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Resonance with Random Walks: From Sticks to Strings

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Abstract—Issues of resonance that appear in non-standard random walk models are discussed. The first walk is called repulsive delayed random walk, which is described in the context of a stick balancing experiment. The second one is called sticky random walk, which is introduced to model string entanglement. Peculiar resonant effects with respect to these random walks are presented.

1. Introduction

A combination of non-linear dynamics and noise gives rise to the phenomena called stochastic resonance, which has been investigated actively [1, 2, 3, 4, 5]. The phenomena has been claimed to appear in a wide variety of things, such as climate change and neural information processing. The main theme of this paper is this type of phenomena in the context of non-standard random walks that we have proposed: repulsive [6] and sticky. The former random walk was mainly derived from a stick balancing experiment[7, 8, 9], while the latter tries to model string entanglement. With both random walks, we observed rather unexpected phenomena that can be viewed as resonance. In the following, we describe each model and its associated behavior.

2. Repulsive Delayed Random Walk

2.1. Model

Delayed random walks have been proposed and studied as one approach to investigate systems with noise and delay[10, 11, 12]. This is a random walk whose transition probability depends on its position at a fixed interval in the past. The focus has been placed on a model that has an attractive bias to a single point. This stable case has been applied to such processes as posture controls[13]. Analytically, the attractive delayed random walk model has shown such behavior as an oscillatory correlation function with increasing delay.

However, such a model is not suitable to model an unstable situation, like balancing a stick in an experiment. Instead, a delayed random walk that has a repulsive point is used. We can consider many different possibilities, but here we test one-dimensional discrete time and step random walk with the origin as a repulsive point. Mathematically, we can define our model as follows. Let the position of the random walker at time step t be given by $X(t)$ and the fixed point be set at the origin, $X = 0$. The delayed random walk is defined by the following conditional probabilities.

$$\begin{aligned} P(X(t+1) = X(t) + 1 | X(t-\tau) > 0) &= p \\ P(X(t+1) = X(t) + 1 | X(t-\tau) = 0) &= \frac{1}{2} \\ P(X(t+1) = X(t) + 1 | X(t-\tau) < 0) &= 1 - p, \end{aligned}$$

where $0 < p < 1$ and τ are the delay. The walker refers to its position in the past with delay to decide on the bias of the next step. The attractive delayed model is a case of $p < 0.5$, where the origin becomes attractive with no delay, $\tau = 0$. However, $p > 0.5$ gives the repulsive case that will be discussed in the rest of this paper. Though this appears to be a small change from the attractive delayed case, we actually observed very different behavior from it. Most of all, when the walker escapes from the origin, we no longer have a stationary probability distribution. This makes an analytical treatment of this repulsive model more difficult compared with the attractive delayed case, particularly with a non-zero delay. Our investigation in this paper was done using computer simulations. The most notable feature of this model is that we can find an optimal combination of the bias p and τ where the random walker can be kept around the origin for the longest duration.

2.2. Simulation Results

As in the stick balance experiment, one of the main interests is how long the walker can be kept around the repulsive fixed point. We investigated this by focusing on the average first passage time L to reach a certain position (a limit point $\pm X^*$) away from the origin. In other words, we measured the average time for the walker, starting from the origin to reach the limit point for the first time, as we changed parameters in the model. The longer average of the first passage time indicates slower diffusion, which corresponds to a situation of longer stick balancing.

Such analytical results have yet to be obtained for the non-zero delay using a computer simulation. We used an ensemble of 10,000 walkers. The initial condition was set so that the walker performed a normal random walk with no bias $p = 0.5$ for a duration of $t = (-\tau, 0)$. The walker's position at $t = 0$ was set as the origin $X = 0$. The limit point was set at $\pm X^*$. We measured the number of steps for each walker to go from the origin to $\pm X^*$ and averaged them. We performed computer simulations for various bias p and delay τ. The representative results are given in Figure 1, where we have plotted the normalized first passage time $L_n \equiv \frac{\langle L(\tau)\rangle}{\langle L(\tau=0)\rangle}$ against the normalized delay $\tau_n \equiv \tau \frac{p-q}{X^*}, (q = 1 - p)$.

