2005 International Symposium on Nonlinear
Theory and its Applications (NOLTA2005)
Bruges, Belgium, October 18-21, 2005

Switching Dynamics in Four Coupled Line Array of Oscillators with Hard-Type Nonlinearity

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Abstract– This paper investigates various bifurcations and related dynamics such as transitional phenomenon in four coupled line array of oscillators with hard-type nonlinearity. This system has some periodic attractors for comparatively large ε value (= a parameter showing the degree of nonlinearity), and they disappear for a certain value of ε via saddle-node(S-N) bifurcation to become quasi-periodic attractors when ε is decreased. Sometimes, there exists a heteroclinic cycle at the bifurcation point. In such cases, the system presents the switching phenomenon right after the S-N bifurcation. We clarify the existence of some heteroclinic cycles by drawing unstable manifold of saddles in Poincare section, and demonstrate that the switching phenomenon is caused by the heteroclinic cycle by computer simulation. At last, we introduce wave propagation phenomenon briefly.

1. Introduction

The line array of coupled oscillators has been investigated for a long time mainly for weakly nonlinear case via averaging method, and the dynamics for weakly nonlinear case are almost elucidated [1]. But, its dynamics for strongly nonlinear case seems not to be investigated so far. In [2] we showed the switching dynamics in two coupled oscillators with hard type nonlinearity for strongly nonlinear case. In this paper, we investigate various dynamics related to global bifurcation in four coupled line array of oscillators with hard-type nonlinearity for comparatively large ε (degree of nonlinearity). In particular, we are interested in periodic attractors observed only for large ε. These periodic attractors disappear via saddle-node bifurcation for a certain value of ε, and for smaller values of ε, quasi-periodic attractors can be observed. At the bifurcation point, unstable manifold of saddles forms a heteroclinic cycles, and switching dynamics along this heteroclinic cycle can be observed right after the disappearance of the periodic attractor. In this system, we have found some types of heteroclinic (homoclinic) cycles, and we will analyze their dynamics in this paper. At last, we introduce wave propagation phenomenon observed for a certain parameter set.

2. Fundamental Equation

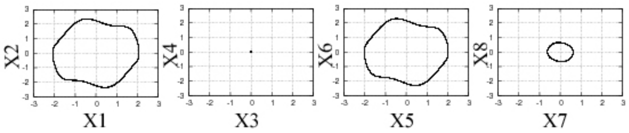
The four inductance-coupled oscillators with hard-type nonlinearity can be written by the following 8th –order autonomous system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\varepsilon(1-\beta x_1{}^2 + x_1{}^4)x_2 - (1+\alpha)x_1 + \alpha x_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -\varepsilon(1-\beta x_3{}^2 + x_3{}^4)x_4 - (1+\alpha)x_3 + \alpha x_1 + \alpha x_5 \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = -\varepsilon(1-\beta x_5{}^2 + x_5{}^4)x_6 - (1+\alpha)x_5 + \alpha x_3 + \alpha x_7 \\ \dot{x}_7 = x_8 \\ \dot{x}_8 = -\varepsilon(1-\beta x_7{}^2 + x_7{}^4)x_8 - (1+\alpha)x_7 + \alpha x_5 \end{cases} \quad (1)$$

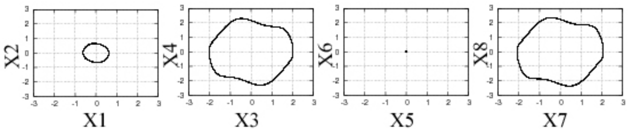
where x_1, x_3, x_5 and x_7 denote the normalized output voltage of the first, second, third and fourth oscillator, x_2, x_4, x_6 and x_8 are their derivatives, respectively. The parameter ε >0 shows the degree of nonlinearity. The parameter 0< α <1 is a coupling factor; namely, α =1 means maximum coupling, and α =0 means no coupling. The parameter β controls amplitude of oscillation. Equation (1) has symmetric nature such that the system is invariant by replacing x_1 by x_7, x_2 by x_8, x_3 by x_5 and x_4 by x_6. In the following sections we take Poincare section at $x_2 = 0$, therefore, a periodic flow in eight dimensional phase space become a discrete point in seven dimensional phase space $(x_1, x_3, x_4, x_5, x_6, x_7, x_8) \in R^7$.

