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Synchronization in Coupled Chaotic Systems by an extended CSM (Cellular Slime Mold) Method

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Abstract—The synchronization method derived from cellular slime mold (CSM) is known as the robust method for synchronization between limit cycle oscillators. In this paper a system of synchronization via an extended CSM method is presented and investigated for cyclically coupled four chaotic circuits. Electronic circuit realization of this system is also presented by using a state-controlled cellular neural network.

1. Introduction

Rhythm synchronization plays important roles in artificial systems and biological systems. Recently a powerful method for synchronization of limit cycles learned by cellular slime mold (CSM) was presented [1] and its engineering applications have been searched. We call this synchronization method the CSM method. The CSM method can synchronize large number of limit cycles with broad different natural frequencies and mechanically quite different types of oscillators [1]. Existence of limit cycles is a necessary condition for synchronization of oscillators by the CSM method. We have a question: Can we synchronize chaotic attractors by extending the CSM method?

In this paper, using numerical experiments, we investigate synchronization in coupled chaotic systems by an extended CSM method. As an example, we present a system of synchronization of cyclically coupled 4 chaotic oscillators using an extended CSM method with allowing non-uniform coupling coefficients and both signs among the coefficients. Some interesting patterns of synchronization of this coupled oscillator are observed by changing couplings. We propose also an electronic circuit realizing this system using a state-controlled neural network (SC-CNN)[2] configuration.

2. The CSM Method and its Extension

In the CSM method, one variable to describe a limit cycle oscillator is replaced with a linear coupling of the same type of variables of individual oscillators [1] . For example, let us consider limit cycle oscillators described by

$$\frac{dx_j}{dt} = X_j(x_j, y_j)\ ,\ \frac{dy_j}{dt} = Y_j(x_j, y_j) \qquad (1)$$

$$j = 1, ..., N$$

The system of synchronizing the limit cycles by the CSM is given by

$$\frac{dx_j}{dt} = X_j(x_j, y_j)\ ,\ \frac{dy_j}{dt} = Y_j(x_j + \gamma_j \sum_l x_l, y_j) \qquad (2)$$

where γ_j is an arbitrary positive number that is chosen to correspond to the sensitivity of a biological receptor. Note that a state variable x_j (not self state variable) was chosen to replace it with linear combination in the differential equation $\frac{dy_j}{dt}$.

The CSM method is powerful for synchronizing coupled- limit cycle oscillators but can not synchronize for coupled- chaotic oscillators as it is. So we extend the CSM method a bit as in the following

$$\frac{dx_j}{dt} = X_j(x_j, y_j)\ ,\ \frac{dy_j}{dt} = Y_j(x_j + \sum_l \gamma_{l_j} x_l, y_j) \qquad (3)$$

where γ_{l_j} is not necessarily uniform as in (2) and allowed to take both signs. There are conventional methods for synchronization of coupled chaotic systems [5]. However, the extended CSM method is not identical to conventional ones.

3. Synchronization of limit cycle oscillators by the extended CSM method

In the beginning we consider synchronization of limit cycles generated by simple cellular neural networks (CNN) [3] by using the extended CSM method.

The CNN limit cycle oscillator is described by

$$\dot{x}_1 = -x_1 + 1.7y_1 - y_2 - 0.2 \qquad (4)$$

$$\dot{x}_2 = -x_2 + y_1 + 1.7y_2 + 0.2 \qquad (5)$$

where y_i is an output of $i-th$ cell and given by a unit gain nonlinear function with saturation:

$$y_i = \frac{1}{2}(|x_i + 1| - |x_i - 1|) \ . \tag{6}$$

The system for synchronization between two CNN oscillators by the extended CSM method is described by

$$\begin{aligned} \dot{x}_1 &= -x_1 + 1.7y_1 - y_2' - 0.2 \\ \dot{x}_2 &= -x_2 + y_1 + 1.7y_2 + 0.2 \\ \dot{x}_3 &= -x_3 + 1.7y_3 - y_4' - 0.2 \\ \dot{x}_4 &= -x_4 + y_3 + 1.7y_4 + 0.2 \end{aligned} \tag{7}$$

where

$$\begin{aligned} y_i &= \frac{1}{2}(|x_i + 1| - |x_i - 1|) \\ y_2' &= \frac{1}{2}(|x_2 + \gamma_1 x_2 + \gamma_2 x_4 + 1| - |x_2 + \gamma_1 x_2 + \gamma_2 x_4 - 1|) \\ y_4' &= \frac{1}{2}(|x_4 + \gamma_1 x_4 + \gamma_2 x_2 + 1| - |x_4 + \gamma_1 x_4 + \gamma_2 x_2 - 1|) \end{aligned}$$

γ_1 is a coupling coefficient from the self cell output and γ_2 is one from the other cell output in a CNN oscillator. Taking proper values and sign as the coupling coefficients, we can determine the pattern of synchronization whether they oscillate in the same phase or constant delayed phase. When $\gamma = \gamma_1 = -\gamma_2$ in the system (7), the relationship between the state of synchronization and the coupling coefficient γ is shown in Table 1. For the parameters $\gamma_1 = 0.4$, $\gamma_2 = -0.4$, the two oscillators evolve with a phase difference π as shown in Figure 1. On the other hand, for parameters $\gamma_1 = \gamma_2$, we can see the two oscillators evolve in phase.

