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Coherent resonance in neuronal networks under external signal influence

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Abstract—We find that the regularity in the spiking behaviour of a neuronal network maximizes at a certain level of environment noise. This effect referred to as coherence resonance is demonstrated in a random complex network of Rulkov neurons. An external stimulus added to some of neurons excites them, and then activates other neurons in the network. The network coherence is also maximized at the certain stimulus amplitude, coupling strength, and the number of stimulated neurons. The coherence enhancement is characterized by the normalized standard deviation from the average inter-spike interval and by the signal-to-noise ratio calculated from power spectra of the excited neurons.

1. Introduction

Noise can lead to more order in the dynamics. To be mentioned here are the effects of noiseinduced order in chaotic dynamics [1], synchronization by external noise, and stochastic resonance [2, 3, 4]. Also, noise has been shown to play a stabilizing role in ensembles of coupled oscillators and maps [5]. Especially interesting is the phenomenon of stochastic resonance, which appears when a nonlinear system is simultaneously driven by noise and a periodic signal. At a certain noise amplitude the periodic response is maximal.

The interest in mathematical modeling of neuronal synchronization has significantly increased after neurobiological experiments with two electrically coupled neurons [6], where various synchronous states have been identified. In order to simulate cooperative neuron dynamics, numerous models based on either iterative maps of differential equations in various coupling configurations have been developed [6]. Depending on the coupling strength and synaptic delay time, coupled neurons generate spike sequences that are matching in their timings, or bursts either with lag or anticipation [7]. When three or more oscillators are accounted for, a large number of coupling configurations can be realized. In the theory of graphs or complex networks, these basic configurations are called network motifs.

We explore a simple neural model, the Rulkov map [8, 9]. Although this model is not explicitly inspired by physiological processes in the membrane, it is capable of generating extraordinary complexity and quite specific neural dynamics (silence, periodic spiking, and chaotic bursting), thus replicating to a great extent most of the experimentally observed regimes [6], including spike adaptation, routes from silence to bursting mediated by subthreshold oscillations, emergent bursting, phase and antiphase synchronization with chaos regularization [8], and complete and burst synchronization.

2. The investigation model

Each neuron-like Rulkov element is described by the following system of equations with synaptic coupling [9]:

$$x_{n+1} = f(x_n, x_{n-1}, y_n + \beta_n), \tag{1}$$

$$y_{n+1} = y_n - \mu(x_n + 1) + \mu\sigma + \mu\sigma_n + \mu A^{\xi}\xi_n, \tag{2}$$

where x is a fast variable associated with membrane potential, y is a slow variable which has some analogy with gating variables, the parameters α, σ and $0 < \mu \leq 1$ control individual dynamics of the system, ξ is a Gaussian noise with a zero mean and standard deviation that equals 1, A^{ξ} is noise amplitude. β_n and σ_n are related to external stimuli, f is a piecewise function defined as

$$f(x_n, x_{n-1}, y_n) = \begin{cases} \alpha/(1 - x_n) + y_n, & \text{if } x_n \leq 0 \\ \alpha + y_n, & \text{if } 0 < x_n < \alpha + y_n \text{ and } \\ & x_{n-1} \leq 0 \\ -1, & \text{if } x_n \geq \alpha + y_n \text{ or } \\ & x_{n-1} > 0 \end{cases} \tag{3}$$

It is constructed in a way to reproduce different regimes of neuron-like activity, such as spiking, bursting and silent regimes.

The parameters β_n and σ_n are defined as

$$\beta_n = \beta^e I_n^{ext} + \beta^{syn} I_n^{syn}, \tag{4}$$

$$\sigma_n = \sigma^e I_n^{ext} + \sigma^{syn} I_n^{syn}. \tag{5}$$

Coefficients β^e and σ^e are used to balance the effect of external current I_n^{ext}. β^{syn} and σ^{syn} are coefficients of synaptic coupling. I_n^{syn} is a synaptic current:

$$I_{n+1}^{syn} = \gamma I_n^{syn} - g_{syn} * \begin{cases} (x_n^{post} - x_{rp}), & \text{spike}^{pre}, \\ 0, & \text{otherwise}, \end{cases} \tag{6}$$

where g_{syn} is the strength of synaptic coupling, $g_{syn} \geq 0$. Indexes pre and $post$ correspond presynaptic and postsynaptic variables respectively. The first condition in (6) corresponds to the presynaptic impulse (spike) generation time moments and defined as $x_n^{pre} \geq \alpha + y_n^{pre} + \beta_n^{pre}$. Parameter γ is a relaxation time of the synapse, $0 \leq \gamma \leq 1$. It defines the part of synaptic current which preserve as in the next iteration. x_{rp} is a reversal potential that determines the type of the synapse: inhibitory or excitatory.