3. Bifurcation of the Periodic Attractors

We will show some examples of this bifurcation. Fig.1 is (a) a periodic and (b) its symmetric counterpart for $\alpha = 0.11, \beta = 3.1$ and $\varepsilon = 0.48$. In Fig.1(a), the first and third oscillators oscillate with large amplitude and



(a)



(b)

Fig.1: A Periodic attractor for comparatively large ε :
$\alpha = 0.11, \beta = 3.1$ and $\varepsilon = 0.48$. Initial condition is
(a) : (2.089,0.000 ,-0.025, 0.014, -2.026, 0.011,0.028,-0.732)
(b) : (0.683,0.000 ,-0.086, -2.109, -0.015, -0.025,-0.0404,2.252)

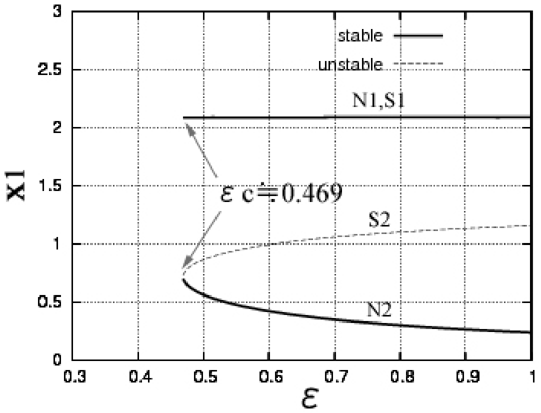
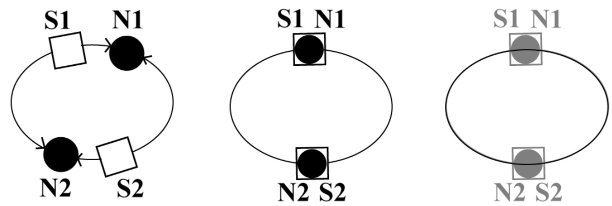


Fig.2: Bifurcation diagram of two symmetric solutions: for $\alpha = 0.11$, $\beta = 3.1$

synchronized with reverse phase, and second oscillator is zero and forth oscillator oscillates with small amplitude. Fig.1(b) is a symmetric counterpart of Fig.1(a). Fig.2 presents bifurcation of these two periodic attractors; namely, the periodic solution in Fig.1(a)(Fig.1(b)) corresponds to a stable node N1(N2), and an associated saddle S1(S2) appears. The periodic solutions N1-S1 and N2-S2 show the saddle-node (S-N) bifurcation at the same point of $\varepsilon = 0.469(\equiv \varepsilon c)$. We want to know what kind of dynamics occur for ε smaller than εc. To predict this we draw schematic diagrams of nodes, saddles with their unstable manifolds(UM's) in Fig.3. In Fig.3(a) one of the unstable manifold of S1(S2) goes to N2(N1). Namely, a cycle connecting two nodes and two saddles is formed for $\varepsilon > \varepsilon c$. For $\varepsilon = \varepsilon c$ this cycle become a "heteroclinic" cycle connecting two (degenerate) saddles. For $\varepsilon < \varepsilon c$ all nodes, saddles and UM's disappear via S-N bifurcation, but their "loci" still survive as far as ε close to εc. Therefore, we can predict the dynamics for ε smaller than but close to εc from the dynamics of UM's for $\varepsilon > \varepsilon c$.

Fig.4 shows nodes, saddles and their UM's in Poincare section (projection) for $\varepsilon = 0.470(> \varepsilon c)$. Note a heteroclinic cycle connecting two (almost degenerate) saddles. The cross marked points show Poincare map of the (quasi-periodic) attractor for $\varepsilon = 0.468(< \varepsilon c)$. It is clear that 1)flow stays around the locus of one node for a long time, and 2) quickly moves along the locus of UM, and 3) stays again around the locus of the other node for a long time, and 4) moves quickly along the locus of UM, vice versa. This is a verification which we predict in Fig.3(c)! We call such dynamics "switching phenomenon". This is observed for ε smaller than but close to εc . When ε become smaller, the switching attractor becomes an ordinary quasi-periodic attractor.



