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Lifting-Based Lossless Image Coding by Discrete-Time Cellular Neural Networks

Hisashi Aomori*, Tsuyoshi Otake†, Nobuaki Takahashi‡ and Mamoru Tanaka*

*Department of Electrical and Electronics Engineering, Sophia University,
7–1, Kioi-cho, Chiyoda-ku, Tokyo 102–8554, Japan
†Department of Media-Network Science, Tamagawa University,
6–1–1, Tamagawagakuen, Machida, Tokyo, 194–8610, Japan
‡IBM Engineering & Technology Services, IBM Japan, Ltd.,
1623–14, Shimotsuruma, Yamato-shi, Kanagawa 242–8502, Japan

Email: aomori@ieee.org

Abstract—The lifting scheme is an efficient and flexible method for the construction of linear and nonlinear wavelet transforms. In the nonlinear lifting scheme, it is difficult to design the optimal update filter corresponding to the nonlinear prediction filter. It is well-known that the combination use of linear filter and nonlinear filter is an efficient filter pair. In this paper, we propose a novel lifting-based lossless image coding method using discrete-time cellular neural networks (DT-CNNs). In our method, the image is interpolated by using the nonlinear interpolative dynamics of DT-CNNs, and the linear 5-tap filter is used for avoiding the aliasing. Since the output function of DT-CNNs works as a multi-level quantizing function, our method composes the integer lifting scheme for lossless image coding. Moreover, our method makes good use of the nonlinear interpolative dynamics by A-template compared with conventional CNN image coding methods using only B-template. The experimental results show a better coding performance compared with the conventional lifting methods.

1. Introduction

The lifting scheme [1],[2] is a general framework for constructing biorthogonal wavelets, and it has been recognized that nonlinear extensions are possible [3]. The main features of the lifting scheme are that it provides entirely the spatial domain interpolation of the transform and it can be extended into the hierarchical structure easily. Since it also provides reversible wavelet transforms for lossless image and signal compression, it has been applied to many applications such as remote sensing and medical imaging. The performance of the lifting method depends on the ability of the filters to interpolate images. In the conventional lossless image coding using the lifting method, the degradations are caused by the use of the integer wavelet transform instead of the discrete wavelet transform [4]. For efficient interpolations, the quantization noises propagated by the rounding operations should be considered.

Discrete-time cellular neural networks (DT-CNNs) [5] have been applied to many applications such as image compression, filtering, and pattern recognition [6]–[8]. The nonlinear interpolative dynamics by feedback A-template is one of the significant characteristics of CNN. However, in some cases, because the model works as a linear filter, the interpolative dynamics of DT-CNN is not used effectively. In [6], the nonlinear interpolative dynamics by feedback A-template of DT-CNN was used for image compression and the image interpolation corresponded to the optimization problem minimizing the Lyapunov energy function. In other words, the DT-CNN is a solver to solve the optimal problem to minimize the Lyapunov energy function.

In our previous work [8], we proposed the effective implementation of nonlinear lifting scheme by using DT-CNNs. Though this method had a good lossless coding performance, since the prediction filter was a spatial IIR filter by the nonlinear interpolative dynamics of DT-CNN, it was difficult to design the optimal spatial FIR update filter by DT-CNN. In this paper, we propose a novel lossless image coding method based on lifting scheme using DT-CNNs. In our method, the image is interpolated by using the nonlinear interpolative effect by feedback A-template, and the aliasing is avoided by using Le Gall 5-tap update filter. Since the output function of DT-CNNs works as a multi-level quantizing function, our method composes the integer lifting scheme for lossless image coding.

The experimental results show that our method introduced DT-CNN based lifting scheme has a good lossless coding efficiency.

2. Discrete-Time Cellular Neural Network

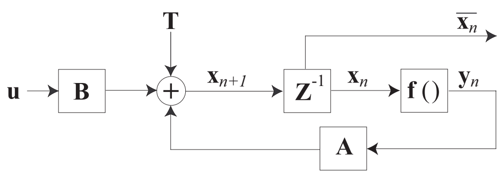

Figure 1: Discrete-time cellular neural network.

