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Relation between Synchronous Rate and Small Variations on an Asymmetrical Coupled Chaotic System

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Abstract—In this study, the relation between synchronous rate and small variations on an asymmetrical coupled chaotic system is investigated. The system is realized by connecting one of two nodes in each chaotic subcircuit. The small variations are given to the subcircuit as an error of the oscillation frequency. In a computer calculation, an interesting phenomenon is observed. The phenomenon is that the synchronous rate of the whole system increases in spite of increasing parameter variations in the system.

1. Introduction

Many researchers have focused on engineering applications of chaos, for instance, chaotic communication systems, chaotic control, chaotic synchronization and so on. Especially, chaotic synchronization is very interesting phenomenon that chaotic subsystems synchronized in spite of different initial values [1]. Additionally, coupled systems of chaotic subsystems generate various kinds of complex higher-dimensional phenomena such as spatio-temporal chaotic phenomena, clustering phenomena, and so on. One of the most studied systems may be the coupled map lattice proposed by Kaneko[2]. The advantage of the coupled map lattice is its simplicity. However, many of nonlinear phenomena generated in nature would be not so simple. Therefore, it is important to investigate the complex phenomena observed in natural physical systems such as electric circuits systems [3]-[5].

In this study, synchronization phenomena in an asymmetrical coupled system of chaotic circuits are investigated. The system is coupled globally and the coupling elements are resistors. This system is the modified version of the symmetrical coupled chaotic system proposed by Miyamura et al.[6]. Each subcircuit has two coupling nodes and selecting one of two coupling nodes realizes the asymmetrical coupled system. The small variation is given to the subcircuit as an error of the oscillation frequency. Namely, the small variation is small parameter mismatches. In computer calculation, the influences of small variation to the system are investigated.

2. System Model

The proposed system is shown in Fig. 1. This system consists of chaotic circuits as shown in Fig. 2 and resistors as coupling elements. The chaotic circuit is a simple three-dimensional autonomous circuit proposed by Mori-group [7][8]. In order to realize the asymmetrical coupling system, we use one of the two coupling nodes. The circuit connected by the node v_{k1} is called as ”A-node circuit”, and the node v_{k2} is called as ”B-node circuit”. The numbers of A-node circuits and B-node circuits are denoted by m and n, respectively. Now, in order to carry out computer calcu-

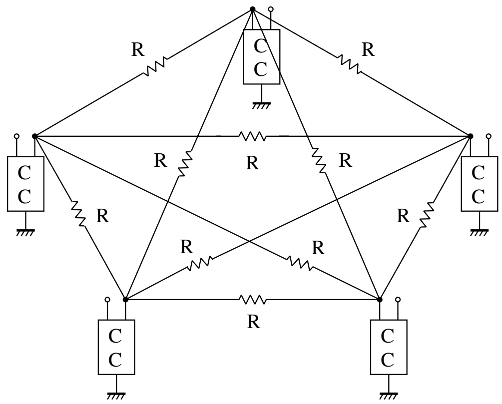

Figure 1: Asymmetrical coupled chaotic system.

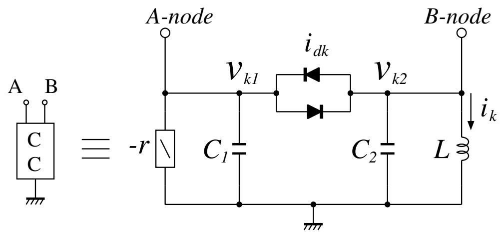

Figure 2: Chaotic subcircuit.

lation, normalized circuit equations are derived. Using the following variables and parameters,

$$x_k = \frac{v_{k1}}{a}, \qquad y_k = \frac{v_{k2}}{a}, \qquad z_k = \frac{1}{a}\sqrt{\frac{L}{C_2}}i_k,$$
$$t = \sqrt{LC_2}\tau, \qquad \text{“}\cdot\text{”} = \frac{d}{d\tau}, \qquad \alpha = \frac{C_2}{C_1}, \tag{1}$$
$$\beta = \frac{1}{r}\sqrt{\frac{L}{C_2}}, \quad \gamma = G_d\sqrt{\frac{L}{C_2}}, \quad \delta = \frac{1}{R}\sqrt{\frac{L}{C_2}},$$

the circuit equations of A-node and B-node circuits group are described as follows:

