

# Associative Memory Operation in a Hopfield-type Spiking Neural Network with Modulation of Resting Membrane Potential

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## Abstract—

This paper proposes a Hopfield-type spiking neural network with modulation of resting membrane potential, and reports retrieval data transition phenomena in its associative memory operation. Spiking neuron models express analog information by the timing of neuronal spike firing events. Since these models operate asynchronously, it is expected that the spiking network operates faster than the conventional synchronous models. We have designed a CMOS spiking neural network circuit. It has been found in the circuit simulation that because of the resting membrane potential modulation with a sinusoidal curve, a retrieval pattern is unstabilized and the network retrieves another memorized pattern.

## 1. Introduction

A biological neuron receives many electric spike impulses via synapses, and it fires by generating a spike impulse. However, the conventional neural network models usually use analog output values based on the neuronal firing rate coding or the firing population coding. Recently, the spiking neuron models, which express analog information by the timing of neuronal spike firing, attract a lot of attention with expectation of their higher information processing ability [1, 2].

From the theoretical neural network model research, it is expected that more advanced neural system can be developed using the spiking neuron models. Additionally, since these models operate asynchronously, it is also expected that spiking neural networks operate faster than the conventional synchronous models. So far, the spiking neuron models have mainly been applied to feedforward networks; for example, image data processing using a spiking feedforward network was reported [3].

However, there have been only a few reports about spiking feedback networks. In order to achieve spiking feedback networks with continuous states, spike outputs expressing the zero value are required. However, a neuron generates no spike unless its internal potential exceeds the threshold, and usually the resting (membrane) potential corresponding to the zero value is below the threshold.

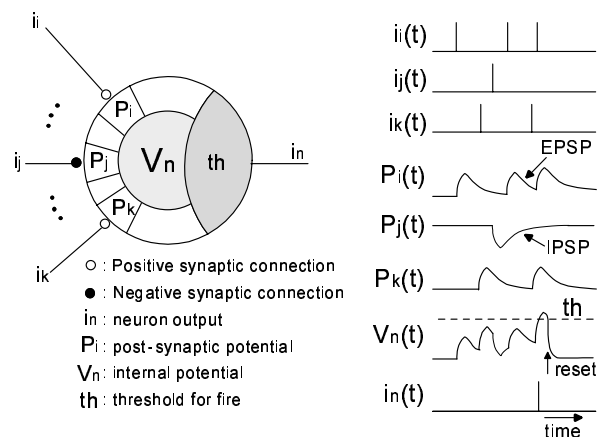


Figure 1: Spiking Neuron Model.

Therefore, a simple information representation scheme by spike timing cannot be applied to such feedback networks.

One of the authors has introduced *negative thresholding* to solve this problem, and this thresholding operation is achieved by introducing a global excitatory unit [4]. It has also reported that feedback (Hopfield-type) spiking neural networks have a retrieval property for associative memory as conventional Hopfield networks with analog internal potential.

On the basis of the previous work [4], this paper reports transition phenomena of retrieval data in associative memory constructed by a newly proposed Hopfield-type spiking neural network with modulation of resting membrane potential. In the model proposed in this paper, resting potential modulation is used as the negative thresholding scheme, and furthermore, the modulation leads to retrieval data transition.

## 2. Spiking Neural Network Model

The integrate-and-fire-type (IF) neuron model is shown in Fig. 1, which is a typical spiking neuron model. When a spike pulse is fed into a neuron via a synapse, a post-synaptic potential (PSP) is generated. A neuronal internal potential is determined by the spatiotemporal summation of

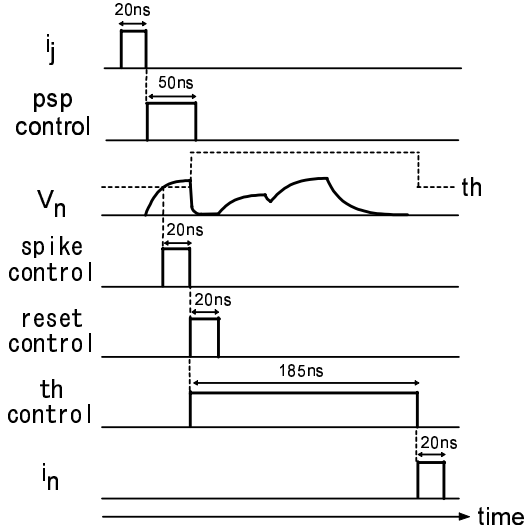


Figure 2: Timing chart of our spiking neuron model.

