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Estimation of coupling between oscillators from short time series via phase dynamics modeling: limitations and application

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We demonstrate in numerical experiments that estimators of strength and directionality of coupling between oscillators based on modeling of their phase dynamics [D.A. Smirnov and B.P. Bezruchko, Phys. Rev. E **68**, 046209 (2003)] are widely applicable. Namely, although the expressions for the estimators and their confidence bands are derived for linear uncoupled oscillators under the influence of independent sources of Gaussian white noise, they turn out to allow reliable characterization of coupling from relatively short time series for different properties of noise, significant phase nonlinearity of the oscillators, and non-vanishing coupling between them. We apply the estimators to analyze a two-channel human intracranial epileptic electroencephalogram (EEG) recording with the purpose of epileptic focus localization.

1. Introduction

Characterization of coupling between two oscillatory systems from their time series is an important task in different fields of scientific research and practice, including climatology, electronics, and physiology, etc. Most of the well-known approaches, such as cross-spectral analysis and information-theoretic characteristics, are often insufficient to detect directional coupling from complex real-world signals. In the last years, new promising techniques are suggested by nonlinear dynamics. One of them if the sensitive approach involves construction of an empiric model for the phase dynamics and calculation of interaction strength from the values of its parameters. The idea is suggested originally in [1] and the technique to realize it is called "evolution map approach" (EMA). It is efficient for analysis of oscillatory processes unsynchronized with each other and exhibiting pronounced main rhythms of oscillations that allows to introduce well-defined phases. In its initial version, EMA provides reliable results for stationary time series of quite a considerable length, such as 5000 characteristic periods under moderate noise levels.

However, in practice one often encounters *nonstationary* signals (e.g., EEG recordings). The special corrections have been introduced into formulas for the EMA coupling estimators, so that the latter become unbiased even in the case of relatively short time series (down to 50 basic periods), and expressions for their confidence bands have been derived in [2]. The modified expressions for the coupling estimators are derived under the assumptions of linear uncoupled phase oscillators influenced by independent sources of Gaussian white noise. Their applicability in other cases has neither been rigorously proven, nor thoroughly investigated experimentally. Our purpose here consists in a systematical investigation of the limits of applicability of the modified EMA estimators.

2. Methods

2.1. Modified evolution map approach

The main idea of the original method is to estimate how strongly future evolution of the phase of one system depends on the current value of the phase of the other system. To achieve this, one obtains time series of the oscillations' phases $\phi_{1,2}(t_i)$ from original time series of the two systems $x_{1,2}(t_i)$. The phases are estimates using the analytic signal concept typically in one of the two ways: via Hilbert transform and complex wavelet transform. The sampling frequency for the original time series is desirable to be not less than 20 points per basic period [3]. In variety of situations, the phase dynamics of oscillators exhibiting a pronounced main rhythm are adequately described with stochastic differential equations of the form

$$d\phi_{1,2}/dt = \omega_{1,2} + G_{1,2}(\phi_1, \phi_2) + \xi_{1,2}(t)\,, \qquad (1)$$

where parameters $\omega_{1,2}$ govern oscillators' frequencies, $\xi_i(t)$ are independent Gaussian white noises. When dealing with discrete time series, it is convenient to consider a difference form of these equations

$$\Delta_{1,2}(t) = F_{1,2}[\phi_1(t), \phi_2(t), \mathbf{a}_{1,2}] + \varepsilon_{1,2}(t), \qquad (2)$$

where $\Delta_i(t) \equiv \phi_i(t+\tau) - \phi_i(t)$ are phase increments over fixed time interval τ, $\varepsilon_i(t)$ noises, F_i trigonometric polynomials, $\mathbf{a}_i$ vectors of their coefficients. The intensity c_1 of the influence of the second system on the first one (2→1) would be defined as the steepness of the dependence $F_1(\phi_2)$, and everything is the same for the intensity c_2 of the influence 1→2:

$$c_{1,2}^2 = \frac{1}{2\pi^2}\int_0^{2\pi}\int_0^{2\pi}\left[\partial F_{1,2}(\phi_1, \phi_2, \mathbf{a}_{1,2})/\partial \phi_{2,1}\right]^2 d\phi_1 d\phi_2\ . \qquad (3)$$

