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Finding the Shortest Path by Using an Excitable Digital Reaction-Diffusion System

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Abstract—This paper presents a shortest path search algorithm using a model of excitable reaction-diffusion dynamics. In our previous work, we have proposed a framework of Digital Reaction-Diffusion System (DRDS) — a model of a discrete-time discrete-space reaction-diffusion system useful for nonlinear signal processing tasks. In this paper, we design a special DRDS, called an "excitable DRDS," which emulates excitable reaction-diffusion dynamics and produces traveling waves. We also demonstrate an application of the excitable DRDS to the shortest path search problem defined on two-dimensional (2-D) space with arbitrary boundary conditions.

1 Introduction

Living organisms can create a remarkable variety of structures to realize intelligent functions. In embryology, the development of patterns and forms is sometimes called *Morphogenesis*. In 1952, Alan Turing suggested that a system of chemical substances, called *morphogens*, reacting together and diffusing through a tissue, is adequate to account for the main phenomena of morphogenesis [1]. Recently, model-based studies of morphogenesis employing computer simulations have begun to attract much attention in mathematical biology [2],[3].

From an engineering viewpoint, the insights into morphogenesis provide important concepts for devising a new class of intelligent signal processing functions inspired by biological pattern formation phenomena [4],[5]. From this viewpoint, we have proposed a framework of *Digital Reaction-Diffusion System* (DRDS) — a discrete-time discrete-space reaction-diffusion dynamical system — for designing signal processing models exhibiting active pattern/texture formation capability. In our previous papers [6],[7], some applications of DRDS to biological texture generation and fingerprint image enhancement/restoration have already been discussed.

The DRDS can simulate a variety of reaction-diffusion dynamics by changing its nonlinear reaction kinetics. This paper describes the design of an *excitable* DRDS based on FitzHugh-Nagumo-type dynamics [2]; the designed DRDS creates excitable traveling waves exhibiting the following characteristics: (i) the waves propagate with a constant velocity, and (ii) they vanish in collisions with other waves without any other interaction. These features suggest a unique algorithm for the shortest path search problem as described in Ref. [8], where the optimal pathways were determined by the collection of time-lapse position information on actual chemical waves propagating through two-dimensional (2-D) mazes prepared with the Belousov-Zhabotinsky (BZ) reaction.

So far, there are some papers discussing the mechanism of finding the collision-free shortest path in a 2-D map using excitable reaction-diffusion dynamics. In the papers [8]–[13], the real chemical reaction, called BZ reaction, is employed as an excitable medium to generate traveling waves for path finding. The use of real chemical media for performing practical computing tasks has the weakness of limited stability in its operation. Also, the size and complexity of maps that can be handled in chemical computers may be limited. The other related papers basically employ continuous-time models of excitable dynamics, including partial differential equation models [8],[14] and circuit models [11],[15]. All these works focus on the mechanism of generating equidistant surfaces for the given map by using excitable chemical waves and describes only simple examples of small maps.

The goal of this paper is to propose a concrete algorithm for shortest path search in 2-D space using the excitable DRDS, including the process of tracing back the equidistant surfaces. The proposed algorithm is based on the discrete-time discrete-space model of DRDS, which is easily implemented in digital computers, and can be applied to arbitrary maps of practical size and complexity.

2 Excitable Digital Reaction-Diffusion System

A Digital Reaction-Diffusion System (DRDS) — a model of a discrete-time discrete-space reaction-diffusion dynamical system — can be naturally derived from the original reaction-diffusion system defined in continuous space and time (see [6] for basic mathematical discussions).

DRDS can simulate various reaction-diffusion dynamics by changing its nonlinear reaction kinetics and parameters. In this paper, we use the FitzHugh-Nagumo (FHN) model, which is one of the most widely studied excitable models [2]. The two-morphogen FHN-based DRDS, called the *excitable* DRDS, is de-

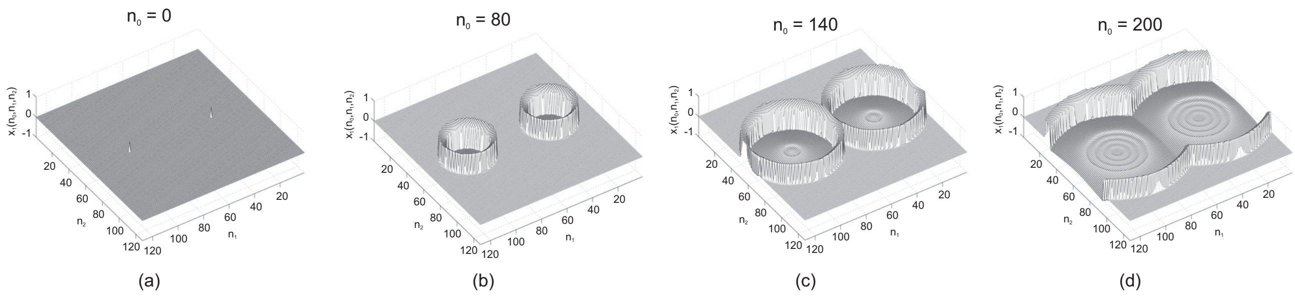

Figure 1: Wave propagation in the 2-D excitable DRDS: (a) initial condition, (b)–(d) snapshots of wave propagation.