PSPs generated by the input spikes. There are two types of synapses: excitatory and inhibitory, according to the sign of the synaptic weight  $w_{ni}$ .

In the simple IF model, the time courses of PSPs are the same, which we call here a unit PSP,  $P(t)$ , as a convolutional kernel, and the PSP from neuron  $i$  to neuron  $n$ ,  $PSP_{ni}(t)$ , is given by the sum of unit PSPs multiplied by the corresponding synaptic weights:

$$PSP_{ni}(t) = \sum_{t_i^{(f)} \in \mathcal{F}_i} w_{ni} P(t - t_i^{(f)}), \quad (1)$$

where  $\mathcal{F}_i = \{t_i^{(1)}, \dots, t_i^{(n)}\}$  is the set of firing times of neuron  $i$ . The internal potential of neuron  $n$ ,  $V_n(t)$ , is given by the sum of  $PSP_{ni}(t)$ :

$$V_n(t) = \sum_{i \in \Gamma_n} PSP_{ni}(t), \quad (2)$$

where  $\Gamma_n$  is the set of input to neuron  $n$ . The effect of PSP is temporary, and the internal potential returns to the resting potential level after the PSP ceases.

The timing chart of our spiking neuron model is shown in Fig. 2. An input spike ( $i_j$ ) generates PSP (*psp control*). When the internal potential ( $V_n$ ) exceeds the threshold, a neuron fires and generates a spike (*spike control*). After firing, the internal potential is reset for a so-called absolute refractory period (*reset control*), and then a relative refractory period follows, when the threshold level is increased (*th control*). A Spike generated at a neuron is transmitted to other neurons or the neuron itself with transmission delay time ( $i_n$ ).

In our model, the resting potential level is modulated with a sinusoidal curve. The period of the sinusoidal modulation is much longer than the inter-spike interval. When the resting potential is nearly maximum, all neuron can fire.

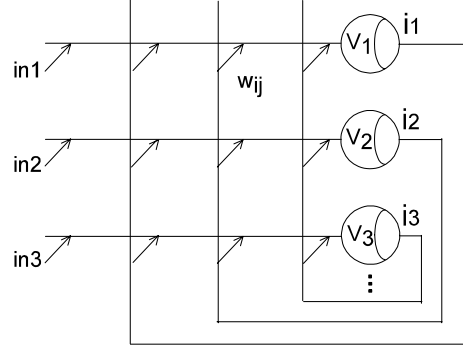


Figure 3: Hopfield network configuration.

Therefore, the relative spike-timing difference can represent the analog information.

### 3. Simulation of Associative Memory Using Spiking Neural Network

#### 3.1. Simulation Condition

We have designed a spiking feedback network LSI circuit using a  $0.35 \mu\text{m}$  CMOS technology. A synapse part was replaced by a constant current source. An input spike pulse turns on a current switch and charges or discharges a capacitor, whose terminal voltage represents the neuronal internal potential, according to the sign of the synaptic weight. The spatiotemporal summation of PSPs by input spikes is performed by this capacitor. In order to compare a neuronal internal potential to the threshold voltage, we used a comparator using a differential pair. In order to make a neuron generate a refractory period, we made the threshold voltage increase after neuronal firing. In order to realize the transmission delay, spike pulses are delayed by using an inverter chain.

We evaluated the performance of our model by circuit simulation of the designed CMOS circuit. The simulations were performed by a high-speed circuit simulator, HSIM, produced by Nassda Corporation. The reason why we used circuit simulation instead of usual software simulation is that it directly leads to the VLSI implementation of our model.