The estimators $\hat{c}_{1,2}$ appear "good" only for very long stationary signals whose length should be about 5000 basic periods for the sampling frequency 10-20 points per a basic period and moderate noise level [2]. For shorter time series often encountered in practice, these estimators turn out to be biased. The modified estimators $\hat{\gamma}_{1,2}$ for $c_{1,2}^2$ and the estimator $\hat{\delta} \equiv \hat{\gamma}_2 - \hat{\gamma}_1$ for the directionality index $\delta = c_2^2 - c_1^2$ are suggested in [2]. Expressions for their 95% confidence bands are derived in the from $[\hat{\gamma}_i - 1.6\hat{\sigma}_{\hat{\gamma}_i}, \hat{\gamma}_i + 1.8\hat{\sigma}_{\hat{\gamma}_i}]$ and $\hat{\delta} \pm 1.6\hat{\sigma}_{\hat{\delta}}$. Under the assumption of linear uncoupled phase oscillators and independent sources of Gaussian white noise, these modified estimators are unbiased and provide the rate of erroneous conclusions about coupling presence and directionality less than 5 % for time series whose length may be as small as 50 basic periods.

2.2. Technique for investigation of applicability limits in numerical experiments

In this work, we vary different properties of oscillators and find out where the estimators $\hat{\gamma}_{1,2}$ and $\hat{\delta}$ are still reliable. Following Refs. [1, 2], we use the third-order polynomials F_i. We calculate also mean phase coherence $\rho = \left| \langle \exp\{ j(\phi_2 - \phi_1)\} \rangle \right|$, where angle brackets stand for the time average, which quantifies the degree of synchrony in the systems' oscillations, to check whether it can always warn about inapplicability of the method. The time series of phases in numerical experiments are of the length $N = 1000$ and we use ensembles of 1000 time series to assess statistical properties.

3. Results

3.1. Influence of noise properties

We apply the method to estimate coupling from time realizations with different properties of $\varepsilon_{1,2}$ for $G_i(\phi_1,\phi_2) \equiv 0$. Noises $\varepsilon_{1,2}$ are taken to be Gaussian with autocorrelation function (ACF) linearly decreasing down to zero over the interval $[0,T]$. Firstly we vary T in the range $[0,10\tau]$ ($\tau = 2\pi$). Noise level $\sigma_1 = \sigma_2 = \sigma$ is varied in the range [0,0.6]. We found that for all T and σ, the number of erroneous conclusions about coupling presence does not exceed 4 % and the estimators $\hat{\gamma}_{1,2}$ and $\hat{\delta}$ remain unbiased. Thus, as one can judge from this particular example, variation of the ACFs of the noises $\varepsilon_{1,2}$ does not itself bound applicability of the estimators $\hat{\gamma}_{1,2}$ and $\hat{\delta}$.

Next, we consider noises $\varepsilon_{1,2}$ with qualitatively different probability density functions (PDFs). We consider the following PDFs: uniform distribution on a finite interval, demeaned chi-square distribution with one degree of freedom, and random alternation of values drawn from two Gaussian distributions with the same variance and different expectations.

The results are practically the same for all PDFs and noise levels. Namely, the estimators are unbiased and the number of erroneous conclusions about coupling presence is less than 5 %. So, the form of the PDFs does not seem to affect applicability of the estimators also.

3.2. Influence of the individual nonlinearities of oscillators

To check to what extent the properties of the estimators deteriorate when oscillators are nonlinear, we calculate $\hat{\gamma}_{1,2}$ and $\hat{\delta}$ from time realizations of the system (1) with $G_i(\phi_1,\phi_2) = \omega_i + b\cos\phi_i$. The coefficient b determines the "phase nonlinearity strength". The results for $\hat{\gamma}_1$ are shown in Fig. 1 (a). They are analogous for $\hat{\gamma}_2$. The estimator $\hat{\gamma}_1$ is unbiased and the probability of erroneous conclusion about coupling presence is less than 5 % in the region to the left from the solid line, i.e. up to sufficiently strong nonlinearity b = 0.3-0.7. The values of mean phase coherence ρ are shown in Fig. 1 (b) with grayscale, ρ increases with nonlinearity to some extent but becomes relatively small.