(a) $\varepsilon > \varepsilon c$ (b) $\varepsilon = \varepsilon c$ (c) $\varepsilon < \varepsilon c$

Fig.3:Schematic diagram of nodes, saddles and UM's

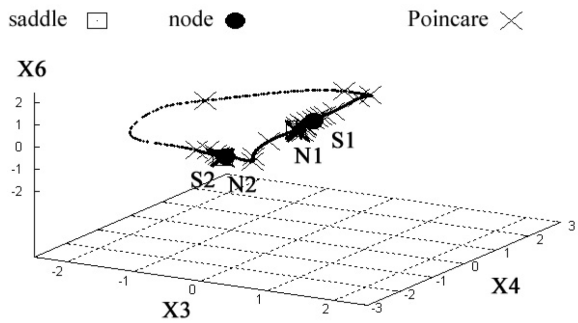


Fig.4: Computer calculation of nodes, saddles and UM's for $\alpha = 0.11$, $\beta = 3.1$ $\varepsilon = 0.470$ Projection onto the (x_3, x_4, x_6) - space. □ : saddle ● : node The cross mark(×) present the Poincare mapped points right after the S-N bifurcation at $\varepsilon = 0.468$.

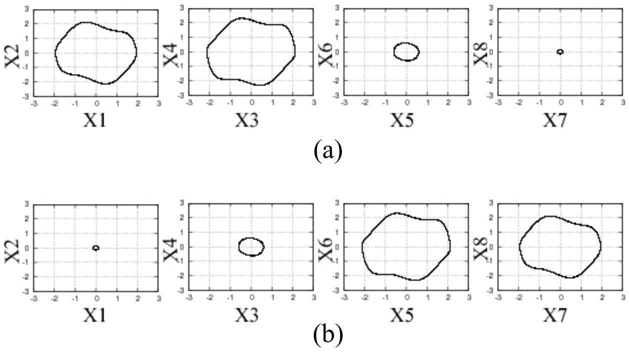


Fig.5: Periodic attractors comparatively large $\varepsilon = 0.453$ for $\alpha = 0.11, \beta = 3.1$. Initial condition is

(a) : (1.931, 0.00 ,1.789, 1.538, 0.219, 0.710,-0.171,-0.015)

(b) : (0.175, 0.00, 0.088, -0.722 -2.013, -0.889,-1.902,0.309)

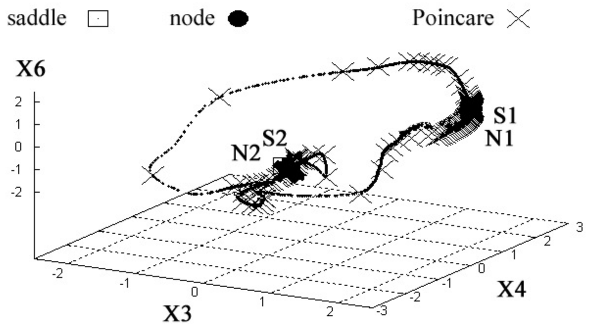


Fig.6: Computer calculation of nodes, saddles and UM's for $\alpha = 0.11$, $\beta = 3.1$ $\varepsilon = 0.453$. Projection onto the (x_3, x_4, x_6) - space. The cross mark(×) present the Poincare mapped points right after the S-N bifurcation at $\varepsilon = 0.451$.

Next, we will show another example. Figures 5(a) and (b) show periodic solutions in which two of four oscillators oscillate with large amplitude with almost the same phase. This pair of periodic attractors disappear via S-N bifurcation for $\varepsilon = 0.452(\equiv \varepsilon c)$, and a heteroclinic cycle is formed at εc . Fig.6 shows nodes, saddles and their UM's in the projected Poincare section for the periodic solutions in Fig.5. The cross marks in the figure show Poincare mapped points for the corresponding switching attractor. Switching dynamics can be observed from condensed points around the loci of nodes and saddles.