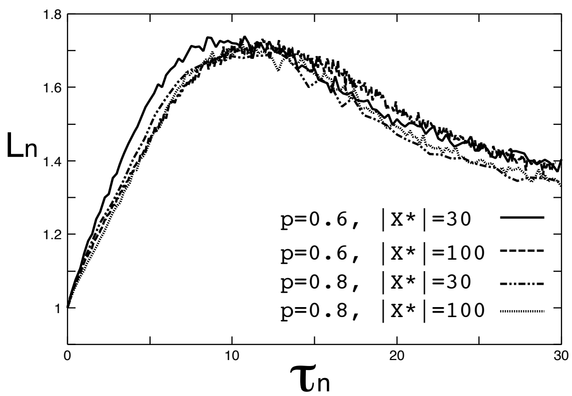

Figure 1: Normalized average first passage time L_n as we changed normalized delay τ_n. The parameter sets (p, X^*) plotted are $(0.6, 30)$, $(0.6, 100)$, $(0.8, 30)$, and $(0.8, 100)$

We can see that the graph goes through a maximum with an optimal balance of τ given p. The peak height is approximately 1.7 times the average first passage time of the zero delay case. This phenomenon can be viewed as a resonance between noise (bias) and delay. Tuning them can help to keep the walker balanced near the origin.

2.3. Delayed Stochastic Control

These theoretical results imply that systems can reach a better balancing performance if an appropriate amount of fluctuation is added given the feedback or reaction delay. We have termed this type of control, which is different from standard feedback or predictive ones, as delayed stochastic control. We performed the following experiment to gain some insight into the existence or utilization of this control scheme. We asked the subjects to sit on a chair and balance a stick, as in the previous stick balancing experiment. But, this time, the subjects were allowed to move their bodies, not just their arms, as they tried to balance the stick. One way to do this is to hold an object with the other hand and move it (Figure 2). Another way is to move their legs. We measured the time for which they could keep the sticks balanced, and compared it with the normal non-movement situations. Out of the six subjects we tested, three subjects showed notable improvement in balancing by reaching their own optimal level of movement (Figure 3).

Figure 2: Picture of a subject balancing a stick on one hand while moving an object in the other.

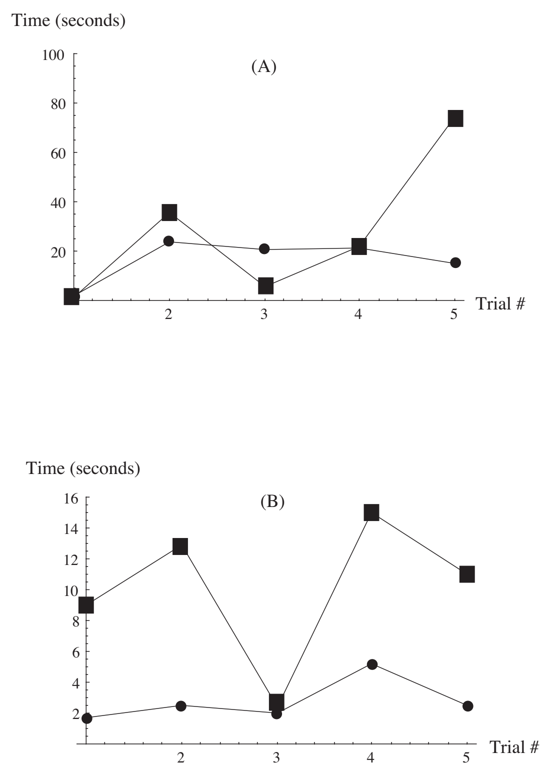

Figure 3: (A) Example of improvement on balancing tasks with (square) and without (dot) moving an object. The subject was given 5 trials without previous practice. By the 5th trial, the improvement was significant. (B) Another subject practiced for a few hours. Here, again improvement with moving the object was evident.