In our modeling we take values of the parameters $\alpha = 3.65$, $\sigma = 0.06$ and $\mu = 0.0005$ so that each neuron being autonomous demonstrates silent regime dynamics. Also we assume $\beta^e = 0.133$, $\sigma^e = 1.0$, $\beta^{syn} = 0.1$, $\sigma^{syn} = 0.5$ and $x_{rp} = 0.0$. Investigation system is a motif of N neurons coupled to each other with a random coupling strength g_{syn} and relaxation time γ. The values of them are randomly chosen from 0.0 to 0.1 and from 0.0 to 0.5 respectively. In the investigating system we apply an external stimulus to Na neurons. Stimulus is a current impulse of the following form: from the start it equals to 0, at the moment t_s when we apply it current starts equal to A. The values of variables are chosen so that without the external stimulus each neuron is in a silent regime but with starting the application of stimulus excited neurons start periodically generate spikes.

3. The analysis

From the system we take signals as time series of fast variable x from all neurons. Additionally we calculate signal averaging over all neurons and analyse them. In figure 1 we can see these signals for systems of 100 neurons for different number of excited neurons. On them we can see phenomenon of grouping. It consists in periodically spiking unexcited neurons so that we can see areas of time on time series (d, e, f) where all unexcited neurons spike and areas where they all are silent and these areas periodically follows one by one. We can notice that for small and big values of Na we don't see grouping.

We analyse influence of amplitudes of external stimulus and internal noise. In figures 2 and 3 we can see dependencies of time series of x from these parameters. Increasing the stimulus amplitude leads to increasing frequency of grouping and grouping durations and decreasing time range between them. Also we can see decreasing oscillation amplitude of average signal. Increasing noise amplitude in its turn leads to decline of grouping effect, signal starts be

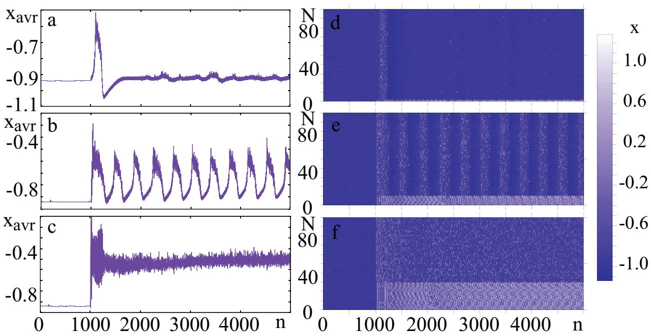

Figure 1: Time series of x variable averaging over all neurons (a), (b), (c) and time series of x variable for all 100 neurons where amplitude x is defined by color (d), (e), (f) for number of neurons being applying by external stimulus $Na = 1, 10$ and 30 respectively, $A^{\xi} = 0.1$, $A = 1.0$.

more noise-like. Also we can see oscillations in time area where external amplitude $A = 0$ so noise starts excite neurons.

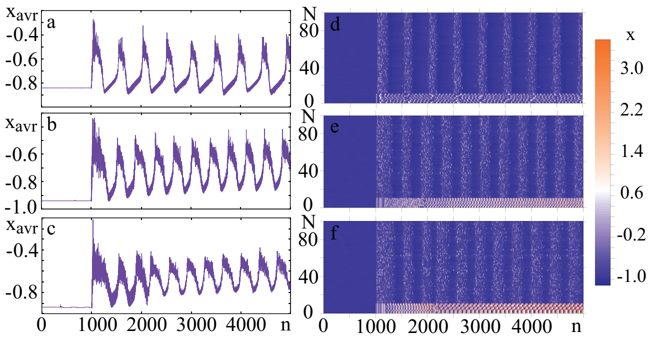

Figure 2: Time series of x variable averaging over all neurons (a), (b), (c) and time series of x variable for all 100 neurons where amplitude x is defined by color (d), (e), (f) for values of external stimulus amplitude $A = 0.5, 1.5$ and 2.5 respectively, $A^{\xi} = 0.1$. We apply the external effect at the first 10 neurons, $N = 100$.

For analyse phenomena of periodical grouping we calculate dependencies of signal-to-noise ration (SNR) from number of neurons in the system N, number of excited neurons Na, amplitude of external stimulus A and amplitude of internal noise A^{ξ}. SNR measured from power spectra of average signal in dB as an excess of main frequency amplitude over background noise [10].

