rapid access to the target attractor of a memory fragment [10].

At least, in realizing recalling process based on chaotic dynamics, it is important to control chaotic wandering state gradually depending on the attribute of the target input or on the input itself. One of most famous studies in controlling chaos was done by Ott, Grebogi and Yorke (OGY) [11]. Inspired by the pioneer work of OGY, various investigations in controlling chaos have been done. For instances, occasional proportional feedback control (OPF) [12], delay feedback control (DFC) [13], pinning control [14], and so on. These controlling methods have ever been applied to systems with small degree of freedoms.

In this paper, we employed the methods into the system with large degree of freedoms. Especially, we focus on neural network model as example of systems with large degree of freedoms. In a few papers, the chaos control in chaotic neural neural network model was investigated by employing different methods [15, 16, 17]. One of them has been done by us, where we proposed a type of delay feedback control of chaotic neural network model [15]. Unfortunately, in the paper, we could not stabilize the orbits which become unstable under bifurcation processes. Stabilization of the unstable orbits is one of important problems in delay feedback control methods of chaotic neural network model.

Therefore, the purposes of the present paper are (i) to propose a novel delay feedback control method in chaotic neural network model, which could stabilize the orbits which become unstable under bifurcation processes, and (ii) to investigate the robustness of the chaos control with the change of the control parameters.

2. Chaotic Neural Network Model

Let us present the chaotic neural network model proposed by Adachi and Aihara [9]. The ith neuron of the network model at time t is given as follows:

$$x_i(t+1) = f(\eta_i(t+1) + \zeta_i(t+1)) \qquad (1)$$

$$\eta_i(t+1) = k_f \eta_i(t) + \sum_{j=1}^{N} w_{ij} x_j \qquad (2)$$

$$\zeta_i(t+1) = k_r\zeta_i(t) - \alpha x_i(t) + a_i \tag{3}$$

where t represents a discrete time ($t = 0, 1, 2, \cdots$), the $x_i(t)$ is the output of the ith neuron at time t, the internal state variable of $\eta_i(t)$ is a feedback input from the other neurons in the network which represents the effect of the associative memory, the internal state variable of $\zeta_i(t)$ is the refractoriness effect of the neuron at time t, a_i is a constant bias input of the ith neuron, and N is the number of neuron in the network. The parameter α is the refractory scaling of neuron. The parameters k_f and k_r are the decay parameters for the feedback inputs and the refractoriness, respectively.

As the output function $f(x)$, we employ sigmoid function with the steepness parameter ϵ as follows:

$$f(x) = \frac{1}{1+\exp(-x/\epsilon)}. \tag{4}$$

The parameters w_{ij} are synaptic weights to the ith neuron from the jth neuron as given,

$$w_{ij} = \frac{1}{n}\sum_{p=P}^{P}(2\xi_i^p - 1)(2\xi_j^p - 1) \tag{5}$$

where ξ_i^p is the binary patterns stored as basal memory patterns in the network and the ith component of the pth binary pattern takes 0 or 1. The parameter P is the total number of stored memory patterns.

3. Delay Feedback Control Method in Chaotic Neural Network

In the chaotic neural network model, the existence of the refractory scaling α and the steepness of the sigmoid function ϵ leads to chaotic dynamics. In usual, the steepness of the sigmoid function ϵ is fixed. In the present study, we control chaotic dynamics of the chaotic neural network model by adjusting the effect of the refractory scaling α through the delay feedback control. Therefore, the novel delay feedback control method is described as follows:

$$x_i(t+1) = f(\eta_i(t+1) + \zeta_i(t+1) + F_i(t+1)) \tag{6}$$

$$F_i(t+1) = k_d F_i(t) + \beta x_i(t-\tau) \tag{7}$$

where $F_i(t)$ is a control signal, β is the strength of the control signal, τ represents the delay time, and the parameter k_d is the decay parameters for the control signal. The variables of $\eta_i(t+1)$ and $\zeta_i(t+1)$ are the same ones as Eq.(2) and (3), respectively.

In the previous work, unfortunately, we could not stabilize the unstable orbits. The reason would be inappropriate choice of the delay feedback signal, in R.H.S of Eq.(7), not $x_i(t-\tau)$ but $x_i(t) - x_i(t-\tau)$. In the case of the delay feedback signal, $x_i(t) - x_i(t-\tau)$, if the output of $x_i(t)$ is correlated with the delayed output of $x_i(t-\tau)$, the effect of the delay feedback signal vanishes, and then the system could not be controlled by the method. In the case of the delay feedback signal, $x_i(t-\tau)$, on the other hands, if the output of $x_i(t)$ is correlated with $x_i(t-\tau)$, the effect of the delay feedback signal could negate the refractory effect of Eq.(3) which is important in giving chaotic dynamics in the system. Thus, the system could be controlled as you like.

