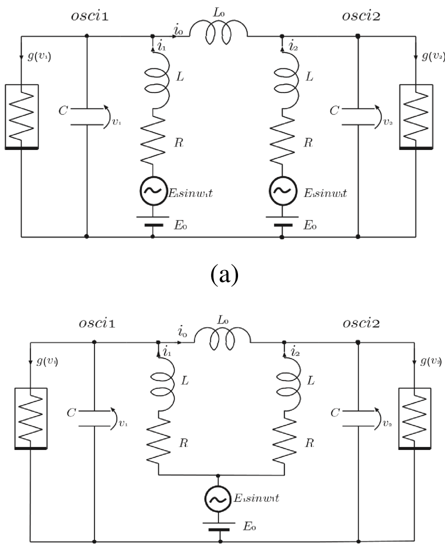
2017 International Symposium on Nonlinear Theory and Its Applications,
NOLTA2017, Cancun, Mexico, December 4-7, 2017

Synchronization of mixed-mode oscillations from a two coupled driven Bonhoeffer-van der Pol Oscillator

Katsuhiro Ogawa†, Naohiko Inaba‡, Kuniyasu Shimizu*, Hidetaka Ito**, Hiroomi Hikawa**, and Tetsuro Endo†

†Department of Electronics and Bioinformatics, Meiji University,
Kawasaki 214–8571, Japan
‡Organization for the Strategic Coordination of Research and Intellectual Properties, Meiji University,
Kawasaki 214–8571, Japan
∗ Department of Information and Communication Systems Engineering,
Chiba Institute of Technology, Narashino 275-0016, Japan
∗∗ Department of Electrical and Electronic Engineering, Kansai University, Suita 564-8680,
Japan
Email: †ce171016@meiji.ac.jp

Abstract—Mixed-mode oscillations (MMOs) are phenomena discovered in chemical experiment. We investigate MMOs generated by a two-coupled weakly driven Bonhoeffer-van der Pol (BVP) oscillator, which is connected by a large inductor. The parameter values of each oscillator are chosen as the same values. Because the two oscillators are weakly coupled chaos synchronization is not observed. However, complete synchronization of MMOs occurs.



(a)

(b)

Figure 1: (a)Circuit diagram of a two-coupled driven Bonhoeffer-van der Pol oscillator, (b)Equivalent circuit.

1. Introduction

Mixed-mode oscillations (MMOs) were phenomena discovered in 1970s and have been studied extensively in recent years [1, 2, 3, 4, 5, 8]. MMOs comprise L large excursions and s small peaks, and according to custom, they are assigned a symbol L^s. At first glance, the definition of MMOs appears to be ambiguous. However, MMOs are universal phenomena such as period-doubling bifurcations or Arnold tongues and have been observed in various dynamics such as BZ-reactions, electro-chemical system, and several electric circuits. Shimizu *et al.* discovered that MMOs and MMO-incrementing bifurcations (MMOIBs) are generated in a weakly driven Bonhoeffer-van der Pol (BVP) oscillator [6, 7]. BVP dynamics are known as a simplified Hodgkin–Huxley model and have been studied with great interesting [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27].

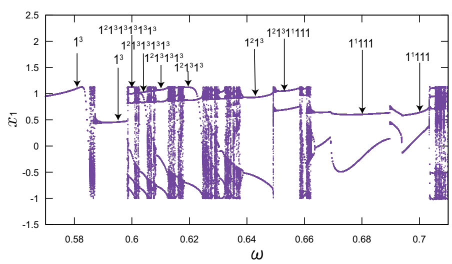


Figure 2: One-parameter bifurcation diagram.

In this study, we propose a two-coupled BVP oscillator that is connected by a large inductor and investigate synchronizations. Our dynamics are represented by a singularly perturbed fifth order nonautonomous differential equation. Because the two oscillators are weakly coupled, chaos synchronization has not been observed in this oscillator. In contrast, complete synchronization of MMOs is

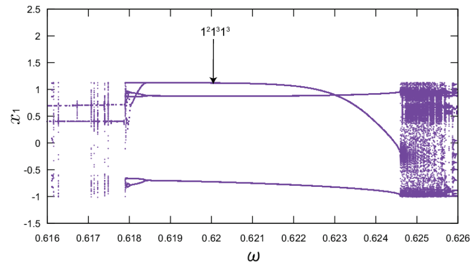


Figure 3: (a)Magnified view of Fig2.

observed. Moreover, completely synchronized MMOIBs occur.