In figure 4, a we can see dependence of SNR from number of neurons in the system when we excite 10 of them. At small values of N (<38) signal-to-noise ratio is small too but for increasing N from 38 leads to rapid increasing SNR from 5 to 30 and then it stays near of this level until $N = 140$ when SNR starts slowly decrease. So for $Na = 10$ we have optimal values of $N = 38 - 140$ at which SNR takes the highest value. In figure 4, b we can see depen-

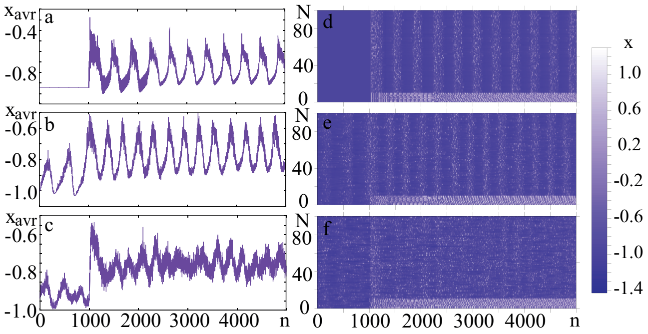

Figure 3: Time series of x variable averaging over all neurons (a), (b), (c) and time series of x variable for all 100 neurons where amplitude x is defined by color (d), (e), (f) for values of noise amplitude $A^{\xi} = 0.0, 1.0$ and 2.0 respectively, $A = 1.0$. We apply the external effect at the first 10 neurons, $N = 100$.

dence of SNR from number of neurons being applying by external stimulus for system of 100 neurons. We can say that optimal values of Na are from 4 to 18. For this area of Na SNR takes the highest values. Moving away from it signal-to-noise ratio value decreases to 0.

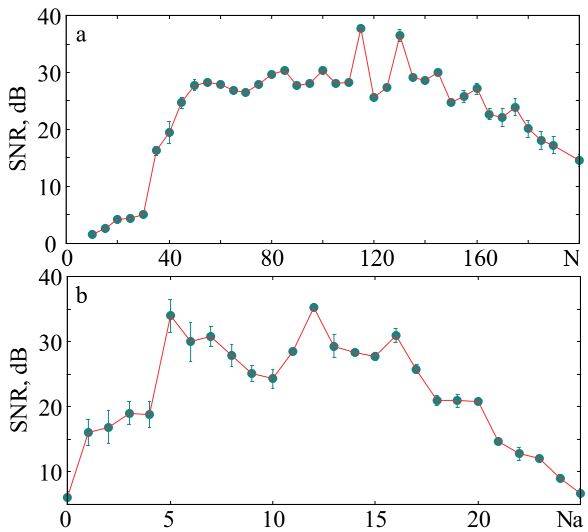

Figure 4: Signal-noise ratio (SNR) versus number of neurons in the system N (a) for $A^{\xi} = 0.1, A = 1.0, Na = 10$ and versus number of neurons being applying by external stimulus Na (b) for $A^{\xi} = 0.1, A = 1.0, N = 100$.

Figure 5, a shows signal-to-noise ratio dependence from external stimulus amplitude, on which we can see the phenomenon of coherent resonance when for a certain values of external stimulus amplitude ($A = 1.3 - 1.6$) SNR takes the maximum value. For $A > 1.6$ signal-to-noise ratio takes the same value. Decreasing external stimulus amplitude from 1.3 to 0 leads to decreasing SNR. In figure 5, b we can see influence of internal noise amplitude to signal-to-

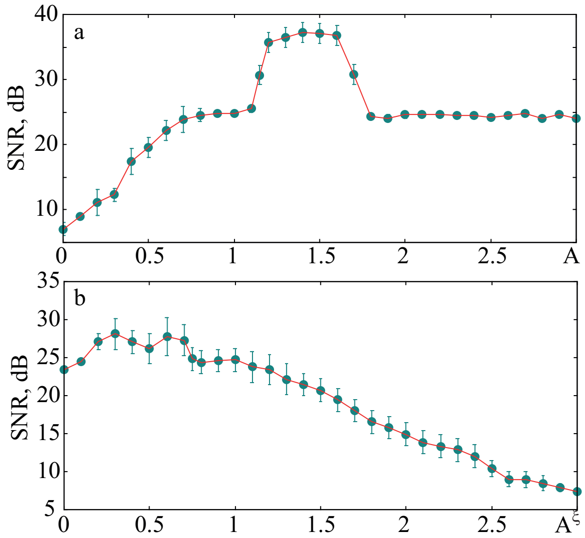

Figure 5: Signal-noise ratio (SNR) versus external stimulus amplitude A (a) for $A^{\xi} = 0.1, Na = 10, N = 100$ and versus noise amplitude A^{ξ} (b) for $A = 1.0, Na = 10, N = 100$.

noise ratio. For $A^{\xi} = 0.3$ SNR takes the maximum value and decreases to 4 with decreasing A^{ξ}.