In our method, the parameters are β, τ and k_d. Especially, the parameter dependence on β and k_d is important. If we can identify the system *a priori*, it is easy to determine the value of the parameters. In general, however, we could not know them *a priori*. In the latter computer experiments, therefore, we investigate the parameter dependence of the controlled system dynamics on β and k_d.

4. Computer Experiments

4.1. Stored Memory Patterns and Experiments Conditions

In the present paper, as shown in Fig.1, four patterns are stored in the network [9]. Each pattern is composed of 10×10 binary pixels, corresponding to the network output with 100 neurons. The pth stored pattern of the ith neuron, xi_i^p, takes 0 or 1. In the Fig.1, the “excited” neuron which takes 1 is represented by a block “■”, and the “restraining” neuron which takes 0 is denoted by a block “□”.

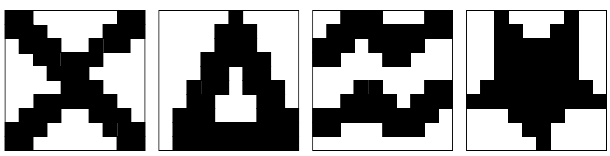
The parameters are employed as $k_r = 0.8$, $k_f = 0.2$, $a_i = 2$ and $\epsilon = 0.1$. In all the following numerical simulations, the same values of parameters are employed without losing generality of our results.

4.2. Chaotic Dynamics in Noncontrolled System

The parameter dependence of the chaotic dynamics of the network model is visualized by observing the following one-dimensional quantity during long time steps with the change of the refractory scaling parameter α.

$$m(t) = \frac{1}{N}\sum_{i=1}^{N}(2\xi_i^p - 1)(2x_i(t) - 1) \tag{8}$$

The variable $m(t)$ is calculated by the inner product between the reference pattern ξ_i^p and the output of the ith neuron of the network at time t. While the network output corresponds to the reference pattern, the value of $m(t)$ takes 1. On the other hand, it become $m(t) = -1$ for the reversal pattern. In these cases, by plotting the value of $m(t)$ on the α-$m(t)$ plane during long time steps, only one



p = 1 p = 2 p = 3 p = 4

Figure 1: Four patterns stored in the model

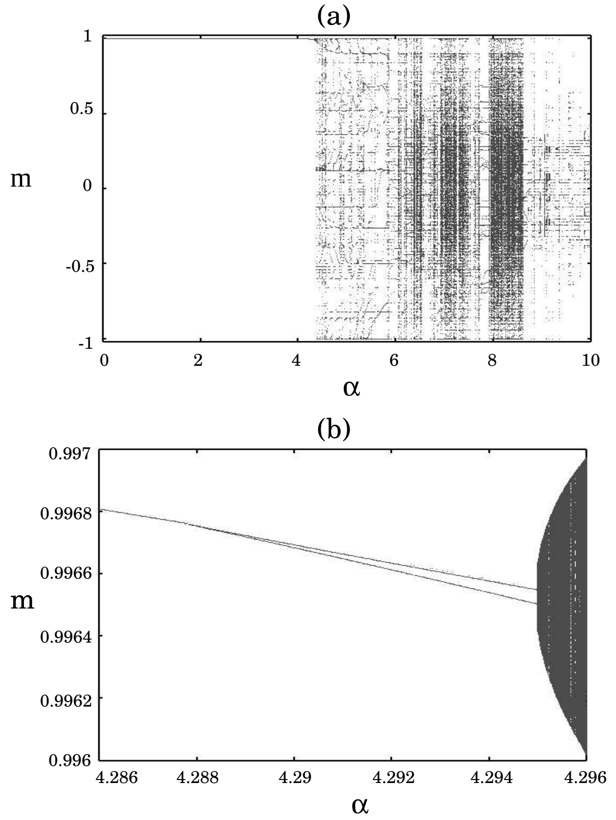


Figure 2: Long time observation of the overlap between the reference pattern and the network output. (a) The change of α from 0 to 10.0. (b) Enlarged view of (a) from 4.286 to 4.296.