Fig. 1 shows the block diagram of the DT-CNN. The state equation of DT-CNN is described in matrix form as

$$\mathbf{x_{n+1}} = \mathbf{Af(x_n)} + \mathbf{Bu} + \mathbf{T}, \tag{1}$$

$$\mathbf{y_{n+1}} = \mathbf{f(x_{n+1})}, \tag{2}$$

where $\mathbf{u}$ is the input matrix, $\mathbf{x}$ is the state variable, $\mathbf{f}()$ is the multi-level quantizing function, and $\mathbf{T}$ is the constant matrix. $\mathbf{A}$ and $\mathbf{B}$ are feed-back and feed-forward template matrices respectively. The Lyapunov energy function of DT-CNN [6] is defined by

$$E_t = -\frac{1}{2}\mathbf{y}^t(\mathbf{A} - \delta\mathbf{I})\mathbf{y} - \mathbf{y}^t\mathbf{Bu} - \mathbf{T}^t\mathbf{y}, \tag{3}$$

where δ is the positive constant value to determine the quantizing region such that $x = \pm\delta$ for $\mathbf{f(x)} = \pm 1$. It is proved that the Lyapunov energy function becomes a monotonically decreasing function, if the $\mathbf{A}$ matrix is symmetric and the diagonal elements are larger than zero [6].

In order to obtain the high quality image, it is necessary that image can be reconstructed considering the distortion. Let $\mathbf{G}$ be a Gaussian filter, the distortion function is defined by

$$dist(\mathbf{y}, \mathbf{u}) = \left\| \frac{1}{2}\mathbf{y}^t(\mathbf{Gy} - \mathbf{u}) \right\|. \tag{4}$$

It means that the difference between the interpolative predict image and the input image should be small. By the comparison between (3) and (4), the $\mathbf{A}$ and $\mathbf{B}$ templates and the parameter $\mathbf{T}$ of interpolative DT-CNN can be determined.

3. Lifting-Based Image Coding Scheme using DT-CNN

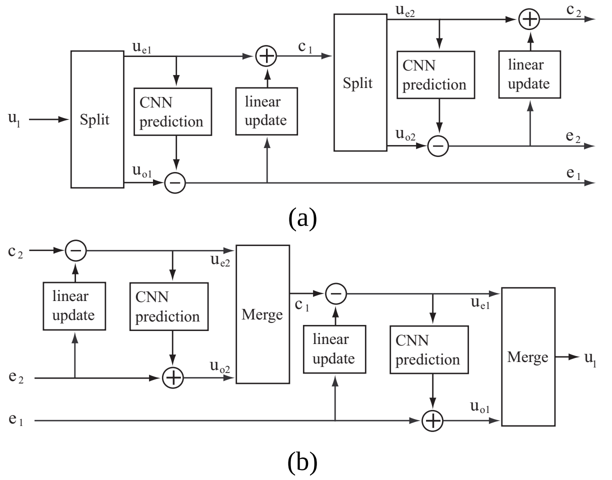

Figure 2: Proposed lossless image coding system : (a) encoder, (b) decoder.

Fig. 2 shows the block diagrams of our proposed system. At the split stage, the original image u_1 is divided into

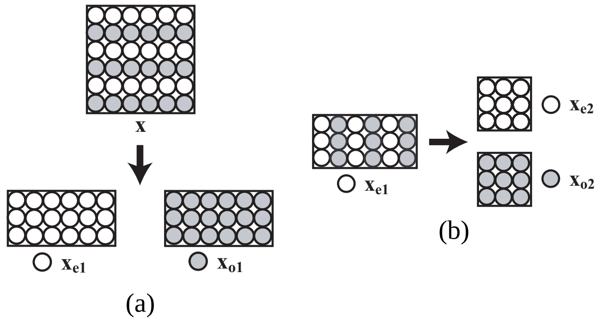

Figure 3: Split step of the lifting scheme: the input image is divided into two polyphase components. (a) Vertical subsampling, (b) Horizontal subsampling.

even polyphase components u_{e1} and odd polyphase components u_{o1} such like Fig. 3. The prediction for each u_{on} is designed by using the two-layered DT-CNN. In the first layer DT-CNN, the distortion function is minimized, and the compressed image for reconstruction can be obtained by the nonlinear interpolative effect of A-template. In the second layer DT-CNN, the interpolated image is obtained by DT-CNN filtering using B-template. Then the prediction residual e_n is transmitted to the decoder. The update for each u_{en} is designed by using the Le Gall linear 5-tap filter. Then we obtain the updated image c_n which is the input of the next stage. In the encoder, these lifting processes using the hybrid filters are applied iteratively. In the decoder, the same lifting rules are applied, and the reconstruction image is gradually improved.

