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Reflective Boolean Network Tomography for Node Failure Detection

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Abstract—Boolean network tomography is a network monitoring scheme to identify failure nodes from end-to-end measurements. We propose a Reflective Boolean Network Tomography scheme for wireless mesh networks. While multiple measurement nodes are deployed to transfer probe packets in existing Boolean network tomography schemes, only one measurement node is used in the proposed scheme. The measurement node establishes round-trip paths to transfer probe packets sequentially according to candidate failure nodes. With simulation experiments, we evaluate the performance of the proposed scheme.

1. Introduction

Network tomography estimates network internal characteristics such as packet loss rates and delays from end-to-end measurements [3]. In this paper, we consider *Boolean network tomography* to identify failure nodes. In Boolean network tomography, the relationship between end-to-end measurements and node states are represented with a system of Boolean equations and failure nodes are identified by solving the equations.

Mukamoto et al. [5] propose an adaptive Boolean network tomography scheme for wireless mesh networks. In this scheme, several mobile measurement nodes, which are referred to as measurement nodes hereafter, are connected to nodes in a wireless mesh network. Measurement paths between measurement nodes are sequentially established according to candidate failure nodes, and probe packets are transmitted on the paths. Although this scheme can reduce the number of measurement paths comparing to non-adaptive network tomography schemes, measurement nodes need to cooperate with each other in order to establish paths and collect measurements.

In [6], in order to eliminate the cooperation among measurement nodes, a reflective network tomography scheme for estimating delays is proposed. In this scheme, only one measurement node is deployed, and measurement round-trip paths are established based on the mutual coherence [4] of the routing matrix. Probe packets are then transmitted on the paths and link delays are estimated by means of compressed sensing. In this paper, based on this idea, we consider a *reflective Boolean network tomography* scheme. The proposed scheme has a *coarse-to-fine* structure as the network tomography scheme proposed in [5]. The proposed scheme first establishes several measurement round-trip paths to roughly estimate a set of failure nodes, which is referred to as a *candidate set* of failure nodes. The candidate set is refined by iteratively adding measurement round-trip paths, and failure nodes are identified from the finally obtained candidate set.

The remainder of the paper is organized as follows. Section 2 describes the network model and the basic idea of the proposed scheme. Section 3 explains the measurement path construction of the proposed scheme. In section 4, we evaluate the performance of the proposed scheme. Finally, we conclude the paper in section 5.

2. Boolean Network Tomography

2.1. Network Model

Let undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ denote a wireless mesh network, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denote the sets of nodes and links, respectively, and $N = |\mathcal{V}|$ denotes the number of nodes. We define node state vector $\boldsymbol{x} = (x_1\ x_2\ \cdots\ x_N)$, where $x_n \in \{0, 1\}$ $(n = 1, 2, \ldots, N)$ denotes a state of node $v_n \in \mathcal{V}$. There are two types of nodes: *normal nodes* and *failure nodes*. We define the set of failure nodes as $\mathcal{V}_\mathrm{F} \subset \mathcal{V}$. x_n is then given by

$$x_n = \begin{cases} 0 & \text{if } v_n \in \mathcal{V} \setminus \mathcal{V}_\mathrm{F} \\ 1 & \text{if } v_n \in \mathcal{V}_\mathrm{F} \end{cases} .$$

We assume that normal nodes successfully transfer packets with probability 1 and failure nodes drop them with probability 1.

We select a node in the network as the measurement node. The measurement node establishes round-trip paths and transmits probe packets on them. We hereafter refer to measurement round-trip paths and probe packets as measurement paths and measurement packets, respectively. We define $\mathcal{W} = \{w_1, w_2, \ldots, w_M\}$ as a set of measurement paths, where each path is defined as a set of nodes on the path and $M = |\mathcal{W}|$ denotes the number of the paths. We also define measurement vector $\boldsymbol{y} = (y_1\ y_2\ \ldots y_M)$, where $y_m = 0$ $(m \in \{1, 2, \ldots, M\})$ if measurement node v_0 successfully receives a packet transmitted on $w_m \in \mathcal{W}$, and $y_m = 1$ otherwise. From the assumption of packet loss probability, a packet transmitted on measurement path w_m is lost if at least one failure node is included on w_m, while it is successfully transferred if there are no failure nodes on w_m.