Table 1: Relationship between the value of the coupling coefficient and synchronization

$0 < \gamma < 0.1$	$0.1 \leq \gamma \leq 1.2$	$1.2 < \gamma$
long transient	synchronized	broken waves

Next we demonstrate synchronization between mechanically different two limit-cycle oscillators (the CNN oscillator and the van der POL's oscillator) by using the extended CSM method. The system for synchronization between the CNN oscillator and the van der Pol's oscillator is given by

$$\begin{aligned} \dot{x}_1 &= -x_1 + 1.7y_1 - y_2' - 0.2 \\ \dot{x}_2 &= -x_2 + y_1 + 1.7y_2 + 0.2 \\ \dot{x}_3 &= x_4 + \gamma_1 x_4 + \gamma_2 x_2 \\ \dot{x}_4 &= -\omega^2 x_3 + (1 - x_3^2)x_4 \end{aligned} \tag{8}$$

where

$$\begin{aligned} y_i &= \frac{1}{2}(|x_i + 1| - |x_i - 1|) \\ y_2' &= \frac{1}{2}(|x_2 + \gamma_1 x_2 + \gamma_2 x_4 + 1| - |x_2 + \gamma_1 x_2 + \gamma_2 x_4 - 1|) \end{aligned}$$

When $\gamma = \gamma_1 = -\gamma_2$ in the system (8), the relationship between the state of synchronization and γ is shown in Table 2.

Table 2: Relationship between the value of the coupling coefficient and synchronization

$0 < \gamma < 0.3$	$0.3 \leq \gamma \leq 1.0$	$1.0 < \gamma$
not synchronized	synchronized	broken waves

The synchronized waves in the same phase (for parameters $\gamma_1 = 0.4$, $\gamma_2 = -0.4$) are shown in Figure 2. For parameters $\gamma_1 = \gamma_2$, we can see the two oscillators evolve with a phase difference π.

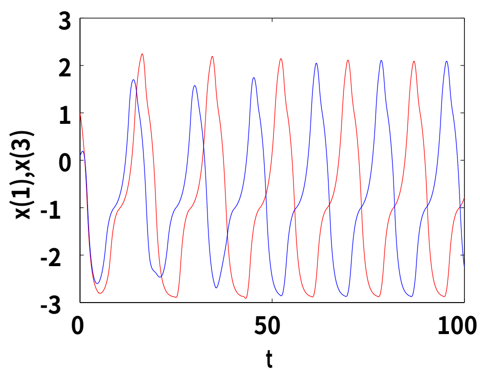


Figure 1: Wave forms of the CNN limit cycle oscillators in synchronized state with the phase 180 deg behind

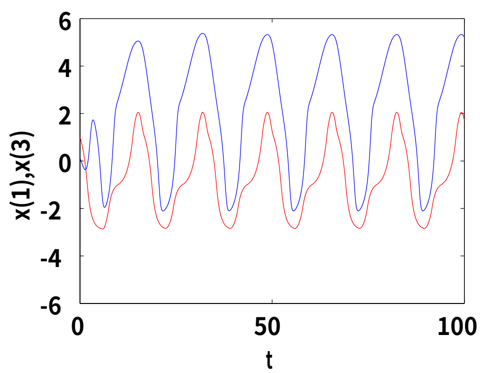


Figure 2: Wave forms of the CNN oscillator and van der Pol's oscillator in synchronized states by the extended CSM method

4. Synchronization of Coupled Chaotic Systems by the Extended CSM Method

In this section we investigate synchronization of cyclically connected 4 chaotic systems as shown in Figure 3 by using the extended CSM method.

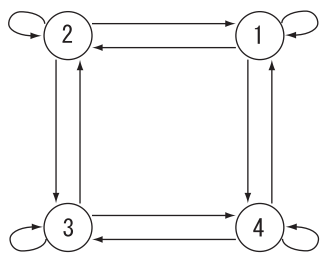


Figure 3: Cyclically connected 4 chaotic systems

In the figure each cell consists of the Chua's oscillator [4] [2] which has the double scroll attractor.