3.1. Image interpolation using two-layered DT-CNN

The subsampled even polyphase images such like Fig. 3 are interpolated using the two-layered DT-CNN. By the comparison between (3) and (4), the templates and the parameters of the first layer DT-CNN are determined as

$$\begin{aligned} \mathbf{A} =& A(i,j;k,l), \qquad C(k,l) \in N_r(i,j) \\ =& \begin{cases} -(1+\lambda) & \text{if } k = i \text{ and } l = j, \\ -\dfrac{1}{2\pi\sigma^2}\exp\left(-\dfrac{\{(k-i)^2 + (l-j)^2\}d_m^2}{2\sigma^2}\right) & \text{otherwise,} \end{cases} \end{aligned} \tag{5}$$

$$\begin{aligned} \mathbf{B} =& B(i,j;k,l), \qquad C(k,l) \in N_r(i,j) \\ =& \begin{cases} 1 & \text{if } k = i \text{ and } l = j \ , \\ 0 & \text{otherwise,} \end{cases} \end{aligned} \tag{6}$$

$$\mathbf{T} = \mathbf{0}, \tag{7}$$

where $N_r(i,j)$ is the r-neighborhood of cell $C(i,j)$ as $N_r(i,j) = \{C(k,l) | \max\{|k-i|, |l-j|\} \leq r\}$, σ is the standard derivative of the Gaussian function, λ is a regularization parameter, and d_m is a sampling interval. Then, we can recall the dynamics of the DT-CNN using the above parameters as follows;

$$x_{ij}(t+1) = \sum_{C(k,l)\in N_r(i,j)} A(i,j;k,l)y_{kl}(t) + u_{ij}, \quad (8)$$

$$y_{ij}(t+1) = f(x_{ij}(t+1)), \quad (9)$$

where $x_{ij}(t)$, $y_{ij}(t)$, and u_{ij} indicate the internal state, the output of the cell, and the input of the cell $C(i, j)$, respectively. The output function $f()$ corresponds to the rounding operation for the integer lifting scheme. It plays an important role that the interpolation is optimized considering the nonlinearity caused by the quantization. At the equilibrium state of the first layer DT-CNN, y^e_{ij} becomes the output and the quantized state variable image is represented by

$$x^{\pm 1.5}_1 = -\mathbf{A}\mathbf{f}^{\pm 1}_{\frac{1}{\delta}}(\mathbf{x}^{\pm 1.5}_1) + \mathbf{u}^{\pm 1}_1, \quad (10)$$

where the subscript and superscript in a variable $\mathbf{v}^{\pm b}_a$ mean "a" is the value of slope and "b" is the value of saturation value of the quantization function, respectively. It is very important that the state variable $x^{\pm 1.5}_1(i, j) \in \mathbf{x}^{\pm\mathbf{1.5}}_{\mathbf{1}}$, which is determined based on minimization of Lyapunov energy function to give an optimized interpolative predict function, is a high quality lossy interpolative DPCM image between the original input $u^{\pm 1}_1(i, j) \in \mathbf{u}^{\pm 1}_1$ and the predict value $\tilde{u}^{\pm 1}_{\frac{1}{\delta}}(i, j) \in \mathbf{A}\mathbf{f}^{\pm 1}_{\frac{1}{\delta}}(\mathbf{x})$. As shown in Fig. 4, the slope $\frac{1}{\delta}$ of the quantizing function $f^{\pm 1}_{\frac{1}{\delta}}(x^{\pm 1.5}_1)$ is larger than that of the quantizing function of the input $u^{\pm 1}_1$ and than that of the transmitted state variable $x^{\pm 1.5}_1$. So the reconstructed image is generated by

$$u^{*\pm 1}_1 = \mathbf{A}\mathbf{f}^{1}_{\frac{1}{\delta}}(\mathbf{x}^{\pm 1.5}_1) + \mathbf{x}^{\pm 1.5}_1 - \delta\mathbf{f}^{1}_{\frac{1}{\delta}}(\mathbf{x}^{\pm 1.5}_1), \quad (11)$$

through the transmission of the quantized state variable image $\mathbf{x}^{\pm 1.5}_1$ for $\left\|\mathbf{x}^{\pm 1.5}_1\right\| < \delta$ and $\mathbf{x}^{\pm 1.5}_1 = 1$ for $\left\|\mathbf{x}^{\pm 1.5}_1\right\| \geq \delta$. The range of the state variable x should be $-\delta \leq x \leq +\delta$ to realize lossless encoding and decoding. If the δ is lager, the state variable x would be in the range but the quantization function $f()$ has high sensitivity. The high sensitivity of the quantization function $f()$ causes falling in local minimum state. The first layer DT-CNN is adopted to obtain the optimal range of the state variable.