To investigate the coherent resonance phenomenon we plotted the 2-dimensional diagram of SNR from amplitudes of external stimulus A and noise A^{ξ} (fig. 6) on which we can see the areas of coherent resonance where SNR values are high. These areas of parameters are colored by red. We can see two blue areas ($A < 0.2, A^{\xi} < 0.25$ and $0.5 < A^{\xi} < 1.0$) where signal-to-noise ratio is the lowest. There are 3 red areas for $0.8 < A < 1.7$ and $0.0 < A^{\xi} < 1.3$. Also we can see that main area of yellow and red colors is located for $A > 0.5$ and $A^{\xi} < 2.4$. And for $A > 1.7$ SNR value does not change for the constant noise amplitude.

We analyse the characteristics of system dynamics such as synchronization degree and coherence. Synchronization degree is defined as

$$S = \sqrt{\frac{1}{T - t_0} \int_{t=t_0}^{T} s(t)dt}, \tag{7}$$

$$s(t) = \frac{1}{N} \sum_{n=1}^{N} [x_n(t)]^2 - \left[\frac{1}{N} \sum_{n=1}^{N} x_n(t) \right]^2, \tag{8}$$

where T is the duration of the time series, t_0 is the duration of transients, N is the number of nodes ($n = 1, 2, ..., N$).

Coherence is defined as

$$H = \frac{1}{N} \sum_{n=1}^{N} h_n^2 - \left(\frac{1}{N} \sum_{n=1}^{N} h_n \right)^2, \tag{9}$$

$$h_n = \sqrt{\frac{1}{M - m_0 + 1} \sum_{m=m_0}^{M} R_m(n)}, \tag{10}$$

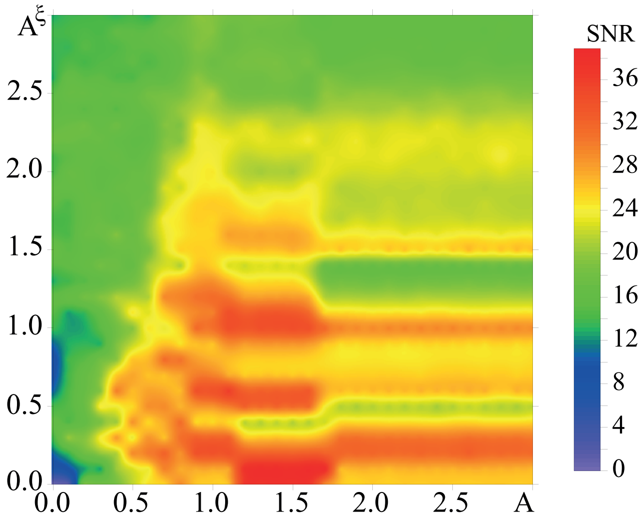

Figure 6: Two-parameter diagram of SNR from amplitudes of external stimulus A and noise A^ξ. SNR amplitude is defined by color.

where R_m is interspike interval (ISI) between m-th and $(m+1)$-th spike, M is the number of spikes ($m = 1, 2, ..., M$), m_0 is the number of transient spikes.

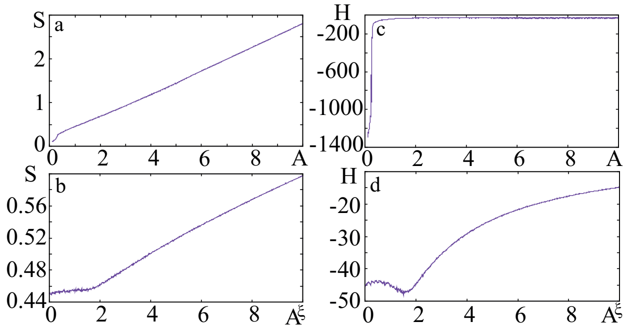

Figure 7: Synchronisation degree (a), (b) and coherence (c), (d) versus external stimulus amplitude and noise respectively. For figures (a) and (c) $A^\xi = 0.1$, for (b) and (d) $A = 1.0$.

On the figure 7 we can see the dependencies of synchronisation degree and coherence from external stimulus amplitude and noise. Increasing of A leads to linear increasing of synchronisation degree (fig. 7, a). Coherence at that time very fast increases and comes to saturation (fig. 7, c). Increasing of A^ξ leads to small linear increasing of synchronisation degree at the amplitude $A^\xi < 2$ and stronger linear increasing for $A^\xi > 2$ (fig. 7, b) with increasing the coherence (fig. 7, d).

4. Conclusion

The macroscopic signal from motif of Rulkov elements with random coupling between them and internal noise presence under external stimulus demonstrates phenomenon of grouping when all unexcited neurons start spiking periodically during the time interval. And at the averaging signal from all neurons we see periodically grouping. Changing such parameters as number of neurons in the system, number of excited neurons, amplitudes of external stimulus and internal noise we can see phenomenon of coherent resonance when at the certain values of these parameters signal-to-noise ratio takes the maximal values.

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