The whole system for synchronization of the chaotic systems using the extended CSM method is described by

$$\begin{aligned}
\dot{x}_1 &= \alpha(x_2 - f(x_1)) \\
\dot{x}_2 &= -x_2 + x_3 + x_1 + (\gamma_2 x_1 + \gamma_1 x_{10} + \gamma_3 x_4) \\
\dot{x}_3 &= -\beta x_2 \\
\dot{x}_4 &= \alpha(x_5 - f(x_4)) \\
\dot{x}_5 &= -x_5 + x_6 + x_4 + (\gamma_2 x_4 + \gamma_1 x_1 + \gamma_3 x_7) \\
\dot{x}_6 &= -\beta x_5 \\
\dot{x}_7 &= \alpha(x_8 - f(x_7)) \\
\dot{x}_8 &= -x_8 + x_9 + x_7 + (\gamma_2 x_7 + \gamma_1 x_4 + \gamma_3 x_{10}) \\
\dot{x}_9 &= -\beta x_8 \\
\dot{x}_{10} &= \alpha(x_{11} - f(x_{10})) \\
\dot{x}_{11} &= -x_{11} + x_{12} + x_{10} + (\gamma_2 x_{10} + \gamma_1 x_7 + \gamma_3 x_1) \\
\dot{x}_{12} &= -\beta x_{11}
\end{aligned} \tag{9}$$

where

$$f(x_i) = \frac{2}{7}x_i + \frac{1}{2}(-\frac{1}{7} - \frac{2}{7})(|x_i + 1| - |x_i - 1|) \ . \tag{10}$$

The wave forms of the state variables x_1, x_4, x_7, and x_{10} are shown in Figure 4 for the parameters $\alpha = 12$, $\beta = 9$, $\gamma_2 = -0.5$, $\gamma_1 = \gamma_3 = \frac{0.5}{2}$. Observing the Lissajou's figure in Figure 5 we find that complete synchronization has been attained among the state variables x_1, x_4, x_7, and x_{10}.

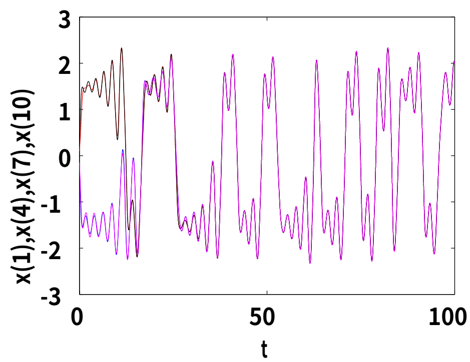


Figure 4: Complete synchronization of the coupled chaotic systems in Figure 3 by the extended CSM method

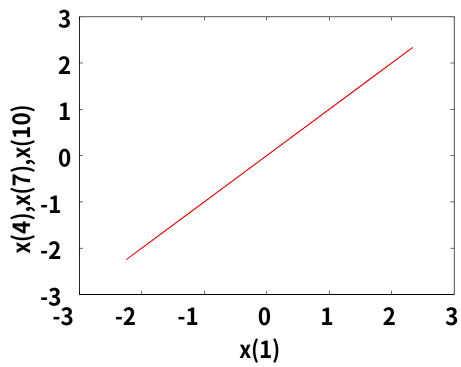


Figure 5: Lissajou's figure : x_4, x_7, x_{10} vs. x_1

Note that the extended CSM method with appropriate coupling coefficients allows the coupled chaotic systems to synchronize as well as the coupled limit cycle systems.

5. Electronic Circuit for Synchronization of Coupled Chaotic Systmes with the Extended CSM Method

In this section we present a state-controlled cellular neural network (SC-CNN[2]) realization of the synchronization system (9) for the coupled chaotic systems. Figure 6 shows one of 4 coupled chaotic systems and the way of couplings by the extended CSM method, which are shown in the broken line boxes. In this op-amp-based circuit the op-amp LF356's are employed. Figure 7 shows a SPICE simulation for the synchronization circuit with coupling coefficients $\gamma_1 = -0.5, \gamma_2 = \gamma_3 = -\frac{0.5}{2}$. These wave forms of voltages correspond to state variables x_1, x_4, x_7, x_{10} in Equations (9). Obseving the Lissajou's figure in Figure 8 we find that adjacent chaotic circuits and chaotic circuits on the diagonal in Figure 3 are synchronized in the same phase and in the opposite phase respectively. Also we note that very precise synchronization has been attained since the trajectories are just on the lines.

