2017 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2017, Cancun, Mexico, December 4-7, 2017

Serial-In Parallel-Out Discrete Fourier Transformer with $O(N)$ Cells

Ena Kohno†, Akiyoshi Yasuda†, Takeshi Kamio†, and Hisato Fujisaka†

†Faculty of Information Science, Hiroshima City University, 731-3194 Japan
Email: yasuo@sp.info.hiroshima-cu.ac.jp

Abstract—We present a tree-structured OFDM demodulator consisting of comb filter blocks. The structure is suitable for demodulating selectively designated sub-band carrier set. Thus, the demodulator can be applicable to adaptive OFDM communications through power lines with fluctuating transmission characteristics. The straightforward execution of the demodulation requires the same order of computational complexity as discrete Fourier transforms. However, exploiting the periodicity of the internal signals of the demodulator reduces the complexity to the same order as that of the fast Fourier transforms.

1. Introduction

Power line communications (PLC) are attracting attention again in the upcoming era of Internet of Things (IoT). Orthogonal frequency division multiplexing (OFDM), e.g. G3-PLC [1], is considered as the most beneficial physical layer protocol for PLC. OFDM PLC communication must be adaptive to the varying transmission and noise characteristics of the power lines. For example, a sub-band of $2^{N_{sub}}$ channels in good condition is selected from a reserved band of 2^N channels, $N > N_{sub}$. For the communication through the selected sub-band, it is required in receivers to execute complicated partial computation of 2^N-point fast Fourier transform (FFT) [2] or $2^{N_{sub}}$-point FFT along with variable frequency shift and sampling rate.

This paper proposes a method of direct extraction of symbols transmitted through channels in the sub-band with digital comb filters connected in a tree-structure. The complexity of the proposed method is compared with that of conventional demodulation using FFT.

Most parts of this paper were presented in [3] in Japanese.

2. Tree-Structured Demodulator

We attempt to extract symbols transmitted through 2^N channels of a baseband OFDM signal $x(n)$, n: time index. Here, we assume that sampling time is 1 and the symbol length is T_{sbl}. Let operators $H_{Tsbl/2,+}$ and $H_{Tsbl/2,-}$ on $x(n)$ be given in the time domain by

$$G_e(n) = H_{Tsbl/2,+} * x(n) = (x(n) + x(n - T_{sbl}/2))/2$$
$$G_o(n) = H_{Tsbl/2,-} * x(n) = (x(n) - x(n - T_{sbl}/2))/2$$

The two operations divide the inputted OFDM signal $x(n)$ into the sum $G_e(n)$ of even channel carriers and the sum $G_o(n)$ of odd channel carriers. The frequency of signal $G_o(n)$ is shifted by the symbol rate $f_{sbl} = 1/T_{sbl}$ by the multiplication by $\exp(-j2\pi f_{sbl} n)$, $j^2 = -1$. The operations in the frequency domain and the function block executing the operations are depicted as shown in Figs. 1 and 2. The block is mainly built of a delay element, adders, and a frequency shifter consisting of an oscillator and a multiplier.

We connect the blocks in tree structure of N stages as shown in Fig. 3. In the blocks of the n-th stage, the delay time of the delay elements is $T_{sbl}/2^n$ and the shift frequency of the frequency shifter is $f_{sbl} \times 2^{n-1}$. The tree of the blocks continues to divide the OFDM signal into two until all the carriers are separated as shown in Fig. 1. Since the carrier frequencies are shifted to zero at the end of the tree, its terminals output the transmitted symbols.

When using $2^{N_{sub}}$ channels in a sub-band, $N_{sub} < N$, only the blocks between the input and the terminals outputting the symbols transmitted through the channels are operated, as exemplified in Fi.g. 4. The blocks to be operated are determined easily.

3. Complexity of Demodulation

A tree extracting symbols transmitted through 2^N channels consists of $2^N - 1$ blocks. The total number of the sampled input values and the sampled internal signals of the tree is $0\mathrm{rd}(2^{2N})$. In each block, additions and the multiplication by $\exp(-j2\pi f_{sbl} n)$ are performed. The tree computation costs the same complexity as 2^N-point discrete Fourier transform.