Some practice was needed for these subjects to reach this better performance. We believe that the subjects were tuning the appropriate level of fluctuation given their reaction times and prediction accuracy. Even though more thorough data needs to be collected, these results may be one supporting example of delayed stochastic control.

3. Sticky Random Walk

3.1. Model

Entangled strings is something we commonly observe. For example, wires for electrical appliances or communication network cords sometimes require us to disentangle them. We describe here a concept of sticky random walk we used to gain some insight into this phenomenon. The model is simple. The strings are represented by the trajectory of a random walker. This random walker leaves sticks or marks at certain time intervals. Therefore, a string is represented by this trajectory with these marks on it. By sending out multiple sticky random walkers, we obtained multiple sticky strings. Furthermore, a string is considered as entangled with another when these marks overlap at the same site in space, and not when they are simply crossed. Thus, the string is considered more sticky when there are more marks on it. We tested a situation having multiple sticky strings in a bounded two-dimensional square grid by sending out sticky random walks in this space. These random walks are discrete time, discrete space walks moving one step to its neighboring grid points. They are bounded by the edge of the square grid. We then pick one string randomly and count the number of strings either directly or indirectly entangled to that string. Indirect entanglement indicates that two strings are entangled through others, i.e., two strings can reach each other by following the chain of directly entangled strings. We performed simulation experiments with various conditions on the number of strings, the number of marks on each string, the length of each string, and the size of the two-dimensional square grid. In particular, we asked the question, if we compare the situation of having more strings with fewer marks and that of having fewer strings and more marks, while keeping the total number of marks in the space constant, which situation gives rise to more entanglement?

3.2. Simulation Results

We kept the total number of sticky marks R and the length of each string L as fixed, and we varied the number of strings S and marks on each string M so that $R = M \times S$. The number of entangled strings was measured both in numbers E and in ratio $e = \frac{E}{S}$. Each part of the data is an average over 100 trials, with various space for N by N square grid. The representative results are shown in Figure 4. We found that an optimal combination of S and M exists. It is given as the highest peak in these graphs. This means that these strings are most entangled when the level of stickiness and the number of strings are optimally tuned. Even more unexpectedly, this optimal combination is independent of the space size N for the ratio e. When N is sufficiently large, it is also independent with respect to E as well. Though it differs from the standard form of stochastic resonance, randomness in the motion of the walkers plays a role in bringing about this resonant behavior. Whether or not this behavior can happen in a real situation requires experimental tests.

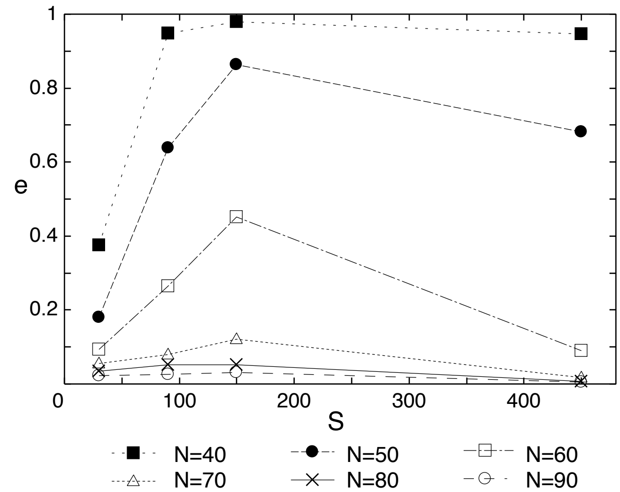

Figure 4: Average ratio, e, of entangled strings e as number of marks on each string is changed. The length of each string is set at $L = 60$, and the total number of marks is set at $R = 1800$. Each line corresponds to a square lattice size N.

4. Discussion

We discussed two non-standard models of stochastic resonance. As a related subject, a binary bit model that shows resonance with noise and delay are proposed and studied [14, 15]. This phenomena was observed in an experiment with solid state laser [16].

Our investigations here with respect to the resonance with random walks are still in the beginning stages. However, they already produced quite unexpected results. Further analysis as well as application with real systems could lead to some additional interesting insights.

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