A-node circuit equations:

$$\begin{cases} \dot{x}_k = & \alpha\beta x_k - \alpha f(x_k - y_k) \\ & +\alpha\delta\{\sum_{i=n+1}^{m+n} x_i + \sum_{j=1}^{n} y_j - (m+n)x_k\}, \\ \dot{y}_k = & f(x_k - y_k) - z_k, \\ \dot{z}_k = & (1+p_k)y_k. \end{cases} \tag{2}$$

B-node circuit equations:

$$\begin{cases} \dot{x}_k &= \alpha\beta x_k - \alpha f(x_k - y_k), \\ \dot{y}_k &= \delta\{\sum_{i=n+1}^{m+n} x_i + \sum_{j=1}^{n} y_j \\ & \quad -(m+n)y_k\} + f(x_k - y_k) - z_k, \\ \dot{z}_k &= (1+q_k)y_k, \end{cases} \tag{3}$$

where $k = 1, 2, 3, \cdots, m+n$ i.e. , the number of A-node circuits is m, the number of B-node circuits is n, and p_k and q_k are introduced to give small variations of the oscillation frequencies. The nonlinear function $f(x_k - y_k)$ corresponding to the characteristics of the diodes is described as follows:

$$f(x_k - y_k) = \begin{cases} \gamma(x_k - y_k - 1) & (x_k - y_k > 1), \\ 0 & (|x_k - y_k| \leq 1), \\ \gamma(x_k - y_k + 1) & (x_k - y_k < -1). \end{cases} \tag{4}$$

Using these equations, computer calculations are carried out in the next section.

3. Computer calculation

3.1. The case of using five chaotic circuits

Figure 3 shows the computer calculated result of the asymmetrical coupled system in the case of $m = 3$ and $n = 2$. The voltages of A-node circuits are y_3, y_4 and, y_5. The voltages of B-node circuits are y_1 and y_2. The horizontal axis is time, and the vertical axes show the voltage differences between two chaotic circuits. Namely, in the case of synchronizing two chaotic circuits, the amplitude

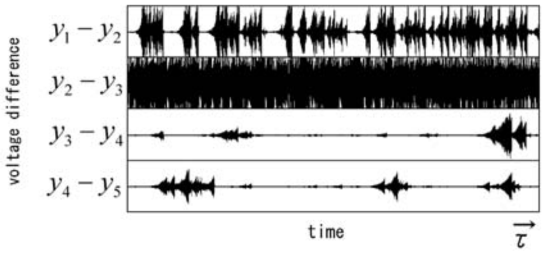

Figure 3: Computer calculated result of the asymmetrical coupled system in the case of $m = 3$ and $n = 2$. $\alpha = 0.4$, $\beta = 0.5$, $\gamma = 20$, $\delta = 0.0702$, and $p_k = q_k = 0.001(k-1)$.

becomes zero. The first graph ($y_1 - y_2$) shows the voltage difference between the two B-node circuits. We can see that synchronization and un-synchronized burst appear alternately in a random way. The second graph ($y_2 - y_3$) shows the voltage difference between an A-node circuit and a B-node circuit. We can see that these two are not synchronized at all. The third and fourth graphs ($y_3 - y_4$ and $y_4 - y_5$) show the voltage differences between the A-node circuits. The synchronous rate is larger than the first graph.

Figure 6 shows the synchronous rate of the system to small variations q_k of the B-node circuits group. We define the synchronous as shown in Fig. 4 and follows:

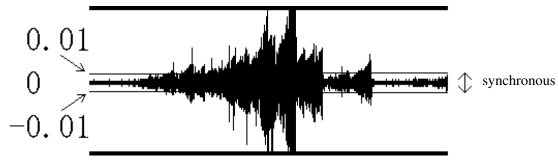

Figure 4: Definition of the synchronous.

$$|y_k - y_{k+1}| < 0.01 \tag{5}$$

where k is number of subsystems.