As shown in Fig. 1, the outputs from the blocks in the n-th stage contain frequency components of DC or integer times of $f_{sbl} \times 2^n$. Then, the outputs are $T_{sbl}/2^n$-periodic. The $T_{sbl}/2^n$-periodic outputs are the inputs to the blocks in the $(n + 1)$-th stage. The period of sinusoidal signals used for the frequency shift in the blocks is $T_{sbl}/2^n$. The first half and the last half of the sinusoidal signals are same in absolute value. Therefore, the block may operate only in the interval of $T_{sbl}/2^{n+1}$. The outputs in the interval are used for the outputs in the other intervals. The tree has 2^{n-1} blocks in the n-th stage and the blocks operate only in the interval of $T_{sbl}/2^n$. Thus, the number of the samples of the signals on which the blocks operate may be $0\mathrm{rd}(N \times 2^N)$ in total. Then, we see that the tree computation cost can be reduced to the same complexity as 2^N-point fast Fourier transform.

There exist blocks which do not need to operate when a sub-band is used for communication. Needless to say, the computation cost for the sub-band communication can be reduced further.

4. Conclusions

A tree-structured OFDM demodulator consisting of comb filters has been presented. The structure is suitable for demodulating carriers of selected sub-band channels. Then, the tree structure is effective for adaptive PLC communication changing its frequency band according to varying transmission characteristics of a power line.

It has been found that the computational cost of the presented tree-structured OFDM demodulator can be reduced to the same complexity as the fast Fourier transform.

References

[1] "PLC G3 Physical layer Specifications" in *G3 Automated Meter Management Technical Specifications*, ERDF, 2011.

[2] R. P. Brent, *Modern Computer Arithmetic*, Cambridge University Press, 2010.

[3] E. Kohno, T. Kamio, and H. Fujisaka, "Minimum Quantity of Computation for FDM Demodulation with Digital Comb Filters ," *IEICE Technical Reports*, NLP2015-89, pp.43-46, 2015.

