Application of hybrid system theory to power system stability analysis

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Abstract—This paper shows a basic framework for applying hybrid system theory to power system stability analysis. Hybrid dynamical systems and control are nowadays developed in the intersection of computer science and control engineering. In this paper we discuss an application of hybrid system theory to stability analysis of power systems, and propose a novel approach to the stability estimation based on the revolution of reachable sets. This paper applies the proposed approach to transient stability analysis of a simple electric power system, thereby showing the effectiveness of the approach.

1. Introduction

Nowadays electric power systems become complicated in terms of their size, configuration, dynamics, operation, and control: see e.g., [1, 2, 3, 4]. In a technical trend, various power apparatuses including HVDC systems [5] and FACTS [6] are installed into conventional ac power systems. Their apparatuses are based on switching operation of power conversion devices, and they are expected to contribute the operation of power systems. On the other hand, as a non-technical issue, regulatory reforms of power markets require a substantial modification of conventional power systems. These technical and non-technical trends obviously cause the dynamics of power systems to be complicated and therefore make it much difficult to analyze and control the dynamics. A comprehensive approach to the analysis and control has been thus strongly required [2].

Hybrid dynamical systems and their control are attracting a lot of interests in the fields of computer science and control engineering: see e.g., [7, 8, 9]. *Hybrid automata* are of central concern with hybrid systems and control. The mathematical formulation is applicable to the analysis of various complicated systems that involve the interaction of continuous and discrete dynamics. *Reachability* analysis [7] of hybrid automata is here of paramount importance for safety specifications of engineering systems: for examples, steam boiler and flight management systems [7, 9].

The objective of this paper is to apply hybrid system theory to power system stability analysis. Power system stability is one of the fundamental concerns in system planning, operation, and control [10, 11]. Recently several researchers have worked on the intersection of power system analysis and hybrid system theory: Hiskens and Pai [12] propose a hybrid modeling of power systems including transformer tap change and

relay operation; DeMarco [13] proposes a phase transition model for cascading failure via hybrid dynamical systems; and Fourlas *et. al* [14] investigate dynamic response of power transmission system via a hybrid automaton model. The present paper discusses a basic framework for stability analysis of complex power systems based on hybrid system theory. We here propose a novel approach to the stability estimation based on the revolution of *reachable sets*. This paper also applies the proposed approach to transient stability analysis of single machine-infinite bus (SMIB) system. Some of the results in this paper are preliminary presented in [15, 16].

2. Basic framework

This section discusses a basic framework for applying hybrid system theory to power system stability analysis based on hybrid system theory [7, 17, 9].

2.1. Hybrid automaton as power system model

A *hybrid automaton H* is defined to be the tuple $H = (Q \times X, U \times D, \Sigma_u \times \Sigma_d, f, E, Inv, I, \Omega)$ with

- $Q \times X$ is the state space, with $Q \triangleq \{q_1, q_1, \cdots, q_m\}$ a finite set of discrete states and X a n-dimensional manifold; a state of the system is a pair $(q_i, x) \in Q \times X$;
- $U \times D \subset \mathbb{R}^u \times \mathbb{R}^d$ is the product of the set of continuous control inputs and the set of continuous disturbances; the space of acceptable control and disturbance trajectories are denoted by $\mathcal{U} \triangleq \{u(\cdot) \in PC^0 \mid u(\tau) \in U \ \forall \tau \in \mathbb{R}\}$ and $\mathcal{D} \triangleq \{d(\cdot) \in PC^0 \mid d(\tau) \in D \ \forall \tau \in \mathbb{R}\}$. PC^0 denotes the space of piecewise continuous functions over $\mathbb{R}$;
- $\Sigma_u \times \Sigma_d$ is the product of the finite set of discrete control actions and the finite set of discrete disturbance actions;
- $f : Q \times X \times U \times D \to TX$ is the vector field which associates a control system $f(q, x, u, d)$ with each discrete state $q \in Q$;
- $E : Q \times X \times \Sigma_u \times \Sigma_d \to 2^{Q \times X}$ is the discrete transition function;
- $Inv \subseteq Q \times X$ is the invariant associated with each discrete state, meaning that the system evolves according to $\dot{x} = f(q, x, u, d)$ only if $(q, x) \in Inv$;
- $I \subseteq Q \times X$ is the set of initial states;
- Ω is the trajectory acceptance condition (here $\Omega = \Box F$ for $F \subseteq Q \times X$. $\Box$ denotes a map,

called *property*, from the set of all executions of H to {True,False}[7].).