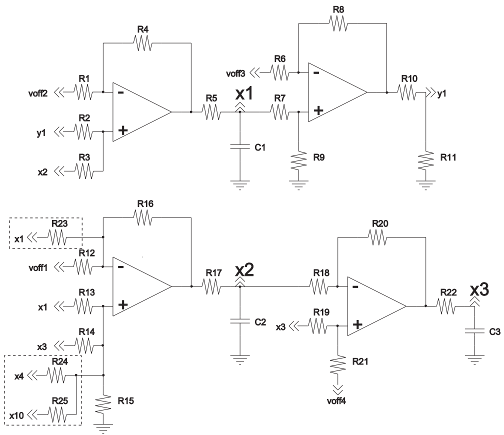


Figure 6: Part of SC-CNN realization for the system (9)

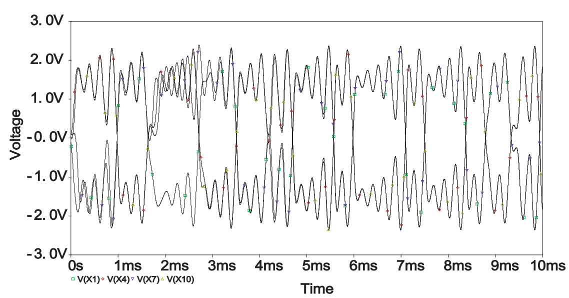


Figure 7: Wave forms of the coupled chaotic circuits

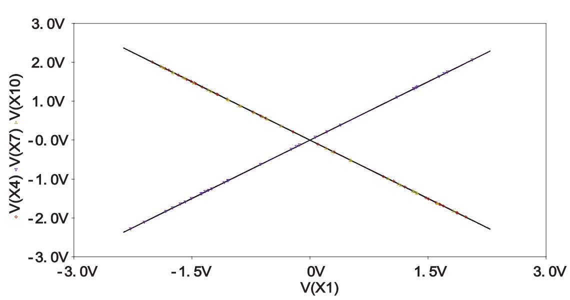
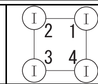
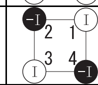
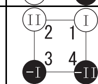
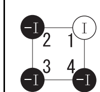
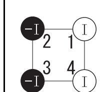


Figure 8: Lissajou's figure for node voltages in the coupled chaotic circuits: x_4, x_7, x_{10} vs. x_1

Table 3 shows relationship among the coupling template, the type of synchronization pattern, and its pattern, which are confirmed in numerical experiments. In the table a coupling template $\mathbf{A_r}$ denotes a vector of $\mathbf{A_r} = [\gamma_2 \quad \gamma_1 \quad \gamma_3]$ in a chaotic cell. The pattern represents configuration of synchronization in chaotic circuits, where I denotes a chaotic oscillation with synchronization in the same phase and -I denotes one in the opposite phase. II and -II denote the other chaotic oscillation synchronizing each other with the opposite phase.

Table 3: Patterns of synchronization in the coupled chaotic systems (9)

coupling template	pattern type	pattern
$\mathbf{A_r} = [\ \frac{0.5}{2} \ -0.5 \ \ \frac{0.5}{2}]$	I	
$\mathbf{A_r} = [-\frac{0.5}{2} \ -0.5 \ -\frac{0.5}{2}]$	I , -I	
$\mathbf{A_r} = [-\frac{0.36}{2} \ -0.6 \ \ \frac{0.36}{2}]$	I , -I , II , -II	
$\mathbf{A_{r1}} = [-\frac{0.5}{2} \ -0.5 \ -\frac{0.5}{2}]$ $\mathbf{A_{r2}} = [\ \frac{0.5}{2} \ -0.5 \ -\frac{0.5}{2}]$ $\mathbf{A_{r3}} = [\ \frac{0.5}{2} \ -0.5 \ \ \frac{0.5}{2}]$ $\mathbf{A_{r4}} = [-\frac{0.5}{2} \ -0.5 \ \ \frac{0.5}{2}]$	I , -I	
$\mathbf{A_{r1}} = [-\frac{0.5}{2} \ -0.5 \ \ \frac{0.5}{2}]$ $\mathbf{A_{r2}} = [\ \frac{0.5}{2} \ -0.5 \ -\frac{0.5}{2}]$ $\mathbf{A_{r3}} = [-\frac{0.5}{2} \ -0.5 \ \ \frac{0.5}{2}]$ $\mathbf{A_{r4}} = [\ \frac{0.5}{2} \ -0.5 \ -\frac{0.5}{2}]$	I , -I	

6. Conclusions

In this paper, via numerical experiments, we have shown that the extended CSM method can synchronize the coupled chaotic oscillations as well as the limit cycles. We have demonstrated them for the cyclically coupled 4 chaotic systems, where we found there are several patterns of synchronization depending on coupling templates. We have also presented a SC(State Controlled)-CNN realization for this system and confirmed its performance by SPICE simulations. Theoretical proof for synchronization by the extend CSM method remains to be done.

References

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