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# Computer Simulations of an Augmented Automatic Choosing Control Using the Weighted Automatic Choosing Functions with Constrained Input

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**Abstract**—In this paper we consider a nonlinear feedback control called augmented automatic choosing control (AACC) for nonlinear systems with constrained inputs. It is designed by making use of the linear quadratic (LQ) controls and the weighted automatic choosing functions. Constant terms which arise from sectionwise linearization of a given nonlinear system are treated as coefficients of a stable zero dynamics. Parameters included in the control are suboptimally selected by minimizing the Hamiltonian and a necessary condition for optimization with the aid of the genetic algorithm. Computer simulations show how this AACC can be well in the improvement of transient stability of electric power system.

## 1. Introduction

Generally, it is easy to design the optimal control laws for linear systems, but it is not so for nonlinear systems, though they have been studied for many years[1]∼[6]. One of most popular and practical nonlinear control laws is synthesized by applying a linearization method by Taylor expansion and the linear optimal control method to a given nonlinear system. This is only effective in a small region around the steady state point or in almost linear systems[1]∼[3].

To overcome these weakness, in this paper we consider an augmented automatic choosing control (AACC) for nonlinear systems with constrained inputs and its design procedure is as follows. Assume that a system is given by a nonlinear differential equation. Choose a separative variable, which makes up nonlinearity of the given system. The domain of the variable is divided into some subdomains. On each subdomain, the system equation is linearized by Taylor expansion around a suitable point so that a constant term is included in it. This constant term is treated as a coefficient of a stable zero dynamics. The given nonlinear system approximately makes up a set of augmented linear systems, to which the optimal linear control theory is applied to get the linear quadratic (LQ) controls. These LQ controls are smoothly united by using the weighted automatic choosing functions of sigmoid type to synthesize a single nonlinear feedback controller.

This controller is of a structure-specified type which has some parameters, such as the number of division of the domain, regions of the subdomains, points of Taylor expansion, weights and gradients of the automatic choosing functions, and coefficients of the zero dynamics. These parameters must be selected optimally to be just the controller's fit. Since they lead to a nonlinear optimization problem, we are able to solve it by using the genetic algorithm(GA)[7] suboptimally and successfully. The suboptimal values of these parameters are obtained by minimizing the Hamiltonian and a necessary condition for optimization in this approach.

This approach is applied to a field excitation control problem of power system to improve the transient stability in both accuracy and stable region. Simulation results show that the new controller using the GA can improve performance remarkably well.

## 2. Augmented Automatic Choosing Control

Assume that a nonlinear system is given by

$$\dot{x} = f(x) + g(x)u, \quad x \in \mathbf{D} \tag{1}$$

subject to

$$u_{j,min} \leq u[j] \leq u_{j,max} \quad (j = 1, \cdots, r) \tag{2}$$

where $\cdot = d/dt$, $x = [x[1], \cdots, x[n]]^T$ is an $n$-dimensional state vector, $u = [u[1], \cdots, u[r]]^T$ is an $r$-dimensional control input vector, $f(x) : \mathbf{D} \to R^n$ is a nonlinear vector-valued function with $f(0) = 0$ and is continuously differentiable, $g(x) : \mathbf{D} \to R^{n\times r}$ is a driving matrix with $g(0) \neq 0$ and is continuously differentiable, $u_{j,min}$ is the minimum value of $u[j]$, $u_{j,max}$ is the maximum value of $u[j]$, $\mathbf{D} \subset R^n$ is a domain , and $T$ denotes transpose.