The output of the first layer DT-CNN becomes the input of the second layer DT-CNN which has no dynamics, and the output of the second layer DT-CNN provides the predicted value of the odd polyphase image. Using the output y^e_{ij} at the equilibrium state of the first layer CNN, the interpolated pixel $\hat{y}_{ij}$ is obtained by

$$\hat{y}_{ij} = \sum_{y_{kl}\in N'(i,j)} \hat{B}(i,j;k,l)y^e_{ij}. \quad (12)$$

At the odd layer stages, we use the $\hat{\mathrm{B}}$-template which is obtained by extending the A-template of the first layer DT-CNN vertically, that is,

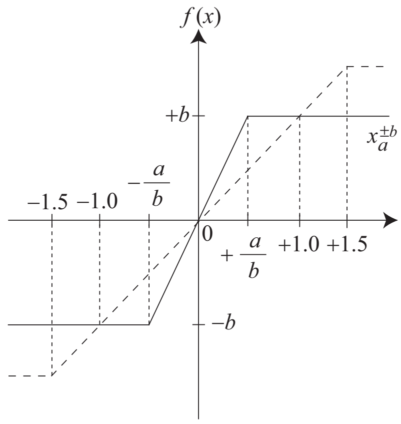

Figure 4: Slopes of quantization functions.

$$\begin{aligned}\hat{\mathbf{B}} =& \hat{B}(i,j;k,l), \qquad C(k,l)\in N'(i,j) \\ =& \frac{1}{2\pi\sigma^2}\exp\left(-\frac{((k-0.5)d_m-i)^2+(l-j)^2}{2\sigma^2}\right), \\ N'(i,j) =& \{C(k,l)|\max\{|(k-0.5)d_m-i|, |l-j|\}\leq rd_m\}.\end{aligned} \quad (13)$$

In the same manner, at the even layer stages, the $\hat{\mathrm{B}}$-template is obtained by extending the A-template of the first layer DT-CNN horizontally;

$$\begin{aligned}\hat{\mathbf{B}} =& \hat{B}(i,j;k,l), \qquad C(k,l)\in N'(i,j) \\ =& \frac{1}{2\pi\sigma^2}\exp\left(-\frac{(k-i)^2+((l-0.5)d_m-j)^2}{2\sigma^2}\right), \\ N'(i,j) =& \{C(k,l)|\max\{|(k-i|, |(l-0.5)d_m-j|\}\leq rd_m\}.\end{aligned} \quad (14)$$

3.2. Image Update by Linear Filter

In order to avoid aliasing, we use the Le Gall 5-tap linear filter for the each update step. The updated image c_{ij} is obtained by

$$c_{ij} = \begin{cases} u_{e\,ij} + \lfloor (e_{i-1\,j} + e_{i\,j} + 2)/4 \rfloor & \text{odd layer stages,} \\ u_{e\,ij} + \lfloor (e_{i\,j-1} + e_{i\,j} + 2)/4 \rfloor & \text{even layer stages,} \end{cases} \quad (15)$$

where $\lfloor\cdot\rfloor$ denotes the round-off operator.

4. Experimental Results

We implemented the coder and decoder of our proposed lossless image coding algorithm based on lifting wavelet using DT-CNNs. We applied our system to the 8-bit standard gray-scale test images; "Aerial," "Barbara," "Boat," "Crowd," and "Lena." The size of all the images is 512×512 pixels.

The performance of the proposed method was compared with the our previous method [8] and the separable 2D lifting method using Le Gall 5-tap/3-tap filters (JPEG 2000) indicated as “5/3 tap” in Table 2 and Table 3. For the simulation, the coding factor is decided experimentally; the number of lifting layers $L = 4$, the r−neighborhood of cell $r = 2$, the regularization parameter $\lambda = -1$. Moreover, the standard deviation of Gaussian σ is decided as Table 1.