2.2. Failure Node Detection

Boolean network tomography is a problem to estimate state vector $\boldsymbol{x}$ from measurement vector $\boldsymbol{y}$. The proposed scheme estimates $\boldsymbol{x}$ by means of CBP (Combinatorial Basis Pursuit)[2] algorithm as the adaptive Boolean network tomography scheme in [5].

We define $\mathcal{V}_{\mathrm{C}}$, and $\mathcal{V}_{\mathrm{I}}$ as a *candidate set* and an *identified set* of failure nodes, respectively. $\mathcal{V}_{\mathrm{C}}$ contains nodes which have not been determined to be failure nodes or not, while $\mathcal{V}_{\mathrm{I}}$ contains nodes determined to be failure nodes. We also define $\mathcal{V}_{\mathrm{S}} = \mathcal{V} \setminus \{\mathcal{V}_{\mathrm{C}} \cup \mathcal{V}_{\mathrm{I}}\}$, which contains nodes determined to be normal nodes. We assume that the measurement node $v_0 \in \mathcal{V}$ is a normal node.

Suppose that a packet is transmitted on a measurement path w. Nodes on w are determined to be normal nodes if the following condition (a) is satisfied and determined to be failure nodes if condition (b) is satisfied.

(a) If the packet is successfully transferred on w, all nodes on w are determined to be normal nodes.

(b) If the packet is lost on w and all the nodes on w but $v \in w$ have been determined to be normal nodes, v is then determined to be a failure node.

The procedure of failure node detection in the proposed scheme consists of *Initial Measurement Phase* and *Additional Measurement Phase*:

Initial Measurement Phase Initially, all the nodes but v_0 are set to be elements of $\mathcal{V}_{\mathrm{C}}$, that is, $\mathcal{V} = \mathcal{V} \setminus \{v_0\}$ and $\mathcal{V}_{\mathrm{C}} = \mathcal{V}$. v_0 constructs a set $\mathcal{W}_{\mathrm{init}}$ of initial measurement paths and transmits packets on the paths. After collecting measurements, nodes satisfying condition (a) are added to $\mathcal{V}_{\mathrm{S}}$ and nodes satisfying condition (b) are added to $\mathcal{V}_{\mathrm{I}}$, and $\mathcal{V}_{\mathrm{C}}$ is updated to $\mathcal{V}_{\mathrm{C}} = \mathcal{V} \setminus \{\mathcal{V}_{\mathrm{S}} \cup \mathcal{V}_{\mathrm{I}}\}$. In section 3.1, we describe the initial measurement path construction scheme in detail.

Additional Measurement Phase After the initial measurement phase, an additional measurement path is computed according to $\mathcal{V}_{\mathrm{C}}$. $\mathcal{V}_{\mathrm{C}}$, $\mathcal{V}_{\mathrm{I}}$, and $\mathcal{V}_{\mathrm{S}}$ are then updated from measurements on the additional paths. Additional paths are iteratively computed until $\mathcal{V}_{\mathrm{C}}$ is not updated. In section 3.2, we describe the additional measurement path construction scheme.

3. Measurement Path Construction

There are two types of round-trip paths: *loopy paths* and *folded paths* [6]. A loopy path is a round-trip path that any nodes on the path do not appear more than once except for the measurement node, while a folded path is a path that any nodes on the path appear twice except for a node, which is referred to as a *return node*. In this paper, we consider that all measurement paths are loopy paths.

3.1. Path Construction in Initial Measurement Phase

In the initial measurement phase, several measurement paths are constructed so that all nodes are included on at least one measurement path. As shown in Fig. 1, each

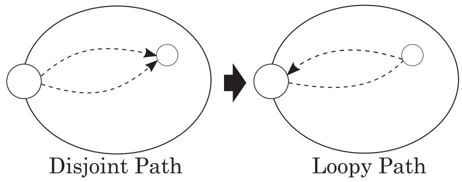


Figure 1: Constructing a loopy path from disjoint paths.

measurement path is construction with two vertex-disjoint paths from the measurement node to a return node. The disjoint paths are computed with the vertex-disjoint shortest pair algorithm [1], and these paths are connected by reversing the direction of one disjoint path.