Using the above spiking neuron model, we constructed a Hopfield-type neural network composed of 36 neurons with symmetric connections as shown in Fig. 3. The synaptic weights  $w_{ij}$ , which are expressed by the sum of autocorrelation matrixes, are given by the following equation:

$$w_{ij} = \sum_{k=1}^N (2I_i^k - 1)(2I_j^k - 1), \quad \text{for } i \neq j, \quad (3)$$

where  $w_{ii} = 0$ , and  $I_i^k$  is the  $i$ -th element of the  $k$ -th stored pattern vector. In the simulation, the number of stored patterns  $N$  was five, and the stored patterns are shown in Fig. 4. The elements  $I_i^k$  were randomly chosen under the

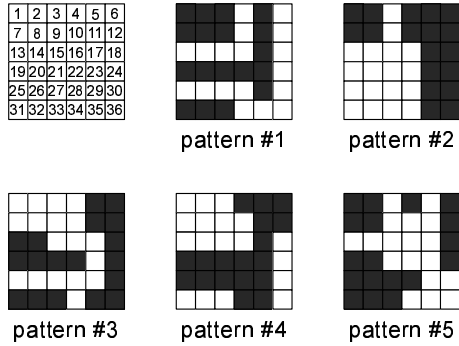


Figure 4: Stored patterns used in the simulations.

condition that the numbers of ‘0’ and ‘1’ are equal. As a result,  $w_{ij} \in \{\pm 5, \pm 3, \pm 1\}$ . We used gray-level (5-levels) input patterns that converge to stored pattern #1 shown in Fig. 4.

In the simulation, the time step corresponding to one level in gray-level patterns was set at 25 ns. Therefore, the firing timing of input spikes was limited in  $\{0, 25, 50, 75, 100\}$  ns, and the time span for receiving input spikes was 100 ns.

### 3.2. Simulation Results

A simulation result about our spiking neural network is shown in Fig. 5, which shows the outputs of twelve neurons corresponding to pixels 1 to 12 in the first and second rows of the pattern shown in Fig. 4. Figure 5(a) also shows the sinusoidal modulation  $\epsilon \sin(\omega t)$  of the resting potential. The parameters of the modulation is as follows:  $\epsilon = 0.06$  V,  $\omega = 2\pi \times 1.67 \times 10^5$  rad/s. Here, modulation phase  $\theta$  is defined as  $\theta = \omega t \bmod 2\pi$ . As can be seen in Fig. 5(a), most neurons generate approximately periodical spikes because of feedback with transmission delay. However, some neurons generate no spikes when  $\theta = [3\pi/2, \pi/2]$ . For a single neuron, it is easy to fire at the highest resting potential ( $\theta = \pi/2$ ), and it is most difficult to fire at the lowest resting potential ( $\theta = 3\pi/2$ ). However, due to collective behavior of the network, the period when neurons are difficult to fire shifts slightly.

Figure 5(b) shows a magnified view of the a spatio-temporal spike pattern at the initial period. In this case, the spike pattern converged to stored pattern #1 because we inputted a pattern similar to pattern #1. However, after some modulation periods, the network converged to a different stored pattern as shown in Fig. 5(c); it was pattern #5 in this case.

Figure 6 shows spike outputs of some neurons and their corresponding internal potentials when the spike pattern becomes unstable. The internal potentials reach the threshold at timing  $\alpha$  and  $\beta$ , and after the predefined transmission delay, the neurons output spikes at timing  $\alpha'$  and  $\beta'$ , respectively.

The spatio-temporal spike pattern transition in Fig. 5(a)

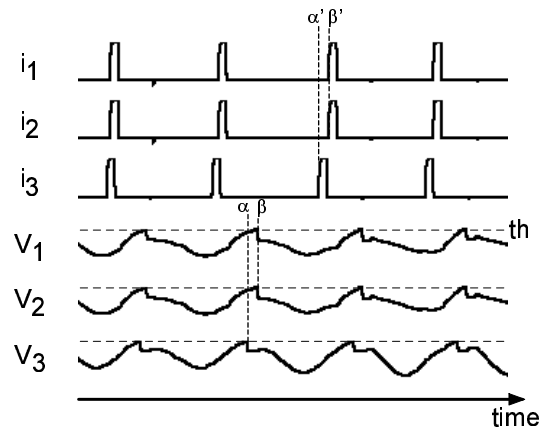


Figure 6: Simulation result.

can be understood as follows. For the first period ( $t = [0, 3] \mu s$ ) (Fig. 5(b)), the network converges to pattern #1, where a rising resting potential works as negative thresholding as proposed in the previous work. For the second and third modulation period ( $t = [3, 17] \mu s$ ), some neurons cannot fire because of a low resting potential and a small value of PSP summation. This unfiring in some neurons leads to a different stored-pattern basin from the first stabilized state (pattern #1). Then, for the fourth modulation period ( $t = [17, 20] \mu s$ ) (Fig. 5(c)), the network reaches a different pattern (#5) state.