2. Circuit setup

Figure 1 shows the circuit diagram of a two-coupled driven BVP oscillator. Two identical driven BVP oscillators are employed and they are connected by a large inductor L_0. Because the voltage souses are identical, the circuit can be re-illustrated as shown in Fig. 1(b). The coupled MMO-generated oscillators are analyzed for the first time. From Kirchhoff's law, the governing equation is represented by the following fifth-order nonautonomous differential equation:

$$\begin{aligned}
C\frac{dv_1}{dt} &= i_1 - g(v_1) + i_3, \\
L\frac{di_1}{dt} &= -v_1 - i_1 R + E_0 + E_1 \sin\omega_1 t, \\
C\frac{dv_2}{dt} &= i_2 - g(v_2) - i_3, \\
L\frac{di_2}{dt} &= -v_2 - i_2 R + E_0 + E_1 \sin\omega_1 t, \\
L_0\frac{di_3}{dt} &= v_2 - v_1.
\end{aligned} \tag{1}$$

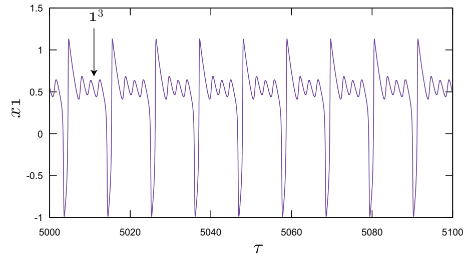
Via rescaling

$$\begin{aligned}
\tau &\equiv \frac{t}{Lg_1}, \quad \varepsilon \equiv \frac{C}{g_1^2 L}, \\
k_1 &\equiv g_1 R, \quad \omega \equiv Lg_1\omega_1, \\
B_0 &\equiv \sqrt{\frac{g_3}{g_1}}E_0, \quad B_1 \equiv \sqrt{\frac{g_3}{g_1}}E_1, \\
x_1 &\equiv \sqrt{\frac{g_3}{g_1}}v_1, \quad x_2 \equiv \sqrt{\frac{g_3}{g_1}}v_2, \\
y &\equiv \sqrt{\frac{g_3}{g_1^3}}i_3, \quad y_1 \equiv \sqrt{\frac{g_3}{g_1^3}}i_1, \\
y_2 &\equiv \sqrt{\frac{g_3}{g_1^3}}i_2, \quad \frac{L_0}{L} \equiv \alpha,
\end{aligned} \tag{2}$$

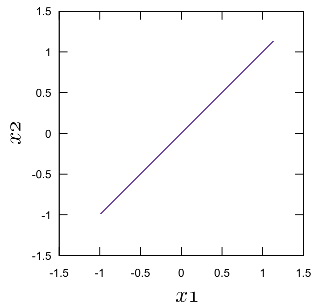
the normalized equation is represented by

$$\begin{aligned}
\alpha\dot{y} &= x_2 - x_1, \\
\varepsilon\dot{x_1} &= y_1 + x_1 - x_1^3 + y, \\
\dot{y_1} &= -x_1 - k_1 y_1 + B_0 + B_1 \sin\omega\tau, \\
\varepsilon\dot{x_2} &= y_2 + x_2 - x_2^3 - y, \\
\dot{y_2} &= -x_2 - k_1 y_2 + B_0 + B_1 \sin\omega\tau. \quad (\tfrac{d}{d\tau} = \text{“}\cdot\text{”})
\end{aligned} \tag{3}$$

Throughout this study, we set $\alpha = 100$. We set constant parameters as in $\varepsilon = 0.1$, $k_1 = 0.9$, $B_0 = 0.207$, $B_1 = 0.0105$ and allow ω to vary.



(a) Time series.