Let $\mathcal{D}(v_0, v)$ denote a pair of disjoint paths between measurement node $v_0 \in \mathcal{V}$ and return node $v \in \mathcal{V} \setminus \{v_0\}$, and $LP(\mathcal{D}(v_0, v))$ denote a round-trip path constructed from the disjoint paths. The procedure for constructing initial measurement paths is as follows.

1. Compute the set $\mathcal{D}_{\mathrm{all}} := \{\mathcal{D}(v_0, v) \mid \forall v \in \mathcal{V} \setminus \{v_0\}\}$ of shortest pairs of disjoint paths.
2. Set $\mathcal{T} := \mathcal{V}$ and $\mathcal{W}_{\mathrm{init}} = \emptyset$.
3. $u := \arg\max_{v \in \mathcal{V} \setminus \{v_0\}} |\mathcal{T} \cap \mathcal{D}(v_0, v)|$.
4. Set $\mathcal{W}_{\mathrm{init}} := \mathcal{W}_{\mathrm{init}} \cup \{LP(\mathcal{D}(v_0, u))\}$ and $\mathcal{T} := \mathcal{T} \setminus \mathcal{D}(v_0, u)$.
5. Go to step 3 until $\mathcal{T} = \emptyset$.

$\mathcal{T}$ denotes the set of nodes which are not included on any paths. Step 3 selects return node u so that $LP(\mathcal{D}(v_0, u))$ includes the maximum number of nodes in $\mathcal{T}$.

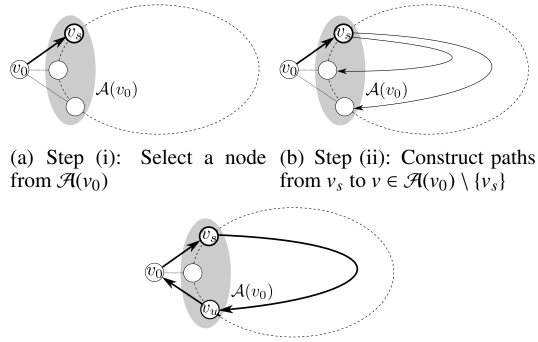
3.2. Path Construction in Additional Measurement Phase

3.2.1. Procedure for Additional Path Construction

In the additional measurement phase, an additional measurement path is computed with two types of costs: *node cost* $NC(v_n)$ and *path cost* $PC(v_0, v_s, v_t)$. We re-define $\mathcal{W}$ as the set of measurement paths that have already constructed. After the initial measurement phase, $\mathcal{W}$ is set to $\mathcal{W} = \mathcal{W}_{\mathrm{init}}$. We also define a set $\mathcal{W}(v_n) \subset \mathcal{W}$ of round-trip paths including $v_n \in \mathcal{V}$. Node cost $NC(v_n)$ for node $v_n \in \mathcal{V}$ is then given by

$$NC(v_n) = \begin{cases} 0 & \text{if } v_n \in \mathcal{V} \setminus \mathcal{V}_{\mathrm{C}} \\ 1 & \text{if } v_n \in \mathcal{V}_{\mathrm{C}} \text{ and} \\ & \displaystyle\sum_{w_p \in \mathcal{W}(v_n)} y_p b_p = 0 \\ 1 + \log(\displaystyle\sum_{p \in \mathcal{W}(v_n)} y_p b_p) & \text{otherwise} \end{cases},$$

where $b_p = 0$ if $w_p \in \mathcal{W}(v_n)$ includes at least one node in $\mathcal{V}_{\mathrm{I}}$, $b_p = 1$ otherwise. We define $\mathcal{P}(v_0, v_s, v_t)$ is a path from v_0 to v_t via node v_s. Path cost $PC(v_0, v_s, v_t)$ is given by the sum of node costs on path $\mathcal{P}(v_0, v_s, v_t)$. Let $\mathcal{A}(v)$ ($v \in \mathcal{V}$) denote the set of neighbor nodes of v. If we select v_s and v_t ($v_s \neq v_t$) from $\mathcal{A}(v_0)$, $PC(v_0, v_s, v_t)$ corresponds to the sum of node costs on a measurement round-trip path.



