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Period-Detection of Pseudo Chaotic signals Based on the Jacobian of the Chaotic Map

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Abstract—In this paper, a rapid method for the periodicity detection of pseudo chaotic signal is proposed. The periodicity of pseudo chaos is generated as a result of the mapping of multiple times. Then, the dynamics of the multiple mapping are first combined; the Jacobian of the dynamics is evaluated.
In this study, 1st and 2nd order chaotic maps, such as Logistic map and Henon map, are evaluated by the proposed method, and the effectiveness of the proposed method is shown by the computer simulation.

1. Introduction

When the chaotic maps are realized by computers, which carry out the finite bit-length arithmetic, the periodicity appears to the generated chaotic signal. The chaotic signal with periodicity is called pseudo chaos; it cannot be avoided to become pseudo chaos as long as the computer is used.

Besides, we have examined the private communication systems with chaos[1]. As an influence by being the pseudo chaos, it seems that the coefficient sensitivity of parameter-mismatching between transmitter and receiver sides is decreased, so the reliability of the system will be decreased. Namely, to detect the periodicity of the pseudo chaotic signal and to find the systems that can generate the signals similar to the true chaos are important problems.

Usually, the autocorrelation function is used for the periodicity detection of signals[2]. The autocorrelation function is a robust method that can be employed to detect the periodicity of various periodic signals. However, the autocorrelation function requires a lot of computational time and memory resources in computers. The length of the period of the pseudo chaotic signal becomes very long when the double-precision arithmetic is used, so the detection of the period by the autocorrelation function might be not practicable. Besides, the periodicity detection method using multi-rate digital signal processing has been proposed[3, 4] in our research. In the proposed method, the number of pseudo chaotic data is reduced by the down-sampling technique with anti-aliasing filter; the characteristic of the chaotic signal of a long time is condensed into the signal of a short time. As the results, the periodicity detection in the high-speed can be achieved even if the autocorrelation function is used. This method is suitable for the periodicity detection of the signal with the extremely long-length period. However, when the fixed-point arithmetic with short bit-length is used, the period of the generated signal does not always become long, so the proposed method cannot be adopted[5].

In this paper, we propose a method to estimate the periodicity of the pseudo chaotic signal based on the chaotic dynamics. The periodicity of the chaotic signal is generated by the mapping of the multiple times. Then the dynamics of the multiple times are first combined, and the Jacobian of the combined mapping is estimated. We show that the periodicity can be estimated by evaluating the tendency of the Jacobian. In this report, the proposed method is adopted to the periodicity detection of 1st and 2nd order chaotic maps, such as Logistic map and Henon map, and the effectiveness of the proposed method is shown by the computer simulation.

2. Evaluation based on the multiple mapping

In general, the form of the 1st order chaotic map is shown as

$$x(n+1) = D_n\left(x(n)\right), \tag{1}$$

where $D_n(\cdot)$ is the dynamics at the time n. In this study, the mapping from $x(n)$ to $x(n+m+1)$ is collectively evaluated, therefore, the total dynamics is expressed as

$$\begin{aligned} x(n+m+1) &= D_{n+m}\left(D_{n+m-1}\left(\cdots D_n\left(x(n)\right)\cdots\right)\right) \\ &= D_n^{m+1}\left(x(n)\right). \end{aligned} \tag{2}$$

If $x(n+m+1)$ is equal to $x(n)$, the signals $x(n) : n = 0, 1, , \cdots$ have the period of m in length. However, the period of the signal $x(n)$ can be found even for the following.

$$\begin{aligned} x(n+m+1) &= Cx(n) \\ x(n+m+2) &= Cx(n+1) \\ &\vdots \\ x(n+m+k+1) &= Cx(n+k) \end{aligned} \tag{3}$$

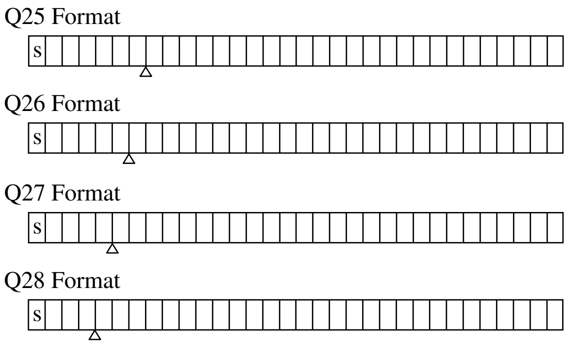

Figure 1: Formats of the fixed-point arithmetic.

```
// VC++ver6.0
#define FXPI 25  // Number of Format
int Ai;
int logistic(int xn)
{
        _int64 acc1,acc2;
        acc1=((_int64)Ai*(_int64)xn)>>FXPI;
        acc2=((((_int64)1)<<(2*FXPI))
                -(((_int64)xn)<<FXPI))>>FXPI;
        return (int)((acc1*acc2)>>FXPI);
}
```

Figure 2: C-Language program of Logistic map by fixed-point arithmetic.

where C is a constant value. Therefore, it is insufficient only to retrieve the point where the orbit is corresponding as the periodicity detection.