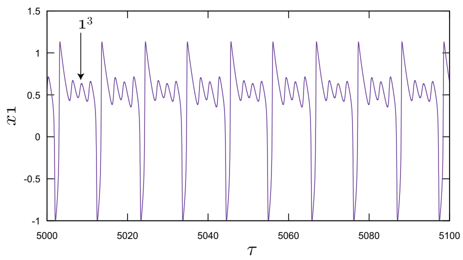
(b) Projection of MMOs onto $x_1 - x_2$ plane.

Figure 4: Example of MMOs generated in a two-coupled driven BVP oscillator ($\omega = 0.58$).

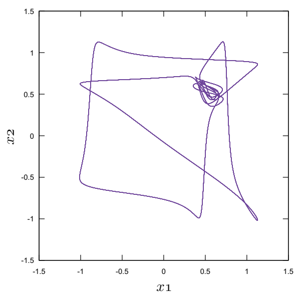
To observe dynamical behavior in detail, Poincaré map F is defined as a stroboscopic section.

Figure 2 shows a one-parameter bifurcation diagram where ω is varied. Complex bifurcation structures are observed. The magnified view of Fig. 2 is show in Fig. 3. Each MMO is surrounded by a saddle-node bifurcation and period bifurcation. According to our numeric, the natural angular frequency is 1.538493. Thus, MMOs occur when ω is less than the natural frequency.

Figure 4(a) shows an example of time series behavior. As observed, the time series observed in a coupled driven BVP oscillator is also MMOs shape. It represents 1^3. Moreover, Fig. 4(b) reveals that the MMO sequence is completely synchronized. According to our numeric, MMOIB-generated MMOs are always completely synchronous.



(a) Time series.



(b) Projection of MMOs onto $x_1 - x_2$ plane.

Figure 5: Example of MMOs generated in a two-coupled driven BVP oscillator (ω = 0.5915).

By contrast, Fig. 5 shows an attractor obtained with ω = 0.5915. As observed from Fig. 5(b), the oscillators are synchronized, but not completely synchronized.

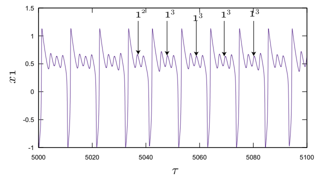
Furthermore, the generation of completely synchronized MMOIBs can be observed as shown in Fig. 6.

It is confirmed that these MMOs are completely synchronized. The projection of attractor $1^2 1^3 1^3 1^3 1^3$ onto $x_1 - x_2$ plane is showed in Fig. 7.

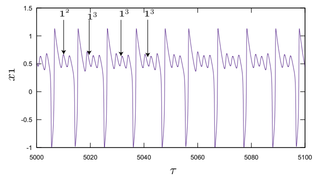
3. Conclusion

We investigated a two-coupled of weakly driven BVP oscillator each of which generates MMOs and MMOIBs. The dynamic showed completely synchronized MMOs and incompletely synchronized MMOs.

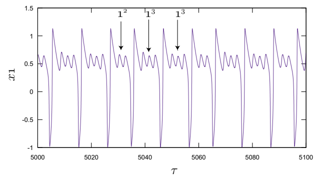
Acknowledgement
This work was supported by JSPS KAKENHI Grant Numbers 16H02876 and 16K06430.



(a)



(b)



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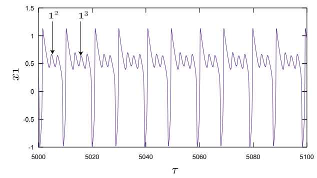

(d)

Figure 6: Example of MMOs generated in a two-coupled driven BVP oscillator (a) $1^2 1^3 1^3 1^3 1^3$ (ω = 0.603), (b) $1^2 1^3 1^3 1^3$(ω = 0.61), (c) $1^2 1^3 1^3$ (ω = 0.62), (d) $1^2 1^3$ (ω = 0.642)

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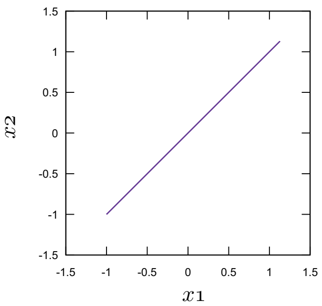

Figure 7: Complete synchronization of $1^2 1^3 1^3 1^3 1^3 (\omega = 0.603)$.

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