Considering the nonlinearity of the system (1), introduce a vector-valued function $C : \mathbf{D} \to R^L$ which defines the separative variables $\{C_j(x)\}$, where $C = [C_1 \cdots C_j \cdots C_L]^T$ is continuously differentiable. Let $D$ be a domain of $C^{-1}$. For example, if $x[2]$ is the element which has the highest nonlinearity of (1), then

$$C(x) = x[2] \in D \subset R \quad (L = 1)$$

(see Section 4). The domain $D$ is divided into some subdomains: $D = \cup_{i=0}^{M} D_i$, where $D_M = D - \cup_{i=0}^{M-1} D_i$ and $C^{-1}(D_0) \ni 0$. $D_i (0 \le i \le M)$ endowed with a lexicographic order is the Cartesian product $D_i = \Pi_{j=1}^{L}[a_{ij}, b_{ij}]$, where $a_{ij} < b_{ij}$.
Introduce a stable zero dynamics :

$$\dot{x}[n+1] = -\sigma_i x[n+1] \tag{3}$$

$$(x[n+1](0) \simeq 1, \quad 0 < \sigma_i < 1),$$

where the value of $\sigma_i$ shall be selected so that $\sigma_i = -\dot{x}[n+1]/x[n+1] \le -\dot{x}[k]/x[k]$ holds for all $k (k = 1, \cdots, n)$. This tries to keep $x[n+1] \simeq 1$ for a good while when the system (1) is not on $C^{-1}(D_0)$.
Combine (1) with (3) to form an augmented system

$$\dot{\mathbf{X}} = \bar{f}(\mathbf{X}) + \bar{g}(\mathbf{X})u \tag{4}$$

where

$$\mathbf{X} = \begin{bmatrix} x \\ x[n+1] \end{bmatrix} \quad \in \mathbf{D} \times R$$

$$\bar{f}(\mathbf{X}) = \begin{bmatrix} f(x) \\ -\sigma_i x[n+1] \end{bmatrix}, \bar{g}(\mathbf{X}) = \begin{bmatrix} g(x) \\ 0 \end{bmatrix}.$$

Let a cost function be

$$J = \frac{1}{2}\int_0^\infty \left(\mathbf{X}^T\mathbf{Q}\mathbf{X} + u^T\mathbf{R}u\right) dt \tag{5}$$

where

$$\mathbf{Q} \quad = \quad \begin{bmatrix} Q & 0 \\ 0 & q \end{bmatrix}, \quad R \ni q > 0,$$

$Q = Q^T > 0$ and $\mathbf{R} = \mathbf{R}^T > 0$ which denote positive symmetrix matrices. Values of $\mathbf{Q}$ and $\mathbf{R}$ are properly determined based on engineering experience.
On each $D_i$, the nonlinear system is linearized by the Taylor expansion truncated at the first order about a point $\hat{X}_i \in C^{-1}(D_i)$ and $\hat{X}_0 = 0$ (see Figure 1):

$$\begin{aligned} f(x) + g(x)u &\simeq A_i x + w_i + B_i u \\ &\simeq A_i x + w_i x[n+1] + B_i u \end{aligned} \tag{6}$$

where

$$\begin{aligned} A_i &= \partial f(x)/\partial x^T|_{x=\hat{X}_i}, \ B_i = g(\hat{X}_i), \\ w_0 &= 0, \ w_i = f(\hat{X}_i) - A_i\hat{X}_i. \end{aligned}$$

That is, an approximation of (4) is

$$\dot{\mathbf{X}} = \bar{A}_i\mathbf{X} + \bar{B}_i u \quad \text{ on } \ C^{-1}(D_i) \times R \tag{7}$$

where

$$\bar{A}_i = \begin{bmatrix} A_i & w_i \\ 0 & -\sigma_i \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}.$$

An application of the linear optimal control theory[2] to (5) and (7) yields

$$u_i(\mathbf{X}) = -\mathbf{R}^{-1}\bar{B}_i^T\mathbf{P}_i\mathbf{X} \tag{8}$$

where the $(n+1) \times (n+1)$ matrix $\mathbf{P}_i$ satisfies the Riccati equation :

$$\mathbf{P}_i\bar{A}_i + \bar{A}_i^T\mathbf{P}_i + \mathbf{Q} - \mathbf{P}_i\bar{B}_i\mathbf{R}^{-1}\bar{B}_i^T\mathbf{P}_i = 0. \tag{9}$$