### 4. Conclusions

We proposed a Hopfield-type spiking neural network with modulation of resting membrane potential, and observed retrieval data transition in its associative memory operation. The transition is caused by some unfiring neurons for a low resting potential period. On the other hand, for a high resting potential period, all neurons generate spikes, and therefore data representation using relative spike-firing timing is achieved. It is a future work to design the network and to determine the synaptic weight values for obtaining desirable data transition.

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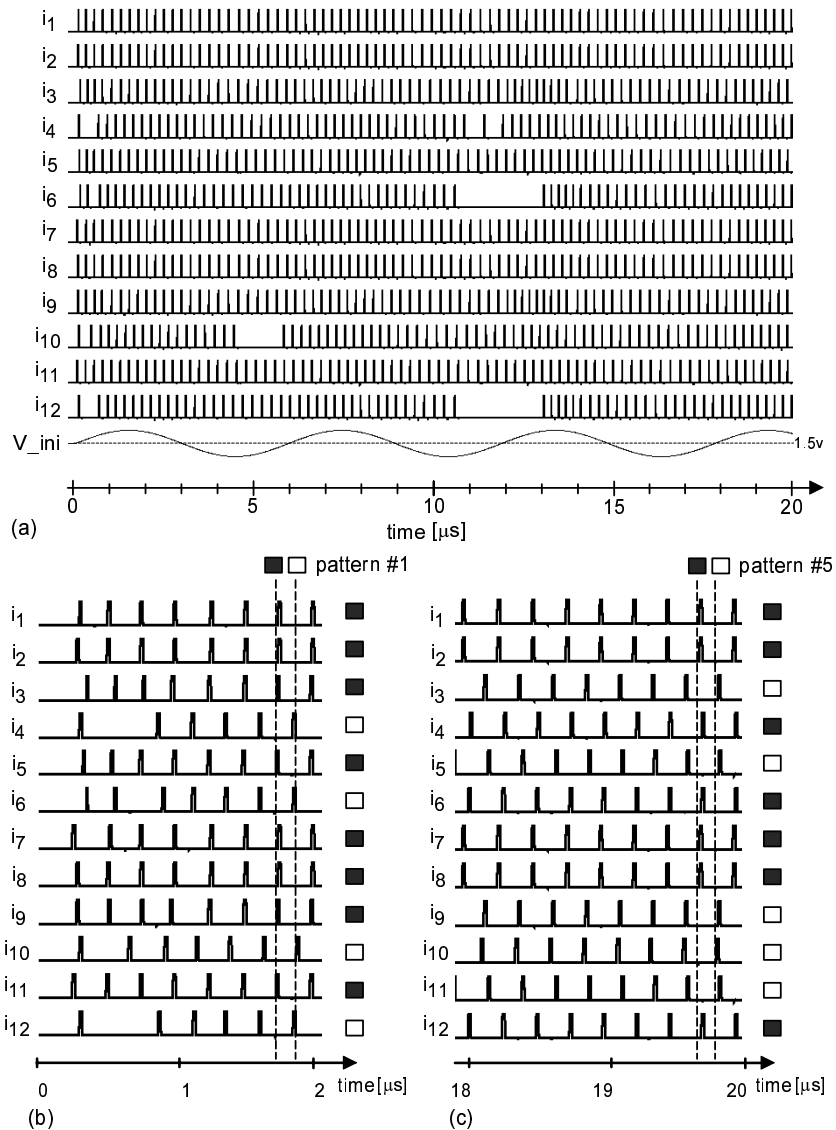


Figure 5: Simulation result.

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