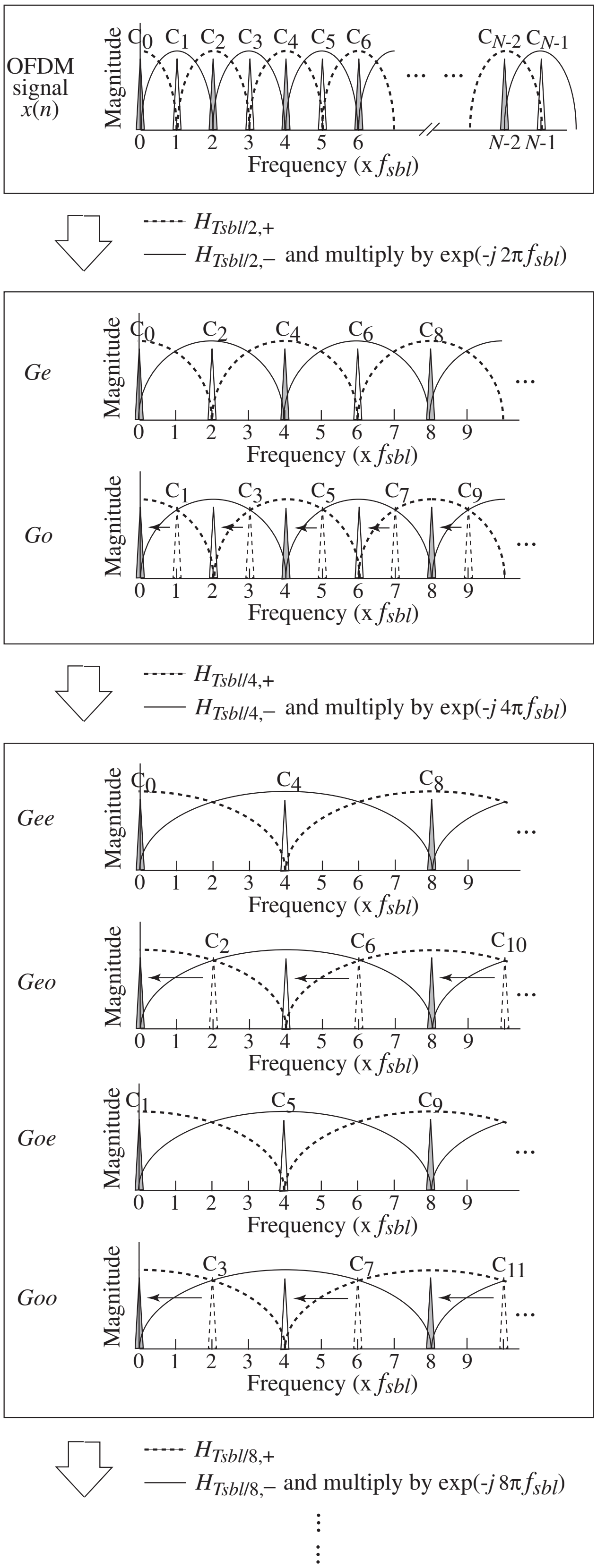

Figure 1: Frequency domain operations of the blocks connected in a tree-structure.

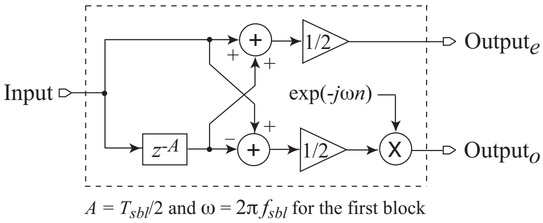

Figure 2: A function block used in the tree.

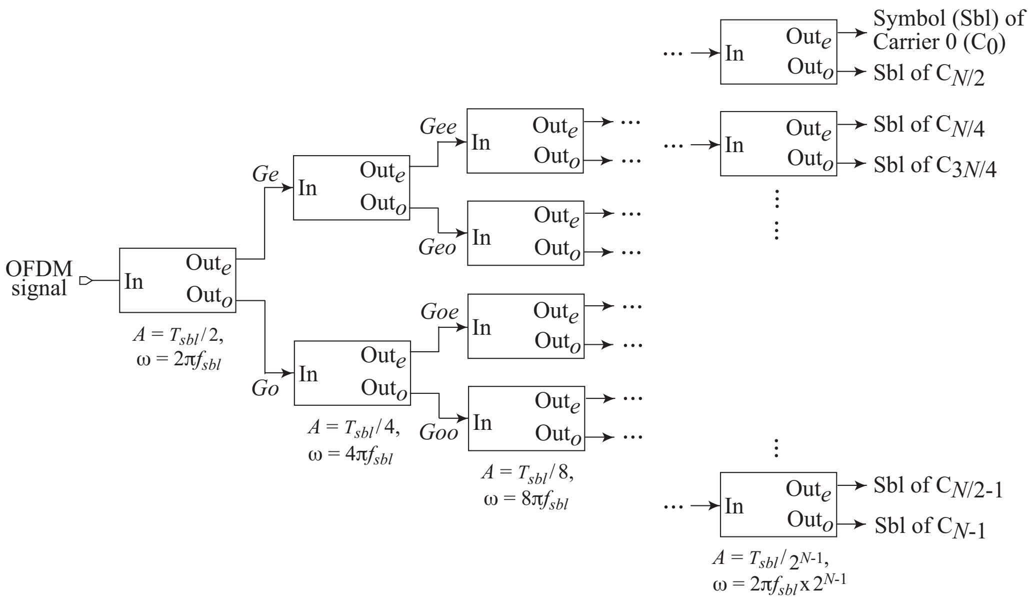

Figure 3: Tree-structured demodulator.

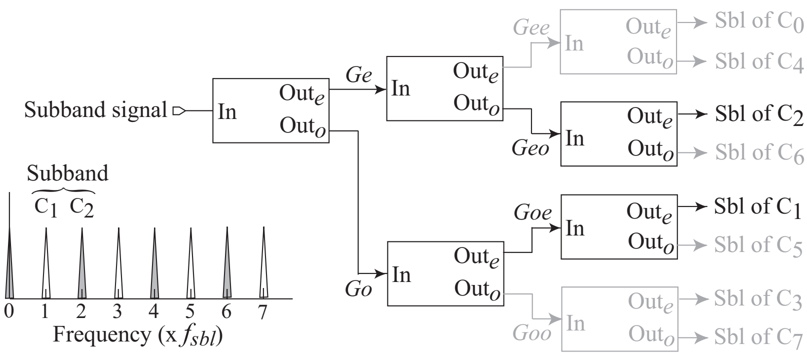

Figure 4: Demodulation of carriers in a sub-band.