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Improvement of High-Accuracy CT Image Reconstruction Using Nonlinear Dynamics

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Abstract– CT scanners are used for imaging the inside of human bodies to detect foci of disease. Issues on performance of CT scanner are to accelerate reconstruction time and improve quality of reconstructed image. We developed a method for CT image reconstruction to solve these issues.

First, the problem of CT image reconstruction can be defined as finding a steady-state solution to a continuous time nonlinear dynamical system with a discrete projection operator and projection data acquired from a CT scanner. Then, Euler and ADI methods were applied to find the solution. We will compare the efficiency of these methods and the quality of reconstructed image.

1. Introduction

CT scanners fulfill a very important role in medical field in the sense that they can visualize the inside of human bodies as a tomographic image. Using 3D images obtained by a CT scanner, a doctor judges whether there is a focus of disease inside subjects' bodies or not.

Filtered back-projection (FBP) and iterative methods have been used to reconstruct computed tomography (CT) images from a projection dataset. The FBP method is widely used in clinical CT scanners, because its speed to reconstruct images is so quick owning to digital hardware technologies. However, artifacts appear in images reconstructed by the FBP method when there exists a metal substance or a substance with high radiation absorption in the inside of a subject.

Compared with the FBP method, iterative methods can produce a high quality image. However, it needs the huge amount of memory and time to reconstruct [1], [2].

In this paper, we improve the reconstruction of the CT image that is obtained by finding the steady-state solution of a set of nonlinear ordinary differential equations [3]. Since the nonlinear equations have a similar form of Lyapunov or Riccati equation in the field of control theory, an alternative direction implicit (ADI) iteration method, which is known as an efficient solver of Lyapunov equation [5], can be introduced as a numerical integration method.

In the illustrative examples, it is confirmed that ADI method gives better quality of the reconstructed image and the computation time is compatible to that with conventional Euler method [3].

2. Image Reconstruction

2.1. Linear System for Image Reconstruction

A CT scanner is composed of an X-ray irradiator and detector. Acquisition of tomographic image by CT scanner is to irradiate the X-ray detector side and detect the X-ray intensity distribution. Then, a tomographic image is obtained by analyzing the irradiation distribution.

We express a projection dataset as $y \in R^P$, which corresponds to the X-ray intensity distribution obtained by the detectors, and $A \in R^{(P \times N)}$ is a projection operator corresponding to the discrete Radon transform that indicates an observation process of projection data, where P is the number of projections and N is the number of pixels. Then, the liner system that we should solve the problem of CT image reconstruction is given as

$$Ax^* = y \tag{1}$$

where $x^* \in R^N$ is the vector of true pixel's values. Hence, the basic problem of CT image reconstruction is to solve the linear system. The aim of our study is to develop an efficient method to find a more accurate solution x to the system than that of conventional iterative methods.

2.2 Nonlinear Dynamics

On the basis of the projection operator, a set of ordinary differential equations for image reconstruction is defined as

$$\frac{dx(t)}{dt} = X(t)A^T(y - Ax(t)) \tag{2}$$

where $x(t)$ corresponds to the reconstructed images at t, and $X(t)$ is the diagonal matrix that the diagonal entries are $x(t)$ [3]. In actual CT scanner, the size of projection matrix becomes huge, for example, P and N correspond to approximately nine million and quarter million. Then the computer memory lacks to reconstruction images, besides it takes long time to obtain a steady-state solution to Eq. (2) even if Eq. (2) is calculable using a numerical integration. Fujimoto et al. [3] have proposed the projection operator A and the projection dataset y are divided into some blocks as

$$\frac{dx(t)}{dt} = X(t)B_m^T(y_m - B_m x(t)) \quad (m = 1,2,3...M) \tag{3}$$

where B_m and y_m are the mth block of A and dataset, respectively. Equation (3) is simulated in the order of m = 1, 2, ..., M until the $x(t)$ reaches the steady-state.

3. Numerical Integrations

3. 1 Euler Method

The Euler method is the simplest method for solving initial value problems of nonlinear ordinary differential equation. By applying the Euler method to Eq. (3), the following updating rule

$$x_{n+1} = x_n + h(X_n B_m^T (y_m - B_m x_n)) \tag{4}$$

is obtained where h is the time step size. The dynamics of $x(t)$ is obtained by applying Eq. (4).