(a) Step (i): Select a node from $\mathcal{A}(v_0)$

(b) Step (ii): Construct paths from v_s to $v \in \mathcal{A}(v_0) \setminus \{v_s\}$

(c) Step (iii): Select a node from $\mathcal{A}(v_0)\setminus\{v_s\}$ and construct a round-trip path

Figure 2: Additional Measurement Path Construction.

Fig. 2 shows the procedure for selecting an additional measurement path. We define $prev(v)$ for $v \in \mathcal{V}$ as the previous node along the current path from v_0 to v. We first choose node $v_s \in \mathcal{A}(v_0)$ randomly (**Step (i)**). Next, we compute path costs $PC(v_0, v_s, v)$ and $prev(v)$ for $\forall v \in \mathcal{V} \setminus \{v_0, v_s\}$ (**Step (ii)**). Finally, node $v_u \in \mathcal{A}(v_0) \setminus \{v_s\}$ to satisfy the following condition is selected (**Step (iii)**).

$$v_u = \underset{\substack{v_t \in \mathcal{A}(v_0)\setminus\{v_s\} \\ prev(v_t) \neq v_0}}{\arg\min} |PC(v_0, v_s, v_t) - d_{\rm th}|$$

Let $v_{i_j} \in \mathcal{V}_{\rm C}$ ($i_j \in \{1, 2, \ldots, N\}, j = 1, 2 \ldots, |\mathcal{V}_{\rm C}|$) denote nodes sorted in the increasing order of node costs, i.e., $NC(v_{i_1}) \leq NC(v_{i_2}) \leq \cdots \leq NC(v_{i_{|\mathcal{V}_{\rm C}|}})$. $d_{\rm th}$ with parameter a ($a > 0$) is defined as

$$d_{\rm th} = \sum_{j=1}^{|\mathcal{V}_{\rm C}|/a+1} NC(v_{i_j}).$$

We obtain the additional measurement path from v_0 via v_s and v_u by recursively following $prev(v_u)$. Steps (i) and (ii) are described in sections 3.2.2 in more detail.

The purpose of the additional measurement phase is to reduce the size of candidate set $\mathcal{V}_{\rm C}$. If a packet is transferred successfully on an additional measurement path including nodes in $\mathcal{V}_{\rm C}$, the size of $\mathcal{V}_{\rm C}$ is reduced. A measurement path with a lower path cost indicates that only a few candidate nodes are included on the path, which is not effective to reduce the size of $\mathcal{V}_{\rm C}$. On the other hand, a measurement path with a higher path cost indicates that failure nodes may be included on the path, which means that a packet is lost with higher probability. In order to select an appropriate measurement path, therefore, we set threshold $d_{\rm th}$ for round-trip paths, and select a measurement path with path cost nearest to $d_{\rm th}$.

3.2.2. Procedure for Computing Path Costs

Step (i) Choose a node $v_s \in \mathcal{A}(v_0)$ randomly, and set $prev(v_s) := v_0$, $\mathcal{N}_{\rm V} := \{v_s\}$, $\mathcal{N}_{\rm C} := \mathcal{A}(v_s) \setminus \{v_0\}$, $PC(v_0, v_s, v_a) := NC(v_s) + NC(v_a)$ and $prev(v_a) := v_s$ for $\forall v_a \in \mathcal{N}_C$, and $PC(v_0, v_s, v) = -1$ for $v \in \mathcal{V}\setminus(\mathcal{N}_C \cup \{v_0, v_s\})$.

Step (ii) Iterate steps (1) and (2) until $\mathcal{N}_{\rm C} = \emptyset$.