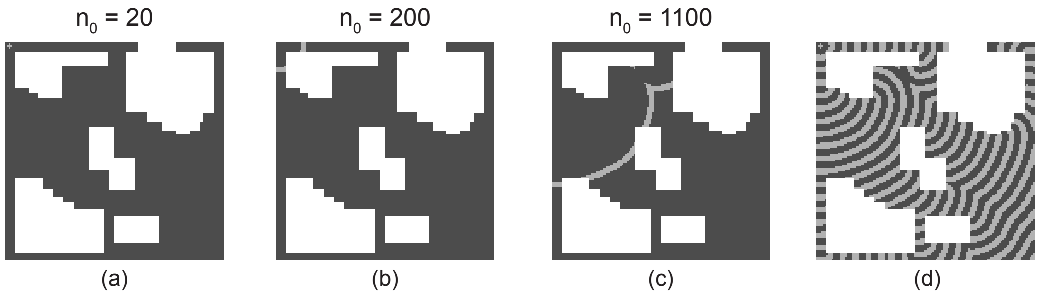

Figure 2: Wave propagation in the 2-D excitable DRDS, with obstacles appearing as white parts: (a)–(c) the snapshots of wave propagation in $x_1(n_0, n_1, n_2)$, (d) superposition of traveling waves in $x_1(n_0, n_1, n_2)$ taken every 100 steps.

fined as follows:

$$\begin{aligned}\left[\begin{array}{c} x_1(n_0{+}1, n_1, n_2) \\ x_2(n_0{+}1, n_1, n_2) \end{array}\right] &= \left[\begin{array}{c} x_1(n_0, n_1, n_2) \\ x_2(n_0, n_1, n_2) \end{array}\right] \\ &+ \left[\begin{array}{c} R_1(x_1(n_0, n_1, n_2), x_2(n_0, n_1, n_2)) \\ R_2(x_1(n_0, n_1, n_2), x_2(n_0, n_1, n_2)) \end{array}\right] \\ &+ \left[\begin{array}{c} D_1(l * x_1)(n_0, n_1, n_2) \\ D_2(l * x_2)(n_0, n_1, n_2) \end{array}\right], \qquad (1)\end{aligned}$$

where n_0 is a time index, n_1 and n_2 are spatial indices, x_1 and x_2 are the concentrations of the morphogens, and D_1 and D_2 are the diffusion coefficients of morphogens x_1 and x_2, respectively. Here R_1 and R_2 are the nonlinear reaction kinetics for morphogens x_1 and x_2, respectively, and are given by

$$\begin{aligned} R_1(x_1, x_2) &= T_0 \left\{ \frac{1}{k_1} \{x_1\{x_1 - k_2\}\{1 - x_1\} - x_2\} \right\}, \\ R_2(x_1, x_2) &= T_0 \{x_1 - k_3 x_2\}, \end{aligned}$$

where T_0 is a time sampling interval. The operator $*$ in Eq. (1) denotes the spatial convolution, and

$$l(n_1, n_2) = \begin{cases} \frac{1}{T_1^2} & (n_1, n_2) = (-1, 0), (1, 0) \\ \frac{1}{T_2^2} & (n_1, n_2) = (0, -1), (0, 1) \\ -2(\frac{1}{T_1^2} + \frac{1}{T_2^2}) & (n_1, n_2) = (0, 0) \\ 0 & \text{otherwise,} \end{cases}$$

where T_1 and T_2 are space sampling intervals. In this paper, we employ the parameter set: $k_1 = 10^{-3}$, $k_2 = 10^{-6}$, $k_3 = 0.1$, $D_1 = 40$, $D_2 = 0$, $T_0 = 10^{-3}$, and $T_1 = T_2 = 1$.

The excitable DRDS exhibits the characteristic behavior of excitable dynamics and generates traveling waves depending on the initial condition. Assume that the initial condition is given by $x_1(0, n_1, n_2) = x_2(0, n_1, n_2) = 0$ except for the starting point (n_1^S, n_2^S). When we give a stimulus above the threshold (~ 0.9 for the above parameter set) at the starting point, for example, $x_1(0, n_1^S, n_2^S) = 0.9$, a traveling wave is initiated from the starting point and propagates with a constant velocity as the time step n_0 increases.