3. 2 ADI method

ADI method is a semi-implicit numerical integration [5], whereas Euler method of (4) uses a n explicit one. Since Eq. (2) has the cross term $X(t)A^TAx(t)$, a large scale of nonlinear equations must be solved, even if an implicit formula is applied to (2). Instead of this, the cross term is alternately replaced as $X_{n+1}A^TAx_n$ and $X_nA^TAx_{n+1}$. This is the reason why this method is called ADI method. The updating two update equations to (4) are given as follows:

$$\frac{x_{n+1} - x_n}{h} = X_{n+1} B_m^T (y_m - B_m x_n) \tag{5}$$

$$\frac{x_{n+1} - x_n}{h} = X_n B_m^T (y_m - B_m x_{n+1}) \tag{6}$$

We noted that it is possible to use the ADI method partially. Namely, either (5) or (6) can be used for the whole simulation.

4. Basic Algorithm

This section explains a foundational algorithm how to obtain the reconstruction of four pixels by the ADI method. First, we here set the true pixel values to (x^*_1, x^*_2, x^*_3, x^*_4) = (5, 3, 4, 9) ^{T}. Next, we define the projection operator and the projection dataset, where it is assumed that the pixel values are reconstructed after six times' projections as shown in Figure 1. Therefore, the projection matrix A is 6 ×4 elements:

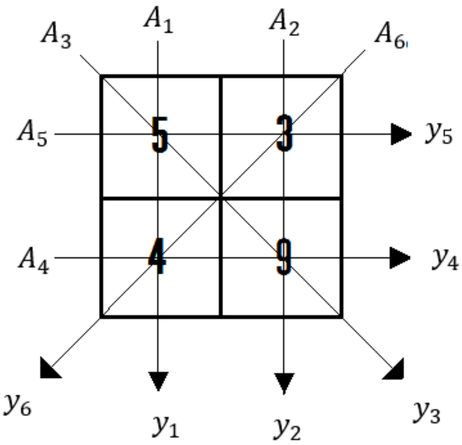

Figure 1: Toy problem of CT-image reconstruction with four-pixel image and six projections.

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} \tag{7}$$

where the column number corresponds to the pixel number, and the row number corresponds to the projection number, respectively. We here have y = (9, 12, 14, 13, 8, 7) ^{T} by applying x* and A to Eq. (1).

Let us consider projection operators that A is divided into two subsets. The projection operators are given as follows:

$$B_1 = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix} \quad B_1 = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} \tag{8}$$

To obtain the pixel values by using B_1 and B_2, the projection dataset is also divided into the two subsets. In this example, the projection datasets y_1 and y_2 is given as

$$y_1 = \begin{pmatrix} 9 \\ 12 \\ 14 \end{pmatrix} \qquad y_2 = \begin{pmatrix} 13 \\ 8 \\ 7 \end{pmatrix} \tag{9}$$

We set the parameter values for simulation of nonlinear dynamics below. The number of time steps is 500, the time step size is 1.0×10^{-2}, and all the initial pixel values are 10. Figure 2 shows the dynamics of x obtained by the ADI method with two subsets, where a numerical analysis software MATLAB is used to implement the ADI method.

As shown in Figure 2, the solution of each pixel value of x converges to the true pixel value of x^* finally, although the value changes abruptly when a subset was switched to the next one.

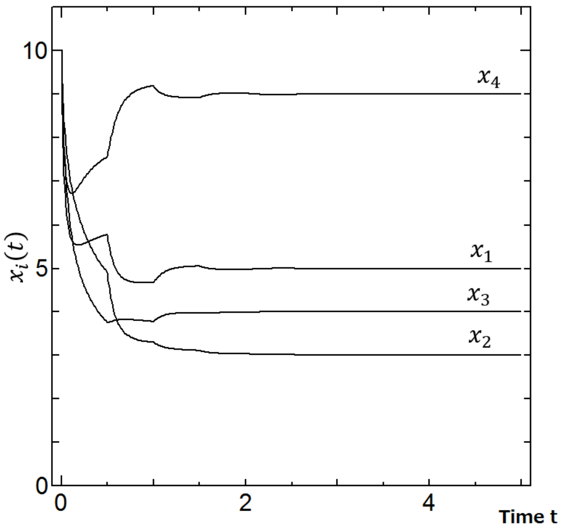

Figure 2: Time evolution of x obtained by ADI method.

5. Results

This section shows the results of reconstructing the images. We used the phantom skull image imitating the human skull that MATLAB has. The image is shown in Figure 3.

Figure 3: Phantom image

The size of the phantom image and reconstructed images is 64×64, the angular range of X-ray detectors is 180 degrees, the number of the projections is 100, the projection operator is divided into two subsets, and the time step size h is 3.0×10^{-4}. We calculate the dynamics of x at 2,000 points. Table 1 shows the results obtained by the Euler method, the two partial ADI methods, and the ADI method. In Table 1, Peak Signal-to-Noise Ratio (PSNR) is an evaluation index for quality of image, where an image with a high value PSNR means better quality in the sense that reconstructed image x is close to the true image x^*. We defined the residual norm on the basis of the Euclidean distance between the original projection dataset y and the back-projection Ax.