Then, we attempt the periodicity detection based on the Jacobian of the dynamics. The Jacobian of the total dynamics D_n^m is calculated as

$$J_n^{m+1} = J_{n+m} J_{n+m-1} \cdots J_n, \tag{4}$$

where $J_{n+k} : k = 0, 1, \cdots, m$ is the Jacobian of $D_{n+k} : k = 0, 1, \cdots, m$, respectively.

Besides, it is well known that the multiplication by a lot of Jacobian causes the overflow in the computation. Actually, we have confirmed that the range which can be set to the variable m is about up to 1000. Then, an actual calculation is executed by the following methods.

$$L_n^{m+1} = \sum_{k=n}^{n+m} \log |J_k| \tag{5}$$

Namely, the information based on the positive / negative signs on the function L_n^{m+1} cannot be considered.

Usually, because of the target map is a 1st order mapping, L_n^{m+1}/m corresponds to the Lyapunov exponent. Lyapunov exponent is a technique to evaluate the divergent / convergent tendency of the map. Namely, if the Lyapunov exponent becomes positive value, it is decided that the map has the chaotic property. However, Lyapunov exponent shows the momentary tendency, so the periodicity included in the pseudo chaos is not reflected in the value of Lyapunov exponent. In this study, we evaluate the tendency based on the function L_n^{m+1} which does not take the time average.

3. Experimental examples using pseudo chaotic signals with short period

In this section, the proposed method is evaluated using pseudo chaotic signals with short period. It is shown that the result of the detected period by the proposed method is corresponding to the result by the autocorrelation function, and the effectiveness of the proposal method is confirmed.

As a chaotic map, the Logistic map shown is first used.

$$x(n+1) = Ax(n)\,(1 - x(n)) \tag{6}$$

As the arithmetical formats, 32-bit fixed-point arithmetic shown in Figure 1 is used. In the computational process, each variable is expanded to 64-bit format to control the arithmetical error. Furthermore, the behavior of the signal $x(n)$ changes depending on the computational sequence. In this experiment, the signals $x(n)$ are generated by the program shown in Figure 2.

Figure 3 shows the examples of the evaluation by L_n^{m+1} on the Logistic map. In this figure, the autocorrelation functions of $x(n)$ are also shown. As the figure shows, there is a point that the values of function L_n^{m+1} corresponds without depending on the value of n. Namely,

$$L_n^{m+1} = L_{n+1}^{m+1} = \cdots = L_{n+k}^{m+1} = \cdots. \tag{7}$$

Moreover, the point of the detected $m+1$ is corresponding to the position of the peak in the autocorrelation function. Therefore, the value of the $m+1$ shows the length of the period of the pseudo chaotic signal.

In this study, the detection of position $m+1$ that shows the period is detected by the iterative calculation. Other detection methods of point $m+1$ to satisfy the Eq. (7) are assumed to be an examination problem in the future.

4. Application into the chaotic map of 2nd or more order

When the proposed method is applied to the chaotic map of 2nd or more order, the calculation of the function L_n^{m+1} shown in Eq. (5) have to be expanded into the matrix computation. However, it is necessary to solve some problems for the achievement.

We assumed that the proposed method for the higher order map becomes as follows.

The unit orthogonal vectors $\mathbf{u}_k : k = 1, 2, \cdots, dim$ are multiplied to the Jacobian $\mathbf{J}_n$.