Consider an excitable DRDS of size 128×128, where n_1 and n_2 are defined as $0 \leq n_1 \leq 127$, $0 \leq n_2 \leq 127$. Figure 1 shows the wave propagation observed in the snapshots of the first morphogen $x_1(n_0, n_1, n_2)$. In this example, we first give initial stimuli as $x_1(0, 32, 64) = x_1(0, 96, 64) = 0.9$ (Fig. 1 (a)). The traveling waves spread in a circular pattern and vanish in collisions with another wave as shown in Figs. 1 (b)–(d).

In the above example, we can observe two important characteristics of excitable waves: (i) the waves propagate with a constant velocity and (ii) they vanish in collisions with other waves. These features suggest a unique algorithm for the shortest path search problem as described in Ref. [8], where snapshots of real propagating chemical waves are considered as a collection

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procedure Forward Operation
  Input
    a starting point (n_1^S, n_2^S), a goal (n_1^G, n_2^G),
    a map (with obstacle information);
  Output
    W(n_0): a list of points (n_1, n_2) in 2-D space at which
            the value of x_1(n_0, n_1, n_2) is higher than a specific
            threshold value 0.9 (that is, W(n_0) stores the list
            of points at which the traveling wave exists),
    n_0^G: the time step when the traveling wave arrives at
           the goal (n_1^G, n_2^G);
begin
  n_0 ← 0; { Initialize the time step }
  x_1(0, n_1, n_2) ← 0 for all the points (n_1, n_2);
  x_2(0, n_1, n_2) ← 0 for all the points (n_1, n_2);
  x_1(0, n_1^S, n_2^S) ← a constant (> 0.9);
  W(0) ← {(n_1^S, n_2^S)};
  repeat
    Compute the excitable DRDS (Eq. (1)) for one step
      assuming the boundary condition defined by the map,
      and derive x_1(n_0 + 1, n_1, n_2) and x_2(n_0 + 1, n_1, n_2);
    Store the points of the wavefronts into W(n_0+1)
      (that is, the points at which the value of x_1(n_0+1, n_1, n_2)
      is higher than the threshold value 0.9);
    n_0 ← n_0 + 1
  until the traveling wave arrives at (n_1^G, n_2^G);
  n_0^G ← n_0
end.
```

Figure 3: Algorithm for *Forward Operation.*

of equidistant surfaces and are useful for finding the shortest path from the starting point to any specified point in 2-D space.

3 Shortest Path Search Algorithm

This section proposes a shortest path search algorithm using the excitable DRDS designed in the above section. The proposed algorithm employs the excitable DRDS for wavefront generation and performs the traceback of traveling wavefronts to find the shortest paths.

Figures 2 (a)–(c) show the wave propagation in a 2-D excitable DRDS of the size 128 × 128 pixels. Note that we employ the boundary condition defined by obstacles in a map (specifying collision-free space and blocked space), where we set fixed concentrations $x_1 = 0$ and $x_2 = 0$ for obstacle locations. In Fig. 2 (d), 29 snapshots of wavefronts of the first morphogen x_1 are superimposed at every 100-step intervals to form a composite image. Each wavefront represents a set of equidistant locations measured from the starting point, and hence we can derive the shortest path by tracing back the history of wavefront position from the goal to the starting point.

The proposed algorithm consists of two operations: *Forward Operation* and *Backward Operation.* Forward Operation is to generate a traveling wave in the excitable DRDS and record snapshots of equidistant wavefronts with specific time intervals. Backward Operation, on the other hand, is to trace back the wavefronts from the goal to the starting point to find the

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procedure Backward Operation
  Input
    (n_1^S, n_2^S), (n_1^G, n_2^G), W(n_0), n_0^G,
    Δ: a resolution of the time step interval for Backward
        Operation (Δ = 4 in our experiments);
  Output
    Paths: a list of points (with their time index) on the
           shortest path from (n_0^G, n_1^G, n_2^G) to (0, n_1^S, n_2^S);
begin
  Search ←< (n_0^G, n_1^G, n_2^G) >;
  Paths ← BackTrace(Search)
end.

function BackTrace(Search)
begin
  (n_0^b, n_1^b, n_2^b) ← the last element of Search;
  if n_0^b − Δ ≤ 0 then
    Search ← append{Search, < (0, n_1^S, n_2^S) >};
    return Search
  else
    begin
      C ← get the points in W(n_0^b − Δ) that have the
          shortest distance from (n_1^b, n_2^b);
      C ← BranchDetection(C);
      Search ←
        ⋃_{(n_1^c, n_2^c)∈C} BackTrace(append{Search, < (n_0^b − Δ, n_1^c, n_2^c) >});
      return Search
    end
end.

function BranchDetection(C)
begin
  C' ← get the points in W(n_0^b − 2Δ) that have the shortest
        distance from each point in C (see Fig. 5);
  Calculate the distance between the two(or more) points in C';
  if the calculated distance is less than Δ then
    Replace the points in C with their middle position;
  return C
end.
```