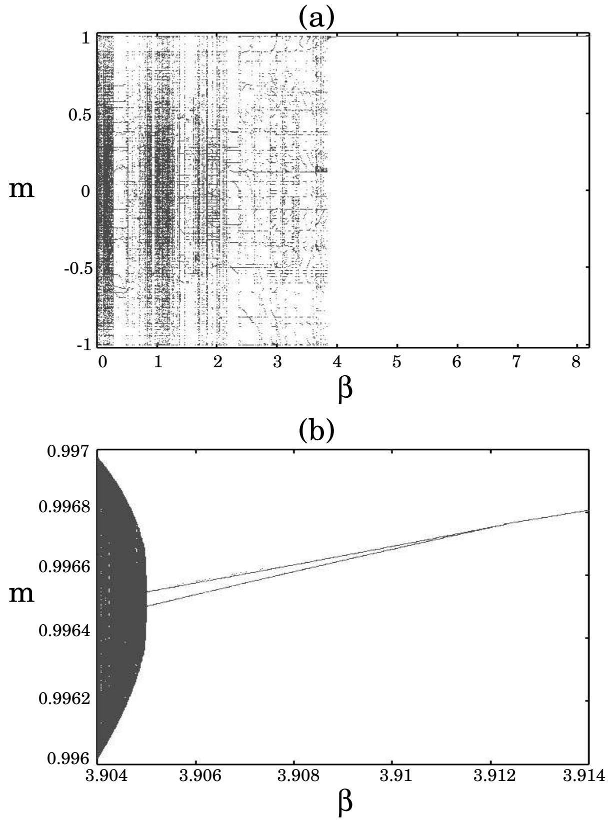


Figure 3: Controlled dynamics. (a) The change of β from 0 to 8.2. (b) Enlarged view of (a) from 3904 to 3914.

point is plotted. For several regions of α, a lot of points are plotted, indicating that chaotic dynamics occurs. Thus, with the change of α, by plotting the value of $m(t)$ during long time steps, we can obtain "bifurcation diagram" with respect to $m(t)$.

The result is given in Fig.2. We evaluate $m(t)$ during the time duration from $t = 11002$ to $t = 13002$ with the change of α from 0 to 10.0 with 0.01 steps. From the result, we can observe that for small value of α, the system is non-chaotic, fixed point or the periodic orbit. As increasing the value of α, through the period doubling bifurcation process, the system becomes chaotic. Around $\alpha = 8.2$, a large number of points are plotted. Thus, the system is in a highly developed chaotic state. In actual, the Lyapunov exponent takes positive value for $\alpha = 8.2$. Therefore, in the latter computer experiments of the chaos control, the refractory scaling parameter is set to be $\alpha = 8.2$.

4.3. Controlled Dynamics

Let us investigate whether the proposed delay feedback control can stabilize the unstable orbits or not, by evaluating the overlap $m(t)$ with the change of the control signal strength of β. For the simplicity of the problem, the decay parameter of the control signal is $k_d = 0.8$ ($= k_r$). In computer experiments, until $t = 11001$ steps, the system is updated without any control signals in order to avoid transient dynamics. At $t = 11002$ steps, we start to inject the control signal. We depict the bifurcation diagram of $m(t)$ from $t = 21002$ to $t = 23002$.

Result is given in Fig.3. We can observe the mirrored-image relationship between Fig.2 and Fig.3. In actual, for the case of $m(t) = 1$, the system converges to the reference pattern. In addition, from Fig.3(b), the control method can stabilize periodic orbits. Therefore, the control method succeeds to stabilize the orbits which become unstable under bifurcation processes. It should be noted that by injecting the control signal at different time steps, the system can converge different stored pattern from the reference pattern.

4.4. Robustness of the Control

In previous subsection, we investigate controlled dynamics with the change of the control signal strength for the same value of the decay parameter k_d as the decay parameter of the refractoriness k_r. In general, however, we could not know system features completely *a priori*. Thus, in the chaos control, the parameter robustness is necessary.

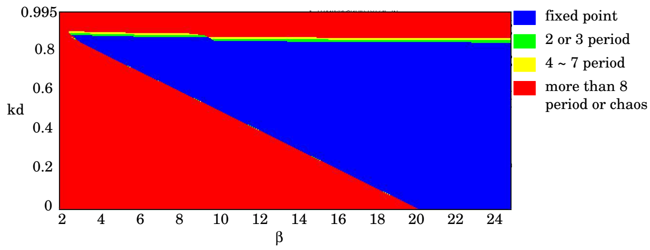


Figure 4: Parameter dependence of the controlled dynamics for β and k_d.

In the Fig.4, the parameter dependence of the controlled dynamics for β and k_d is drawn. In the case of the decay parameter of the refractoriness $k_r = 0.8$, the system can converge to the reference pattern by means of the control method for the region over $0.58 < k_d$. The result suggests that the parameter dependence of the controlled dynamics is not so sensitive to β and k_d. Thus, the control method has the parameter robustness.

5. Conclusions

In the present paper, we investigate the delay feedback control of associative memory dynamics in chaotic neural network model. Results are as follows:

- The proposed control method can stabilize the orbits which become unstable under the bifurcation processes.
- The method has the parameter robustness for the strength of the control signal β and the decay parameter of the control signal k_d. Thus, the method is applicable to various chaotic neural network models even if we could not know system features completely *a priori*.

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