Table 2 shows the the energy of the difference images for the each layer. As shown in Table 2, our proposed method has a better prediction performance comparing with the conventional lifting methods.

Table 3 shows the coding performance for the proposed method. The proposed method consistently outperforms the conventional lifting methods.

Table 1: σ of Gaussian function.

Aerial	Barbara	Boat	Crowd	Lena
0.575	0.565	0.572	0.600	0.600

Table 2: Energy of the difference images for the each layer.

Image	Method	1st Layer	2nd Layer	3rd Layer	4th Layer
Aerial	Proposed	**151.0**	**553.2**	**1018.7**	**1309.4**
	Previous	152.9	603.0	1192.3	1634.4
	5/3	161.0	605.6	1173.9	1496.9
Barbara	Proposed	201.9	**378.4**	**323.8**	**592.9**
	Previous	199.5	491.7	501.8	852.8
	5/3	201.3	412.2	404.5	704.3
Boat	Proposed	**67.9**	**218.4**	**432.3**	**632.0**
	Previous	69.0	243.3	529.2	895.0
	5/3	73.0	239.2	499.8	684.6
Crowd	Proposed	**39.8**	**207.9**	**537.1**	**992.3**
	Previous	40.4	223.7	606.4	1218.7
	5/3	46.2	236.6	660.6	1164.4
Lena	Proposed	**25.9**	**103.7**	**258.8**	**466.7**
	Previous	26.4	113.0	308.3	578.3
	5/3	29.6	125.2	333.6	585.4

Table 3: Comparison of coding performance in terms of lossless rates (bits/pixel).

Image	Aerial	Barbara	Boat	Crowd	Lena
Proposed	**5.50**	**5.22**	**5.02**	**4.56**	**4.38**
Previous	5.55	5.30	5.08	4.59	4.44
5/3	5.55	5.32	5.07	4.65	4.44

5. Conclusion

The lifting-based lossless image coding method using DT-CNN has been proposed. In our method, the dynamics of the DT-CNN is exploited to obtain the optimal prediction considering the nonlinear quantization error. The experimental results show that our proposed method has a better coding performance which is compared with conventional lifting methods. In future, we will design and implement hardware to realize the proposed lossless image coding system in order to accelerate the processing time of the conversion of DT-CNN.

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References

[1] W. Sweldens, “The lifting scheme: A custom-design construction of biorthogonal wavelets,” *Appl., Comput. Harmon. Anal.*, vol. 3, no. 2, pp. 186–200, 1996.

[2] W. Sweldens, “The lifting scheme: A construction of second generation wavelets,” *SIAM J. Math. Anal.*, vol. 29, no. 2, pp. 511–546, 1997.

[3] R. L. Claypoole Jr., G. M. Davis and W. Sweldens, “Nonlinear wavelet transforms for image coding via lifting,” *IEEE Trans. Image Processing*, vol. 12, no. 12, pp. 1449–1459, Aug. 1999.

[4] J. Reichel, G. Menegaz, M.J. Nadenau and M. Kunt, “Integer Wavelet Transform for Embedded Lossy to Lossless Image Compression,” *IEEE Trans. Image Processing*, vol. 10, no. 3, pp. 383–392, Mar. 2000.

[5] L. O. Chua and L. Yang, “Cellular neural networks: theory,” *IEEE Trans. Circuits Syst.*, vol. 35, no. 10, pp. 1257–1272, Oct. 1988.

[6] N. Takahashi, T. Otake, and M. Tanaka, “Nonlinear Interpolative Effect of Feedback Template for Image Processing by Discrete-Time Cellular Neural Network,” *Journal of Circuits, Systems, and Computers*, World Scientific Publishing Company, vol. 12, no. 4, Aug. 2003.

[7] P. L. Venetianer and T. Roska, “Image Compression by Cellular Neural Networks,” *IEEE Trans. Circuits Syst.*, vol. 45, no. 3, pp. 205–215, June 1998.

[8] H. Aomori, K. Kawakami, T. Otake, N. Takahashi, M. Yamauchi, and M. Tanaka, “Separable 2D Lifting using Discrete Time Cellular Neural Networks for Lossless Image Coding,” *IEICE Trans. Fundamentals*, (in press).