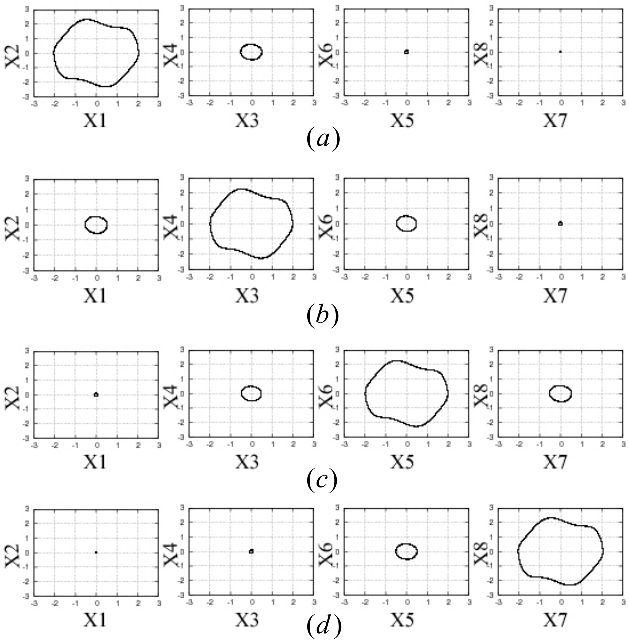


Fig.7: Periodic attractors exist for comparatively large ε : $\alpha = 0.11, \beta = 3.1$ and $\varepsilon = 0.48$. Initial condition is

(a) : (2.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0)

(b) : (0.0, 0.0, 2.0, 0.0, 0.0, 0.0, 0.0, 0.0)

(c) : (0.0, 0.0, 0.0, 0.0, 2.0, 0.0, 0.0, 0.0)

(d) : (0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 2.0, 0.0)

At last, we will show an example of switching dynamics where two S-N bifurcation values are not the same. Fig.7 shows four periodic solutions where only one of four oscillators oscillates with large amplitude. The second and third periodic solutions disappear via S-N bifurcation at $\varepsilon = 0.445(\equiv \varepsilon c1)$. The first and fourth ones at $\varepsilon = 0.425(\equiv \varepsilon c2)$. For ε smaller but close to $\varepsilon c2$, the switching dynamics between loci of attractors in Figs.7(a) and (b) occurs. Similarly, those between loci of attractors in Figs.7(c) and (d) occur via symmetry of equation. Fig.8 shows nodes, saddles and their UM's in the projected Poincare section for the periodic solutions in Fig.7(a) and (b). The cross marks in the figure show Poincare mapped points for the corresponding switching attractor. In these cases two S-N bifurcation values are close but not the same (It is differ from the above cases); namely, $\varepsilon c1 = 0.445$ and $\varepsilon c2 = 0.425$. Therefore, no heteroclinic cycle is formed between two nodes. This dynamics can be understood from the schematic diagram of Fig.9. For $\varepsilon > \varepsilon c1$ there exist stable nodes N1, N2 and saddles S1,S2, and a cycle is formed by UM's as shown in Fig.9(a). For $\varepsilon = \varepsilon c1$ the N2 and S2 degenerate as in Fig.9(b). For $\varepsilon c2 < \varepsilon < \varepsilon c1$ N2 and S2 disappear but

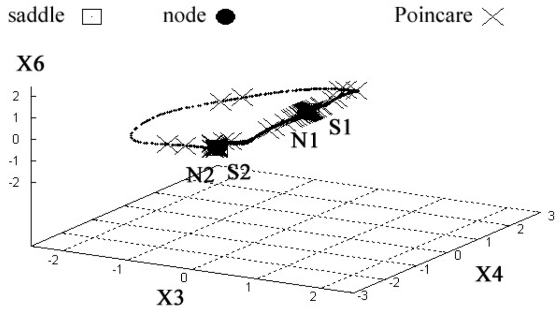


Fig.8: Computer calculation of nodes, saddles and UM's for $\alpha = 0.11$, $\beta = 3.1$ $\varepsilon = 0.426$. Projection onto the (x_3, x_4, x_6) - space. The cross mark(×) present the Poincare mapped points right after the S-N bifurcation at $\varepsilon = 0.424$.