Introduce an automatic choosing function of sigmoid type with weight $d_i$:

$$\begin{aligned} I_i(x) &= d_i \prod_{j=1}^{L}\left\{1 - \frac{1}{1+\exp\left(2N\left(C_j(x) - a_{ij}\right)\right)}\right. \\ &\quad \left. - \frac{1}{1+\exp\left(-2N\left(C_j(x) - b_{ij}\right)\right)}\right\} \end{aligned} \tag{10}$$

where $d_i$ and $N$ are positive real values, $-\infty \le a_{ij} < b_{ij} \le \infty$. $I_i(x)$ is analytic and almost unity on $C^{-1}(D_i)$, otherwise almost zero(see Figure 2).
Uniting $\{u_i(\mathbf{X})\}$ of (8) with $\{I_i(x)\}$ of (10) yields

$$\begin{aligned} \hat{u}(\mathbf{X}) &= [\hat{u}(\mathbf{X})[1], \cdots, \hat{u}(\mathbf{X})[j], \cdots, \hat{u}(\mathbf{X})[r]]^T \\ &= \sum_{i=0}^{M} u_i(\mathbf{X})I_i(x). \end{aligned}$$

We finally have an augmented automatic choosing control which is satisfied with the constraint condition (2) by

$$u(\mathbf{X}) = [u(\mathbf{X})[1], \cdots, u(\mathbf{X})[j], \cdots, u(\mathbf{X})[r]]^T \tag{11}$$

where

$$u(\mathbf{X})[j] = \begin{cases} u_{j,max} & \text{if } \ \hat{u}(\mathbf{X})[j] \ge u_{j,max} \\ u_{j,min} & \text{if } \ \hat{u}(\mathbf{X})[j] \le u_{j,min} \\ \hat{u}(\mathbf{X})[j] & \text{otherwise} \end{cases}$$

$$(1 \le j \le r).$$

## 3. Parameter Selection by GA

The Hamiltonian for Eqs.(4) and (5) is given by

$$\begin{aligned} H(\mathbf{X}, u, \lambda) &= \frac{1}{2}\left(\mathbf{X}^T\mathbf{Q}\mathbf{X} + u^T\mathbf{R}u\right) \\ &\quad + \lambda^T\left(\bar{f}(\mathbf{X}) + \bar{g}(\mathbf{X})u\right). \end{aligned} \tag{12}$$

Assume that the adjoint vector $\lambda(\mathbf{X}) \in R^{n+1}$ is defined by

$$\lambda(\mathbf{X}) = [\lambda^I(\mathbf{X})^T, \lambda^{II}(\mathbf{X})^T]^T \tag{13}$$

where $\lambda^I(\mathbf{X}) = [\lambda[1], \cdots, \lambda[r]]^T = -(G^T(x))^{-1}\mathbf{R}u(\mathbf{X})$

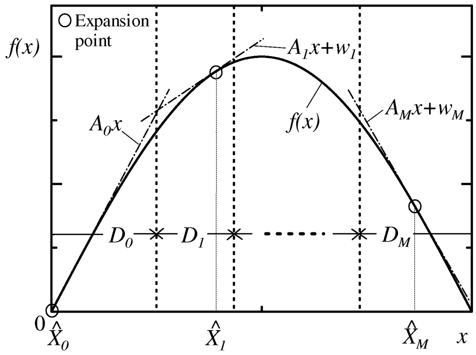

Figure 1: Sectionwize linearization

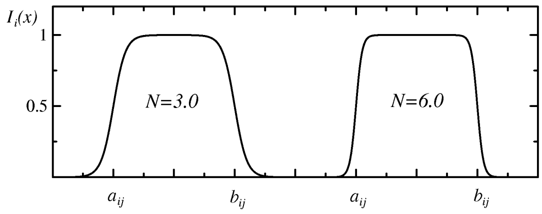

Figure 2: Automatic Choosing Function(N=3.0,6.0)