1. Select a node $v_b \in \mathcal{N}_{\rm C}$ according to the procedure described in section 3.2.3, and set $\mathcal{N}_{\rm C} := \mathcal{N}_{\rm C} \setminus \{v_b\}$ and $\mathcal{N}_{\rm V} := \mathcal{N}_{\rm V} \cup \{v_b\}$.
2. Set $\mathcal{N}_{\rm C} := \mathcal{N}_{\rm C} \cup (\mathcal{A}(v_b) \setminus \mathcal{N}_{\rm V})$. For $\forall v_c \in \mathcal{N}_{\rm C}$, if $PC(v_0, v_s, v_c) < 0$ or $|PC(v_0, v_s, v_c) - d_{\rm th}| > |PC(v_0, v_s, v_b) + NC(v_c) - d_{\rm th}|$, path costs $PC(v_0, v_s, v_c)$ and $prev(v_c)$ are updated to

$$\begin{aligned} PC(v_0, v_s, v_c) &:= PC(v_0, v_s, v_b) + NC(v_c), \\ prev(v_c) &:= v_b. \end{aligned}$$

3.2.3. Node Selection

$\mathcal{N}_{\rm C}$ is divided into subsets $\mathcal{N}_{\rm C}^{\rm (s)} \subset \mathcal{V}_{\rm S}$ and $\mathcal{N}_{\rm C}^{\rm (c)} \subset \mathcal{V}_{\rm C}$. A node v_b is selected according to the following procedure.

1. If $\mathcal{N}_{\rm C}^{\rm (c)} = \emptyset$ (i.e., all nodes in $\mathcal{N}_{\rm C}$ are normal nodes): If $PC(v_0, v_s, v) = 0$ for $\forall v \in \mathcal{N}_{\rm C}$, a node is chosen randomly. Otherwise, the node with the minimum path cost is selected from nodes with positive node costs.
2. If $\mathcal{N}_{\rm C}^{\rm (s)} = \emptyset$ (i.e., all nodes in $\mathcal{N}_{\rm C}$ are candidate nodes), the node with the smallest node cost is selected. If there are several nodes with the smallest cost, a node is chosen randomly from them.
3. If $\mathcal{N}_{\rm C}^{\rm (s)} \neq \emptyset$ and $\mathcal{N}_{\rm C}^{\rm (c)} \neq \emptyset$: Two nodes v_α and v_β are selected according to steps 1 and 2, respectively. v_b is set to $v_b = v_\alpha$ with probability of $1 - p_{\rm path}(PC(v_0, v_s, v_\beta))p_{\rm node}(NC(v_\beta))$ and $v_b = v_\beta$ with probability of $p_{\rm path}(PC(v_0, v_s, v_\beta))p_{\rm node}(NC(v_\beta))$. For $x, y \geq 0$, $p_{\rm path}(x)$ and $p_{\rm node}(y)$ are given by

$$p_{\rm path}(x) = \begin{cases} 1 & (x < d_{\rm th} \text{ or } d_{\rm th} = d_{\rm sum}) \\ \dfrac{1}{2}\dfrac{d_{\rm sum} - x}{d_{\rm sum} - d_{\rm th}} & (x \geq d_{\rm th} \text{ and } d_{\rm th} \neq d_{\rm sum}) \end{cases},$$

$$p_{\rm node}(y) = \begin{cases} 1 & (d_1 = d_{|\mathcal{V}_{\rm C}|}) \\ \dfrac{d_{|\mathcal{V}_{\rm C}|} - y}{d_{|\mathcal{V}_{\rm C}|} - d_1} & (d_1 \neq d_{|\mathcal{V}_{\rm C}|}) \end{cases},$$

where $d_{\rm sum} = \sum_{j \in \mathcal{V}_{\rm C}} NC(v_j)$, $d_1 = NC(v_{i_1})$, and $d_{|\mathcal{V}_{\rm C}|} = NC(v_{i_{|\mathcal{V}_{\rm C}|}})$.

4. Performance Evaluation

4.1. Simulation Environment

Fig. 3 shows the network topology for the simulation experiments, where there are $N = 27$ nodes and $v_1 \in \mathcal{V}$ is set to be the measurement node v_0. We set parameter a for $d_{\rm th}$ to $a = 2$. We evaluate the proposed scheme with two metrics: the numbers of false positive errors and measurement paths. The number of false positive errors are defined as the number of normal nodes in the candidate set that is finally obtained and the number of measurement paths is given by the sum of the numbers of initial and additional measurement paths. We select one node in $\mathcal{V} \setminus \{v_0\}$ and set it to be the failure node. For each set of a failure node and normal nodes, we conduct 100 simulation experiments.