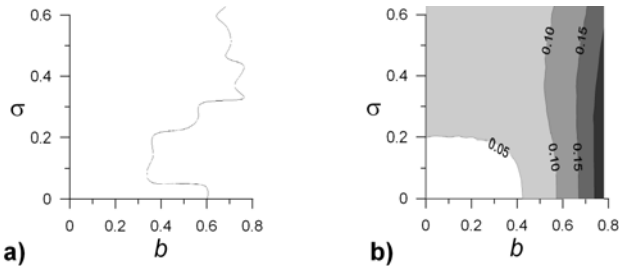

FIG. 1. a) Regions of the coupling estimators applicability on the plane "nonlinearity – noise" for uncoupled oscillators. b) Mean phase coherence values in grayscale.

3.3. Influence of coupling strength

We calculate $\hat{\gamma}_{1,2}$ and $\hat{\delta}$ from time realizations of the system (1) with $G_{1,2}(\phi_1,\phi_2) = \omega_{1,2} + k_{1,2}\sin(\phi_{2,1} - \phi_{1,2})$. The coefficients k_1, k_2 determine the coupling strengths.

Unidirectional coupling ($k_1 = 0, k_2 = k$) In Fig. 2 (a) we show the "triangle" region where the estimates $\hat{\gamma}_{1,2}$ are unbiased (this condition determines the right boundary which is close to vertical straight line) and the number of correct conclusions about coupling strength is greater than 75% (this condition determines the curved left boundary which makes sense as a minimal reliably identifiable coupling strength for a given noise level). The estimators are erroneous if $\rho > 0.8$, see Fig. 2 (a).

The causes of bias in the estimates in the case of large k are following: (i) synchronization for low noise levels [Fig. 2 (a)], (ii) nonlinearity of the phase dynamics induced by the presence of coupling for high noise levels. At a given noise level, the best situation is an intermediate strength of unidirectional coupling, since at weak coupling the probability of correct conclusion is low due to noise and at strong coupling the estimates become biased due to synchronization or just phase nonlinearity.

Bidirectional coupling.($k_1 = k$, $k_2 = k + 0.02$). The value of coupling asymmetry $k_2 - k_1$ is held constant. The results of calculations are shown in Fig. 2 (b). The region of the coupling estimators efficiency is bounded on the right (i.e. for large coupling strength). ρ reaches a value of 0.8 within this region. Again, there are the same two causes that limit the estimators' applicability.

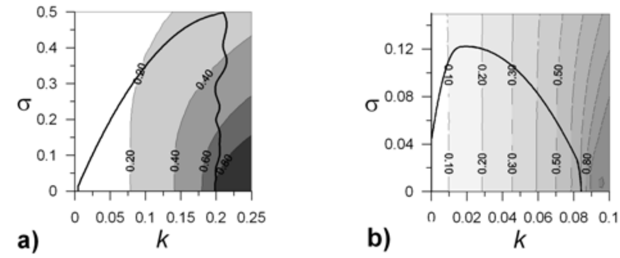

FIG. 2. Regions of the coupling estimators applicability on the plane "coupling – noise" for coupled phase oscillators a) unidirectional coupling, b) bidirectional asymmetrical coupling

3.4. Van der Pol oscillators

More realistic is a situation where one observes not phases directly but rather some variables from which one needs to calculate phases and, hence, may introduce some additional errors. We take coupled van der Pol oscillators as an object:

$$d^2 x_{1,2} / dt^2 = 0.2(1 - x_{1,2}^2)\, dx_{1,2} / dt - \omega_{1,2}^2 x_{1,2} + + k_{1,2}(x_{2,1} - x_{1,2}) + \xi_{1,2} . \quad (4)$$

We calculate the phases of variables x_1, x_2 with the aid of Hilbert transform The oscillators possess individual phase nonlinearity. Noise in the phase dynamics equations is not precisely Gaussian and white. So, this object represents simultaneous violation of several conditions for the estimators applicability. We consider unidirectional coupling: $k_1 = 0$, the value of $k_2 = k$ In Fig. 3 (a) we present the region where the estimators are unbiased (right boundary) and the probability of correct conclusion about coupling presence is greater than 75 % (left boundary). ρ reaches approximately 0.7 within the region. The results are quite analogous to Fig. 2 (a) in Sec.3.3.