, $\lambda^{II}(\mathbf{X}) = [\lambda[r+1], \cdots, \lambda[n+1]]^T = [\mathbf{0}, E]\hat{\lambda}$, $\hat{\lambda} = \sum_{i=0}^{M}\{(\bar{B}_i - \bar{g}(\mathbf{X}))\bar{g}(\mathbf{X})^{\dagger} + E\}^T\mathbf{P}_i\mathbf{X}I_i(x) \in R^{n+1}$, $\bar{g}(\mathbf{X})^{\dagger}\bar{g}(\mathbf{X}) = E$, $E$ is an appropriate-dimentional unit matrix, and † denotes pseudo inverse.
There are two necessary conditions of the optimality. One of them is $\partial H/\partial u = 0$ or $u = -\mathbf{R}^{-1}\bar{g}(\mathbf{X})^T\lambda = -\mathbf{R}^{-1}G^T(x)\lambda^I(\mathbf{X})$ , which is satisfied with Eq.(11) from Eq.(13). By it, Eq.(12) becomes

$$H(\mathbf{X}, u, \lambda) = \frac{1}{2}\mathbf{X}^T\mathbf{Q}\mathbf{X} - \frac{1}{2}u^T\mathbf{R}u + \bar{f}^T(\mathbf{X})\lambda. \quad (14)$$

The other one is $\dot{\lambda} = \partial\lambda(\mathbf{X})/\partial X^{\mathrm{T}} \cdot (\bar{f}(\mathbf{X}) + \bar{g}(\mathbf{X})u(\mathbf{X})) = -\partial H/\partial\mathbf{X}$.
Thus we can define a performance

$$\begin{aligned} PI \;=\; & \int_{\mathbf{D}} \left| H(\mathbf{X}, u, \lambda) \right| / \mathbf{X}^{\mathrm{T}}\mathbf{X} d\mathbf{X} \\ & + \int_{\mathbf{D}} \left\| \partial\lambda(\mathbf{X})/\partial X^{\mathrm{T}} \cdot (\bar{f}(\mathbf{X}) + \bar{g}(\mathbf{X})u(\mathbf{X})) \right. \\ & \left. + \partial H/\partial\mathbf{X} \right\|^2 / \mathbf{X}^{\mathrm{T}}\mathbf{X} d\mathbf{X} \quad (15) \end{aligned}$$

which aims at minimizing the mean values of the Hamiltonian and the necessary conditions on the domain **D**.
A set of parameters included in the control (11):

$$\bar{\Omega} = \left\{M, N, d_i, a_{ij}, b_{ij}, \hat{X}_i\right\}$$

is suboptimally selected by minimizing $PI$ with the aid of GA[7] as follows.

**<ALGORITHM>**
**step1:A-priori:** Set values $\bar{\Omega}_{apriori}$ appropriately.
**step2:Parameter:** Choose a subset $\Omega \subset \bar{\Omega}$ to be improved and rewrite it by $\Omega = \{M, N, d_i \cdots\} = \{\alpha_k : k = 1, \cdot\cdot, K\}$.
**step3:Coding:** Represent each $\alpha_k$ with a binary bit string of $\widetilde{L}$ bits and then arrange them into one string of $\widetilde{L}K$ bits.
**step4:Initialization:** Randomly generate an initial population of $\widetilde{q}$ strings $\{\Omega_p : p = 1, \cdot\cdot, \widetilde{q}\}$.
**step5:Decoding:** Decode each element $\alpha_k$ of $\Omega_p$ by $\alpha_k = (\alpha_{k,max} - \alpha_{k,min})\, A_k / \left(2^{\widetilde{L}} - 1\right) + \alpha_{k,min}$ where $\alpha_{k,max}$:maximum, $\alpha_{k,min}$:minimum, and $A_k$:decimal value of $\alpha_k$.
**step6:Control:** Design $u = u(\mathbf{X})_p \quad (p = 1, \cdots, q)$ for $\Omega_p$ by using Eq.(11).
**step7:Adjoint:**Make $\lambda = \lambda(\mathbf{X})_p \quad (p = 1, \cdots, q)$ for $\Omega_p$ by using Eq.(13).
**step8:Fitness value calculation:** Calculate $PI_p$ by Eq.(15), or fitness $F_p = -PI_p$. Integration of $F_p$ is approximated by a finite sum.
**step9:Reproduction:** Reproduce each of individual strings with the probability of $F_p / \sum_{j=1}^{\widetilde{q}} F_j$.
**step10:Crossover:** Pick up two strings and exchange them at a crossing position by a crossover probability $P_c$.
**step11:Mutation:** Alter a bit of string (0 or 1) by a mutation probability $P_m$.
**step12:Repetition:** Repeat step5∼step11 until prespecified G-th generation. If unsatisfied, go to step2.