$$\mathbf{e}_k(n) = \mathbf{J}_n \mathbf{u}_k : k = 1, 2, \cdots, dim \tag{8}$$

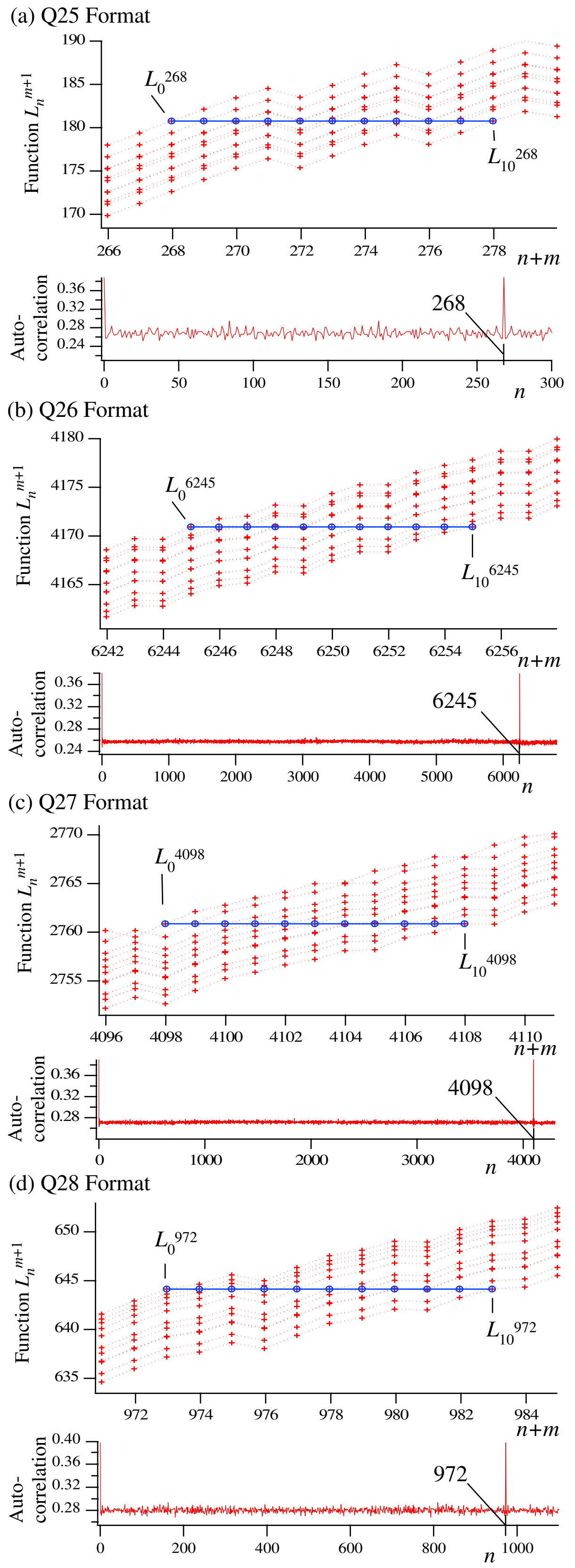

Figure 3: The behavior of the functions L_n^{m+1} of Logistic map and the autocorrelation function of chaotic signal. The parameters: $A = 3.998$, $x(0) = 0.1$. The first 10,000,000[$samples$] of $x(n)$ are omitted to eliminate the transition.

When the analysis method of the Lyapunov exponent is considered, the obtained vectors $\mathbf{e}_k(n) : k = 1, 2, \cdots, dim$ should be orthogonalized using Gram-Schmidt orthogonalization. However, because of the proposed method evaluates the tendency shown in Eq. (5), we propose that the Gram-Schmidt orthogonalization is not employed, and just the length of obtained vectors $\mathbf{e}_k(n)$ are evaluated. Namely, based on the concept of Eq. (5),

$$L_{n,k}^{m+1} = \sum_{i=n}^{n+m} \log |\mathbf{e}_k(i)| \tag{9}$$

is computed. In the proposed method, the unit vectors $\mathbf{u}_k : k = 1, 2$ are fixed, such as $\mathbf{u}_1 = [1, 0]$, $\mathbf{u}_2 = [0, 1]$. This viewpoint is obviously different from the practical Lyapunov exponents analysis.
Now, Henon map is evaluated by the proposed method.

$$x_1(n+1) = x_2(n) + 1 - Ax_1^2(n) \tag{10}$$
$$x_2(n+1) = Bx_1(n) \tag{11}$$

The Jacobian of the Henon map is shown as

$$\mathbf{J}_n = \begin{bmatrix} -2Ax_1(n) & 1 \\ B & 0 \end{bmatrix}. \tag{12}$$

In the case that the unit vectors $\mathbf{u}_2 = [0, 1]$ are multiplied to the Jacobian $\mathbf{J}_n$, $\mathbf{e}_2(n+1)$ is fixed as

$$\mathbf{e}_2(n) = \mathbf{J}_n \mathbf{u}_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}^T, \tag{13}$$

therefore, only the vector $\mathbf{e}_1(n)$ is evaluated.
Figure 4 shows the C++ program for the Henon map using 32-bit fixed-point arithmetic, and Figure 5 shows the estimated function $L_{n,1}^{m+1}$ in the Henon map. As the figure 5 shows, there is a point where the estimated vectors $L_{n,1}^{m+1}$ approach one another, and the detected point $m + 1$ corresponds to the period of pseudo chaos.