As shown in Table 1, we reconstructed images by each method. Although there is no big difference between the values of PSNRs and the residual norms, we found the difference in reconstruction times with the respective method. It took long periods in the case of "only Eq. (5)" and the use of the ADI method comparing to that of the other methods. This is because they involved the calculation of the inverse matrix.

Table 1 PSNR and residual norm of the obtained image and computation time needed to reconstruct the image.

	PSNR [dB]	Reconstruction time [sec]	Residual norm
Euler method	23.71	168.9	7.607
Only Eq.(5)	23.70	169.9	7.636
Only Eq.(6)	23.90	14403	7.641
ADI method	23.80	7355	8.445

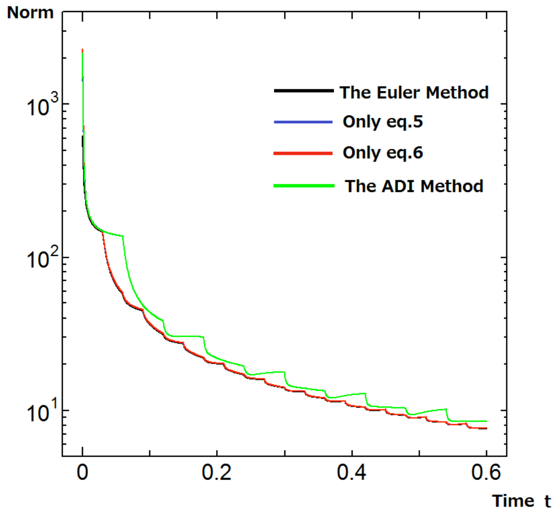

Figure 4: Change of the residual norm

When the time step size was changed to 3.0×10^{-3}, we were able to reconstruct the image quickly by using only Eq. (6) with the aforementioned parameter values. As shown in Table 2, we obtained better results with high PSNRs and small residual norms comparing to Table 1.

Table 2: Experimental results when using (6) only with h=3.0×10^{-3}.

	PSNR [dB]	Reconstruction time [sec]	Residual norm
100 steps	23.34	19.35	20.01
300 steps	29.45	59.95	6.537
1000 steps	37.49	169.9	1.948

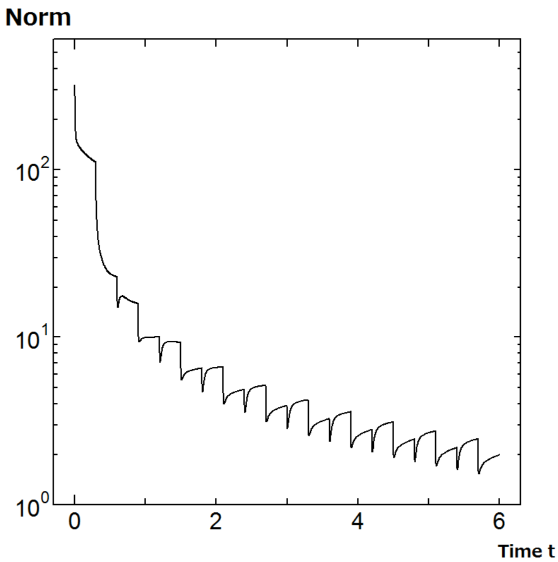

Figure 5: Change of residual norm when using (6) only with h=3.0×10^{-3}.

When the projection operator A was not divided into the subsets (namely, m=1), we were able to reconstruct image even if we set the time step size 10,000 in using only Eq. (6). Then, the reconstruction process was completed with just one step as shown in Table 3. The reconstruction time was so short, and the values of PSNR and the residual norm were also good.

Table 3: The experimental result obtained by only Eq. (6), where A was not divided and a time step size was set to 10,000.

	PSNR [dB]	Reconstruction time [sec]	Residual norm
1 step	58.38	11.56	0.002

6. Conclusion

On the basis of ADI approach, we proposed methods to reconstruct high quality CT-images in a short time. From the numerical examples, we found the fact that the use of only Eq. (6) without dividing the projection operator is the best in the proposed methods. However, it seems that this method is to calculate equation close to $x=A^{-1}y$ by making the step size h large value. Moreover, it is difficult to reconstruct CT images with the real size in clinical CT scanners because the size of the projection operator becomes large in proportion as the number of projections and pixels. We will study on its more improvement.

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