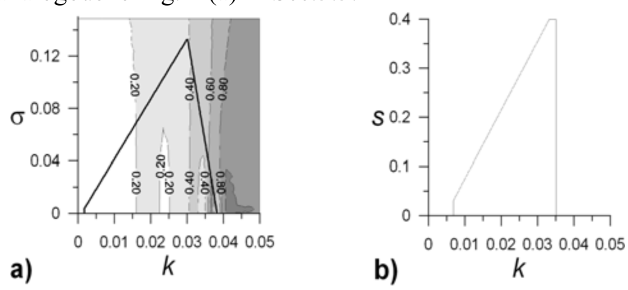

FIG. 3. Regions of the coupling estimators applicability on the plane "coupling – noise" for unidirectionally coupled van der Pol. a) Van der Pol oscillators with dynamical noise. b) Van der Pol oscillators with dynamical and observation noises

Then we consider the presence of observational Gaussian white noises in systems (the standard deviation *s*) that are added to the variables $x_{1,2}$. Dynamical noise level is fixed to be σ = 0.025. Range of the method efficiency is shown in Fig. 3 (b). Again, the range of the method applicability is not infinitesimally small but rather significant.

3.5. Application to electroencephalogram (EEG) data

We apply the technique to the analysis of two-channel intracranial human EEG with the temporal lobe epilepsy. The first channel corresponds to the left hippocampus, the second one to the left temporal neocortex. The time series are presented in Fig.4 (a). The seizure starts approximately at the 100th second and finishes approximately at the 220th second (the dashed lines). We have computed coupling characteristics in a running window of the length of 24 seconds. The results are presented in Fig. 4 (b), where gray tail denotes 95% confidence bands for δ. One can observe a long interval (30 second length) of significant predominant coupling direction neocortex $\rightarrow$ hippocampus before the seizure. So it is possible to perform epileptic focus localization.

This is only the first attempt and the results should not be overestimated, being rather an illustration of the way how to apply the method in practice and what kind of information one can expect from it.

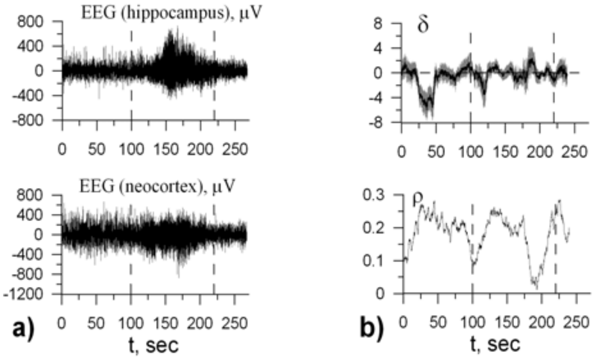

FIG. 4. a) EEG recordings from hippocampus (top) and neocortex (bottom). c) Coupling directionality index and mean phase coherence. Negative delta values correspond to coupling direction neocortex $\rightarrow$ hippocampus (approximately from the 20th second to the 50th second)

4. Conclusion

Numerical experiments demonstrate that the estimators of coupling between oscillatory systems based on phase dynamics modeling are sufficiently widely applicable. Although they are derived under the strict assumption of linear uncoupled oscillators and independent sources of Gaussian white noise, they are valid for various dynamical noise properties including the case of common noise and finite (not negligibly small) strengths of nonlinearity, coupling, and observational noise.

The modified EMA analyzed here is the extension of the EMA to short time series so that it seems to be a very powerful method and deserves special attention. Based on considering several exemplary oscillators, we formulated empiric conditions for applicability of the corresponding coupling estimators. We confirm the potential for the application of the estimators in practice to analyze real-world complex systems. In particular, our first attempt to apply them for epileptic focus localization from multichannel intracranial EEG recordings illustrated in the present paper looks promising.

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