5. Conclusions

A method for the periodicity detection based on the Jacobian of the chaotic dynamics has been proposed in this study. The proposed method is similar to the principle of the Lyapunov exponent analysis. However, the proposed method is not taking the time-average of the Jacobian, as the result, the detection of the periodicity of the pseudo chaotic signal that is generated by multiple mapping is enabled.
In this paper, the proposed method is applied to detection of the period of the signal generated by Logistic map and Henon map, and is able to be detected the period in high-speed and high accuracy.
Besides, the period of the chaotic signal is detected by the iterative computation in this research. However, the condition of detection of the period based on Jacobian has been able to be specified. In the future, we will attempt to find other analysis methods based on this condition.

```
// VC++ ver 6.0
#define FXPI 25 // Number of Format
class ivector{
public:
        int v[2];
};
int Ahi,Bhi;

ivector henon(const ivector xn)
{
        _int64 acc1,acc2;
        ivector oxn;

        acc1=((_int64)Ahi*(_int64)xn.v[0])>>FXPI;
        acc1*=(_int64)xn.v[0];
        acc2=(((_int64)1)<<(2*FXPI))
              +(((_int64)xn.v[1])<<FXPI);
        acc2-=acc1; oxn.v[0]=(int)(acc2>>FXPI);
        acc1=(_int64)Bhi*(_int64)xn.v[0];
        oxn.v[1]=(int)(acc1>>FXPI);
        return oxn;
}
```

Figure 4: C++ program of Henon map by fixed-point computation.

References

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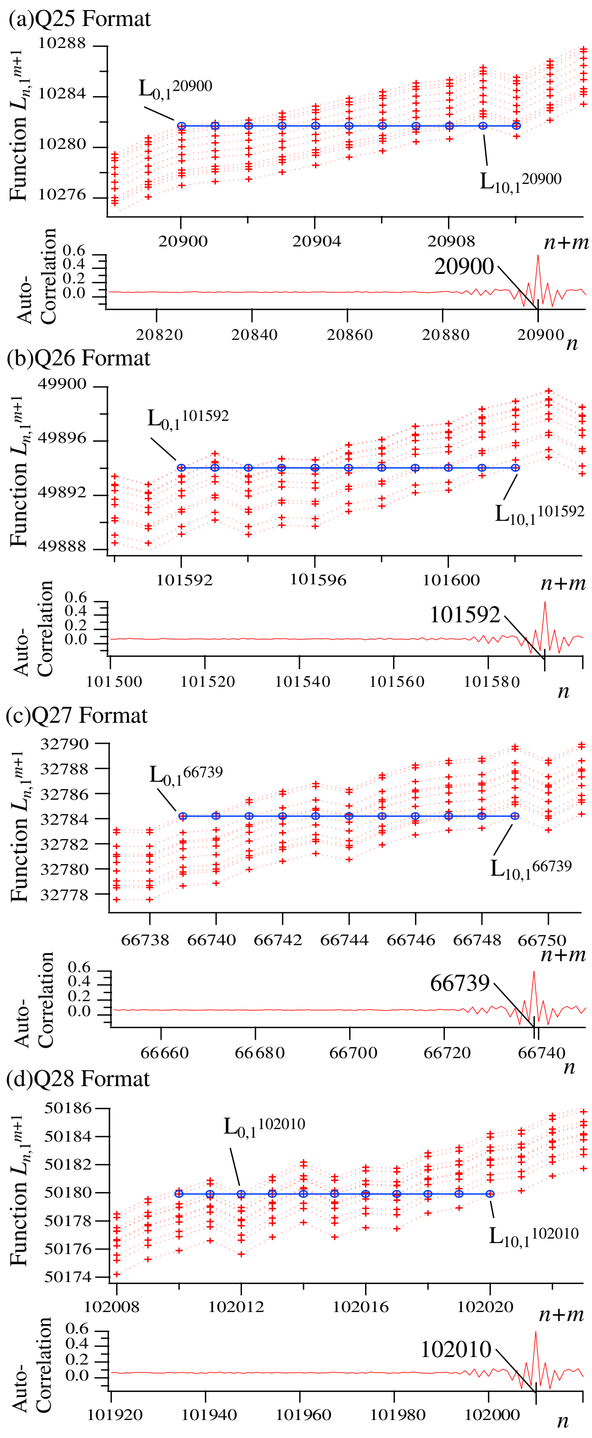

Figure 5: The behavior of the functions $L_{n,1}^{m+1}$ of Henon map and the autocorrelation function of chaotic signal. The parameters: $A = 1.4$, $B = 0.3$, $x_1(0) = 0.1$, $x_2(0) = 0$. The first 10,000,000[*samples*] of $x(n)$ are omitted to eliminate the transition.