Figure 5 shows the influence of the number of B-node circuits to the synchronous rates of A-node circuits group and B-node circuits group. The horizontal axis is the number of B-node circuits, and the vertical axis is the synchronous rate of the groups.

As the number of the B-node circuits increases, the synchronous rate of the whole system decreases. Therefore, we can say that the B-node circuits group is more difficult to be synchronized than the A-node circuits group.

Next, the influence of the synchronous rate between A-node circuits group and B-node circuits group is investigated. Note that only q_k is varied and p_k of the A-node circuit group is constant. The horizontal axis is Q corresponding to the variation of the B-node circuits group, and the vertical axis is the synchronous rate of the system. When the variations Q increases, the synchronous rate of

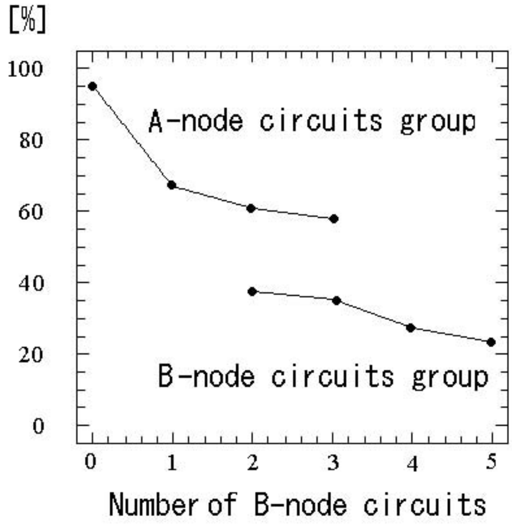

Figure 5: Synchronous rate of the groups to the number of the B-node circuits. $\alpha = 0.4, \beta = 0.5, \gamma = 20, \delta = 0.07$, and $p_k = q_k = 0.001(k-1)$.

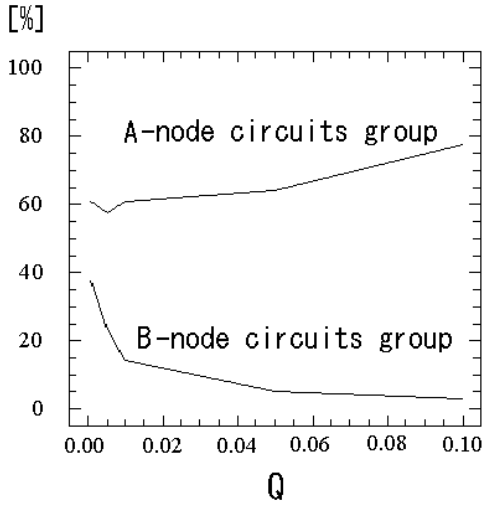

Figure 6: Synchronous rate of the groups to the variation of the B-node circuits. $\alpha = 0.4, \beta = 0.5, \gamma = 20, \delta = 0.07$, $p_k = 0.001(k-1)$, and $q_k = Q(k-1)$.

the B-node circuits group decreases. However, in spite of increasing the variations, the synchronous rate of the A-node circuits group increases.

3.2. Consideration

In the case of all B-node circuit, low synchronization rate is observed as line first of Fig. 3 and, in the case of all A-node circuit, very high synchronization rate is observed as line third of Fig. 3. Additionally, these two type circuits do not synchronous each other as Fig. 3. Therefore, interesting phenomenon that in spite of increasing variations which constrict the synchronization of the whole system, the synchronous rate of the subgroup increased can be explained as follows:

(1) The synchronizations of the A-node circuits group and B-node circuits group are constricted each other.

(2) Decreasing the synchronization of one group decrease an influence to the other group.

(3) Therefore, the synchronization of the other group increases.