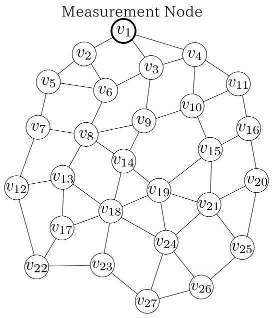


Figure 3: Network topology for simulation experiments. v_1 is set to be measurement node v_0.

4.2. Simulation Results

Fig. 4 shows the performance of the proposed scheme and the Boolean network tomography scheme proposed in [5]. In the figure, "Proposal" and "Conventional" correspond to the performance of the proposed scheme and the conventional scheme in [5], respectively.

Fig. 4(a) shows the average number of false positive errors for each failure node. We observe that the average number of false positive errors is less than 1 in most cases. In the case of node failure at v_2 or v_4, however, many false positive errors occur in the proposed scheme. The reason is that these nodes are neighbors of the measurement node v_1. Namely, because these nodes are included in many round-trip paths, it is difficult to refine the candidate set even with many additional paths. These positive errors, however, can be identified by establishing additional measurement paths from the measurement node to these nodes.

Fig. 4(b) shows the average number of measurement paths. We observe that the proposed scheme can identify with about 10 measurement paths, which is much less than the number N of nodes, and the performance of the proposed scheme is comparable to that of the conventional scheme. In the cases of node failure at v_2 and v_3, however, many measurement paths are required because of the same reason for the false positive errors.

5. Conclusion

In this paper, we proposed a *Reflective Boolean Network Tomography* scheme for node failure detection in wireless mesh networks. In the proposed scheme, one node is set to be a measurement node and failure nodes are detected by sequentially establishing measurement paths according to the candidate set of failure nodes. In a future work, we will consider a selecting scheme of a measurement node.

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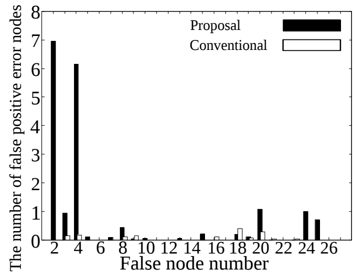
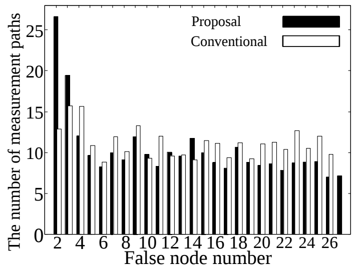


(a) Number of false positive errors.



(b) Number of measurement paths.

Figure 4: Simulation results. v_1 is set to be the measurement node in the proposed scheme and v_1 and v_{27} are set to be the measurement nodes in the conventional scheme.

References

[1] R. Bhandari, *Survivable Networks: Algorithms for Diverse Routing*, Springer, Jan. 1999.

[2] C. L. Chan, P. H. Che, S. Jaggi, and V. Saligrama, "Non-adaptive probabilistic group testing with noisy measurements: near-optimal bounds with efficient algorithms," *Proc. the 49th Annual Allerton Conference on Communication, Control, and Computing*, pp. 1832–1839, Sep. 2011.

[3] M. Coates, A. O. Hero Ⅲ, R. Nowak, and B. Yu, "Internet tomography," *IEEE Signal Processing Magazine*, vol. 19, no. 3, pp. 47–65, May 2002.

[4] Y. C. Eldar and G. Kutyniok, *Compressed Sensing: Theory and Applications*, Cambridge University Press, 2012.

[5] M. Mukamoto, T. Matsuda, S. Hara, K. Takizawa, F. Ono, and R. Miura, "Adaptive Boolean network tomography for link failure detection," *Proc. IFIP/IEEE International Symposium on Integrated Network Management (IM2015)*, pp. 646–651, May 2015.

[6] K. Nakanishi, S. Hara, T. Matsuda, K. Takizawa, F. Ono, and R. Miura, "Reflective network tomography based on compressed sensing," *Proc. the 6th International Conference on Ambient Systems, Networks and Technologies (ANT 2015)*, vol. 52, pp. 186–193, Jun. 2015.