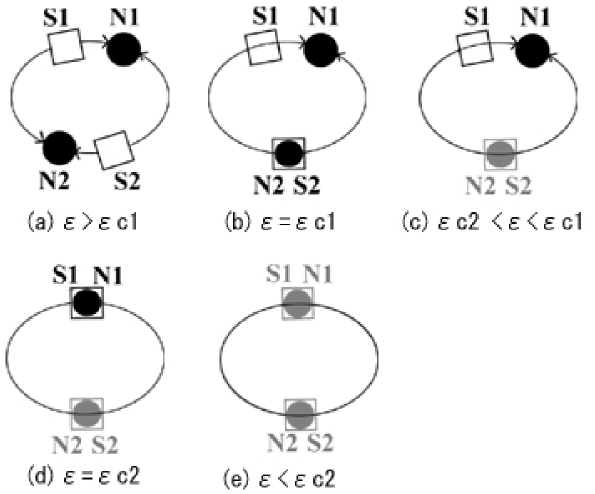


Fig.9:Schematic diagram of nodes, saddles and UM's

their loci still exist as in Fig.9(c). A cycle connecting N1 to itself is formed. For $\varepsilon = \varepsilon c2$ N1 and S1 degenerate as in Fig.9(d). A homoclinic cycle connecting (degenerate) N1 to itself is formed. For $\varepsilon < \varepsilon c2$ N1, S1 and N2,S2 and the homoclinic cycle disappear but their loci still exist as in Fig.9(e). Therefore, switching dynamics between the loci of N1 and N2 occur.

From this example, we notice that bifurcation values are not necessarily equal for switching dynamics to occur.

4. Wave Propagation Phenomenon [3]

We can observe wave propagation phenomenon for comparatively large ε. Fig.10 shows an example. We can see the oscillation with large amplitude propagates from the first oscillator to the fourth oscillator successively. This wave is observed for $\varepsilon = 0.09 \sim 0.35$ for $\alpha = 0.11, \beta = 3.1$.

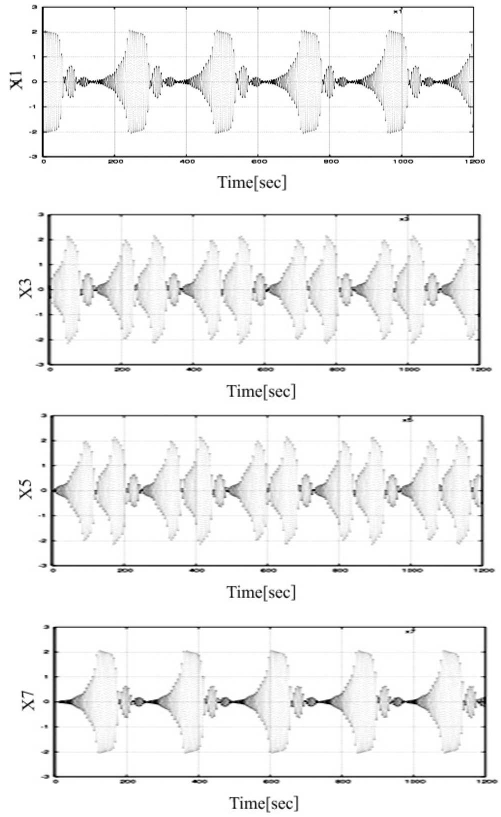


Fig.10: An example of wave propagation phenomenon observed for $\alpha = 0.11, \beta = 3.1$ and $\varepsilon = 0.35$ from initial condition $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = (2,0,0,0,0,0,0,0)$

5. Conclusions

We investigate various switching dynamics in four coupled line array of oscillators with hard type nonlinearity. This switching dynamics are related to the saddle-node bifurcation and behavior of unstable manifold of saddles. We will investigate more through dynamics of unstable manifold in the near future. We also investigate wave propagation phenomenon.

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