The hybrid automaton H is relevant for modeling an electric power system that includes relay operations, switching operation of power conversion apparatuses, and possible disturbances caused by deregulation of power markets. In the continuous part in H, f describes the dynamics of rotor angles and voltages, for which we usually use swing equations [10, 11] and differential-algebraic equations [18, 10]; $u(\cdot) \in U$ is regarded as the control input such as dc links, SVCs, and so on; and $d(\cdot) \in D$ is as a possible and irregular disturbance caused by deregulation of power markets. On the other hand, in the discrete part in H, E represents the discrete transition of system states (modes); $(\sigma_u[\cdot], \sigma_d[\cdot]) \in \Sigma_u \times \Sigma_d$ also implies the controlled and uncontrolled line switching by relay operation, accidental faults, and so on. The hybrid automaton H is thus applicable to complex power systems which involve the interaction of continuous dynamics and discrete events. Note that the above description is relevant to any aspect of power system analysis: transient stability, voltage stability, multi-swing instability, and so on. In the next section we will deal with transient stability analysis of a simple power system.

2.2. Reachable sets for stability analysis

Here we introduce a new approach for stability estimation via hybrid system theory. Let us define an *unsafe set* $G \subset Q \times X$ for the hybrid automaton H. The unsafe set is interpreted as a subset in which the power system cannot be safely operated: large rotor swing, stepping-out, low voltage amplitude, and so on. A reachable set $R_t(G)$ $(t < 0)$ for the hybrid automaton H is roughly defined to be a subset of $Q \times X$ in which any system state reaches to the boundary ∂G of G until at least $|t|$ time despite of the control $(u(\cdot), \sigma_u[\cdot])$: precise discussion of hybrid reachable sets is in [17]. Fig. 1 shows the concept of reachable sets for continuous systems. An *usable part* in the figure, for continuous systems, is a part of the boundary ∂G for which there exists a disturbance $d \in D$ such that for all inputs $u \in U$ the vector field points into G. The usable part is utilized to calculate the reachable set in Section 3. The concept of the reachable sets is much important for validating the stability of power systems, because, if a system state exists in $R_t(G)$, then we can evaluate that the system will reach to an unacceptable operation for any control input. In particular the reachable sets have a great potential to contribute the stability analysis of hybrid power systems caused by line switching, etc. Note that an interesting method for calculating time-growth of reachable sets has been proposed based on a Hamilton-Jacobi equation and a level set method in [7, 17, 19, 9].

3. Transient stability estimation via reachable sets

This section analyzes transient stability of a single machine-infinite bus (SMIB) system, shown in Fig. 2, via reachable sets. The SMIB system consists of a

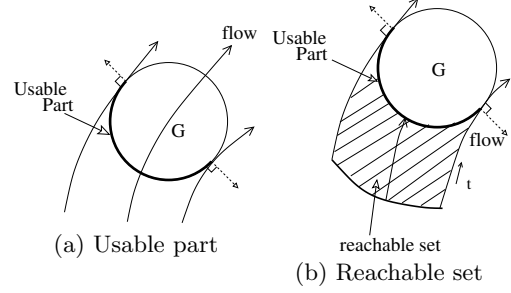

(a) Usable part

(b) Reachable set

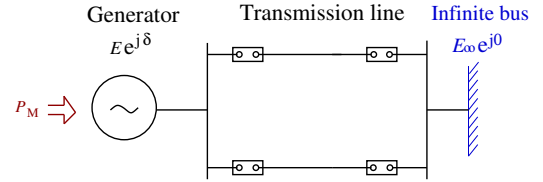Figure 1 Concept of reachable sets for continuous systems

Figure 2 Single machine-infinite bus (SMIB) system

synchronous machine, an infinite bus, and two parallel transmission lines.