Figure 4: Algorithm for *Backward Operation.*

shortest pathways. We can obtain the shortest path by connecting every pair of two points on the adjacent equidistant wavefronts with the shortest distance. Figures 3 and 4 show the detailed algorithms for the Forward and Backward Operations, respectively.

4 Experiments

This section presents some experiments of shortest path search using the proposed algorithm. The traveling wave initiated from the starting point (n_1^S, n_2^S) propagates in the Forward Operation. The shortest path from the starting point to the goal is obtained in the Backward Operation.

Figure 6 shows the experimental result on the map with symmetric obstacles. In this experiment, we obtain two shortest paths, since obstacles are symmetric to the line between the starting point and the goal. By using the proposed algorithm, we can obtain exactly two shortest paths which have equal distances.

Figure 7 shows a typical example of shortest path search in a maze, where five different goals are specified in advance. We can obtain all the paths for each

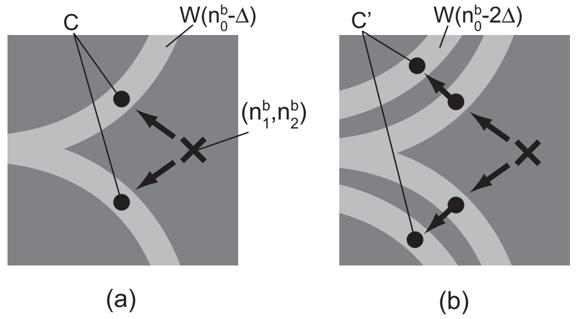

Figure 5: Branch detection. There is a possibility that the shortest path has multiple branches at time $n_0^b - \Delta$ (a). To verify whether the points in C are true branches or not, the *BranchDetection* procedure further traces the wavefront backward to the time step $n_0^b - 2\Delta$ (b).

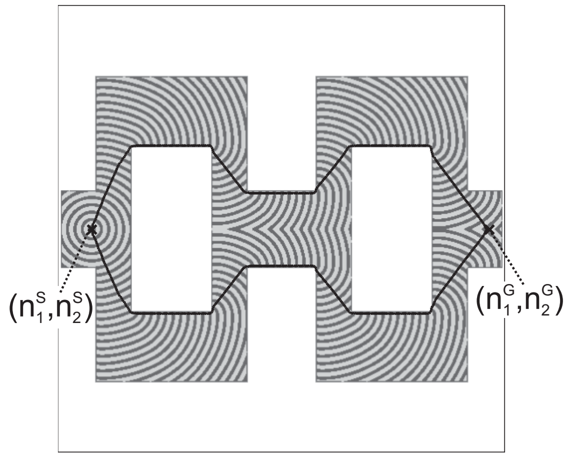

Figure 6: Shortest path search on the map with symmetric obstacles.

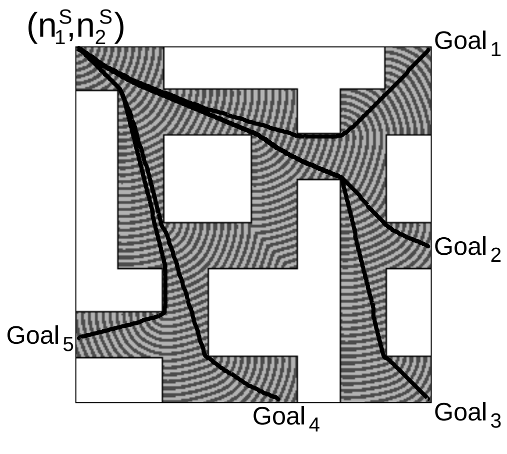

Figure 7: Shortest path search in a maze.

specified goal.

In the above experiments, we can observe that all the obtained paths from the starting point to the goal are shortest in terms of Euclidean distance.

5 Conclusion

This paper presents a shortest path search algorithm in 2-D space using the excitable Digital Reaction-Diffusion System (DRDS). The proposed algorithm could be applied to various navigation tasks defined in 2-D space, and could also be extended to shortest path search algorithms for higher-dimensional space.

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