3.3. The case of using thirty chaotic circuits

In order to reveal the relation among the small variations, the ratio of the number of A-node and B-node circuits, and the synchronous rate, the coupled system of thirty chaotic circuits is investigated. Figure 7 shows one example of the simulated result in the case of $m = 25$ and $n = 5$. The viewpoint of the figure is the same as Fig. 3, the horizontal axis is time, and the vertical axes are the voltage difference between two chaotic circuits. From the first to the fourth graphs show voltage differences between two B-node circuits. The fifth graph shows the voltage difference between an A-node circuit and a B-node circuit. The other graphs show the voltage differences between two A-node circuits.

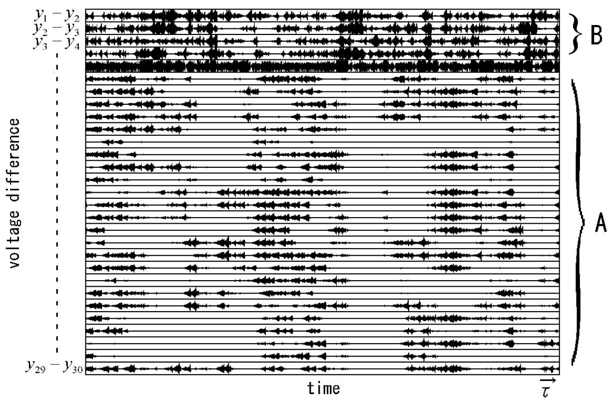

Figure 7: Simulated result of the asymmetrical coupled system in the case of $m = 25$ and $n = 5$. $\alpha = 0.5, \beta = 0.5, \gamma = 20, \delta = 0.01$, and $p_k = q_k = 0.0001(k-1)$.

Figure 8 shows the relation among the synchronous rate of the A-node circuits group, the small variations Q of the B-node circuits, and the number of the B-node circuits n. The synchronous rate of the A-node group approaches to the complete synchronization as the variations and the number of the B-node circuits increase. Drastic increase of the synchronous rate is observed around $Q = 0.01$. Further, for the cases that the number of the B-node circuits is less than five, there are few influences on the synchronization of the A-node circuits group.

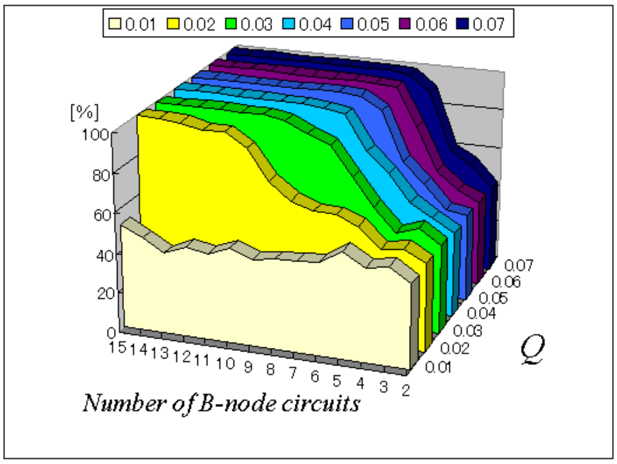

Figure 8: Synchronous rate of the A-node group to the variations Q and the number n of the B-node circuits.

4. CONCLUSION

In this study, the relation between synchronous rate and small variations on an asymmetrical coupled chaotic system is investigated. In spite of increasing variations which constrict the synchronization of the whole system, the synchronous rate of the subgroup increased. This interesting phenomenon can be explained as follows:

(1) The synchronizations of the A-node circuits group and B-node circuits group are constricted each other.

(2) Decreasing the synchronization of one group decrease an influence to the other group.

(3) Therefore, the synchronization of the other group increases.

Furthermore, the synchronous rate of the A-node circuits group was investigated as changing the small variations and the number of the B-node circuits for the case of thirty chaotic circuits. We found that there are some limits for the groups to be synchronized. The detailed analysis of the influence of the synchronous state of each node group is a future research subject.

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