3.1. Mathematical model

The electro-mechanical dynamics of synchronous machine is described by the following swing equation system:

$$\begin{cases} \dot{\delta} &= \omega, \\ \dot{\omega} &= P_{\mathrm{M}} - b\sin\delta - k\omega, \end{cases} \quad (1)$$

where δ is the rotor position with respect to synchronous reference axis, and ω the rotor speed deviation relative to system angular frequency. P_{M} denotes the mechanical input power to generator, b the critical transmission power of SMIB system, and k the damping coefficient in generator. The derivation of the system (1) is given in [11]. This section uses the following parameters setting [20]:

$$b = 0.7, \quad k = 0.05, \quad P_{\mathrm{M}} = 0.2. \quad (2)$$

3.2. Numerical simulation

3.2.1. Continuous case

First of all we examine reachable sets for continuous swing equation system (1). Let us define the unsafe set G with the boundary ∂G as follows:

$$\begin{cases} G &\triangleq \{(\delta, \omega) \in S^1 \times \mathbb{R}^1 \mid \omega_{\mathrm{c}}^2 - \omega^2 \leq 0\}, \\ \partial G &\triangleq \{(\delta, \omega) \in S^1 \times \mathbb{R}^1 \mid \omega_{\mathrm{c}}^2 - \omega^2 = 0\}, \end{cases} \quad (3)$$

where $\omega_{\mathrm{c}} = \pi$. Any state in G physically implies an unacceptable operation of the SMIB system because of the large value of ω. The usable part UP of ∂G is then derived as follows:

$$\begin{aligned} UP = \ & \{(\delta, \omega) \in \partial G \mid \\ & \mp 2\pi(P_{\mathrm{M}} - b\sin\delta \mp \pi k) < 0\}. \quad (4) \end{aligned}$$

(a) $T_R = 1.37$ s   (b) $T_R = 1.65$ s
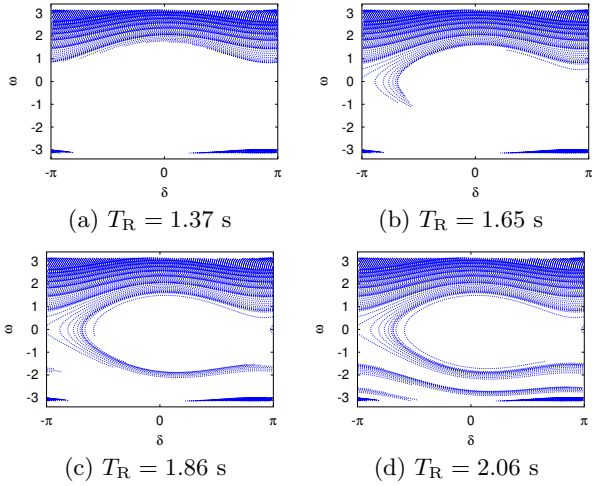
(c) $T_R = 1.86$ s   (d) $T_R = 2.06$ s

Figure 3   Growth of reachable set in continuous swing equation system
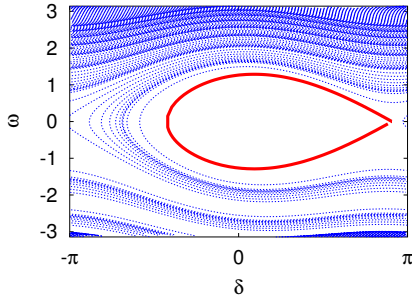

Figure 4   Reachable set and stability region in continuous swing equation system. The *red* dotted closed loop is an analytical criterion for stability based on closest u.e.p. method.
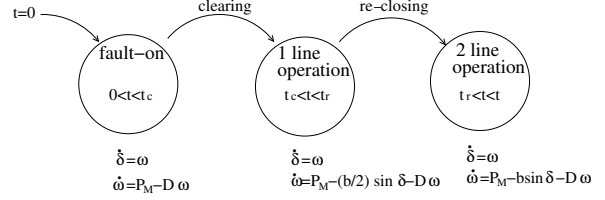

Figure 5   Switching sequence governing SMIB system with clearing and re-closing operations

after re-closing the line. t_c denotes the fault-clearing time, and t_r the re-closing time. In the figure, we can regard the fault clearing and circuit re-closing as the discrete transitions, which drive depends on time variable only, in the hybrid automaton. In the swing equation system (1), the fault-on, 1 line operation, and 2 line operation modes coincide with the parameter settings: $b = 0.0$, 0.35, 0.7, respectively.

We are in a position to investigate hybrid reachable sets of the unsafe set G at $\omega_c = 2.0$ for each mode. The transmission line is here re-closed after the re-closing period $T_{rc} \triangleq t_r - t_c$. The derived reachable set is then decomposed into the two subsets R_{before} and R_{after}; R_{before} is the subset of $S^1 \times \mathbb{R}^1$ from which any state reaches to ∂G *before* the re-closing; and R_{after} is the one from which any state reaches to ∂G *after* the re-closing. Fig. 6 shows the two reachable subsets R_{before} and R_{after} under the condition $T_{rc} = 0.5$ s. The product of the two subsets corresponds to the hybrid reachable set of G.

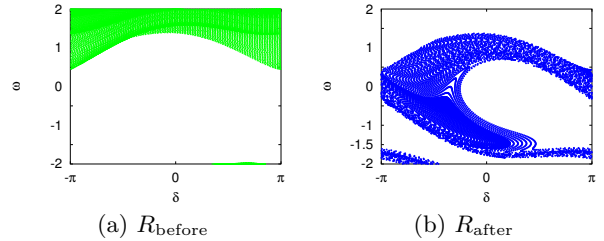(a) R_{before} (b) R_{after}

Figure 6   Reachable set in swing equation system under re-closing time $T_{rc} = 0.5$ s

The derivation of UP is based on [7, 17, 9]. Fig. 3 shows the growth of reachable set of G for continuous swing equation system (1). T_R in the figure denotes the time until at least which any state in the reachable set reaches to ∂G. As T_R increases, the reachable set expands into the phase space; actually the complement of the reachable set approximates the stability region of an asymptotically stable equilibrium point $(\sin^{-1}(P_M/b), 0)$ as time goes to infinity. This is confirmed in Fig. 4; the *red* closed loop is the sufficient condition for transient stability based on closest u.e.p. method [10]. The complement of the obtained reachable set in Fig. 3 hence corresponds to a necessary condition for the transient stability of the SMIB system.

3.2.2. Hybrid case

This section discusses hybrid reachable sets related to re-closing operation of transmission lines. Fig. 5 shows a switching sequence for the SMIB system. The three modes are as follows: (i) fault-on is the system state during a sustained fault on one line; (ii) 1 line operation is the state after clearing the fault line by relay operation; and (iii) 2 line operation is the state

Figure 7 describes how the reachable set changes as the re-closing period T_{rc} increases. The *red* closed loop stands for the sufficient condition for transient stability of 1 line operation. In Fig. 7 we can observe that some of the system states can survive by the *slow* re-closing $T_{cr} = 0.3$ s rather than the *fast* one $T_{cr} = 0.1$ s: see the neighborhoods of $(-\pi/2, -0.5)$ in Figs. 7 (a) and (b). This observation is not given by any classical method of transient analysis. Fig. 7 thus suggests that the reachability analysis makes it possible for us to estimate the transient stability taking the switching operation into account.

4. Summary

This paper showed a basic framework for applying hybrid system theory to power system stability anal-
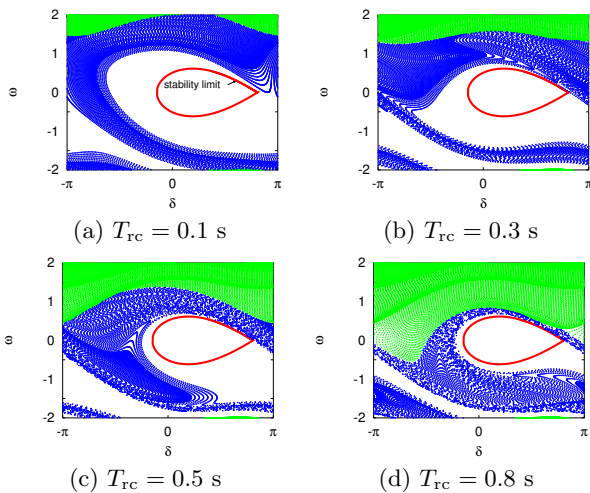
(a) $T_{rc} = 0.1$ s   (b) $T_{rc} = 0.3$ s

(c) $T_{rc} = 0.5$ s   (d) $T_{rc} = 0.8$ s

Figure 7   Hybrid reachable sets in swing equation system. The *red* closed loop stands for the sufficient condition based on closest u.e.p. method.

ysis. The mathematical model via hybrid automata is much relevant to the analysis and control of future power networks involving power conversion apparatuses, deregulation of power markets, and so on. This paper also performed the transient stability estimation based on the revolution of reachable sets. Our future direction is to apply the present approach to practical system analysis [21].

Acknowledgments

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