As a result, we have a suboptimal control $u(\mathbf{X})$ for the string with the best performance over all the past generations.

## 4. Numerical Example

Consider a field excitation control problem of power system which is described[5][6] by

$$\begin{aligned} & \widetilde{M}\frac{d^2\delta}{dt^2} + \widetilde{D}(\delta)\frac{d\delta}{dt} + P_e(\delta) = P_{in} \\ & P_e(\delta) = E_I^2 Y_{11}\cos\theta_{11} + E_I\widetilde{V}Y_{12}\cos(\theta_{12} - \delta) \\ & E_I + T'_{d0}\frac{dE'_q}{dt} = E_{fd} \\ & E_I = E'_q + (X_d - X'_d)I_d(\delta) \\ & I_d(\delta) = -E_I Y_{11}\sin\theta_{11} - \widetilde{V}Y_{12}\sin(\theta_{12} - \delta) \\ & \widetilde{D}(\delta) = \widetilde{V}^2\left\{\frac{T''_{d0}(X'_d - X''_d)}{(X'_d + X_e)^2}\sin^2\delta \right. \\ & \qquad\qquad \left. + \frac{T''_{q0}(X_q - X''_q)}{(X_q + X_e)^2}\cos^2\delta\right\}, \end{aligned}$$

where $\delta$: phase angle, $\dot{\delta}$: rotor speed, $\widetilde{M}$: inertia coefficient, $\widetilde{D}(\delta)$: damping coefficient, $P_{in}$: mechanical input power, $P_e(\delta)$: generator output power, $\widetilde{V}$: reference bus voltage, $E_I$: open circuit voltage, and $E_{fd}$: field excitation voltage. Put

Table 1: Performances $\widetilde{J}$

| Method | $x^T(0)$ | | | | |
|---|---|---|---|---|---|
| | [0, 0.4, 0] | [0, 0.6, 0] | [0, 1.0, 0] | [0, 1.37, 0] | [0, 1.385, 0] |
| LOC | 0.954 | × | × | × | × |
| AACC($d_i$:fix) | 1.127 | 1.994 | 2.684 | 3.053 | × |
| AACC($d_i$:GA) | 1.177 | 2.069 | 2.889 | 3.057 | 3.430 |

× : very large value

$x=[x[1],x[2],x[3]]^T=[E_I-\hat{E}_I,\delta-\hat{\delta}_0,\dot{\delta}]^T$ and $u=E_{fd}-\hat{E}_{fd}$, so that

$$\begin{bmatrix}\dot{x}[1]\\ \dot{x}[2]\\ \dot{x}[3]\end{bmatrix}=\begin{bmatrix}f_1(x)\\ f_2(x)\\ f_3(x)\end{bmatrix}+\begin{bmatrix}g_1(x)\\ 0\\ 0\end{bmatrix}u \quad (16)$$

where

$$\begin{aligned}
f_1(x) &= -\frac{1}{kT'_{d0}}\left(x[1]+\hat{E}_I\right)\\
&\quad+\frac{(X_d-X'_d)\widetilde{V}Y_{12}}{k}x[3]\cos\left(\theta_{12}-x[2]-\hat{\delta}_0\right)\\
f_2(x) &= x[3]\\
f_3(x) &= -\frac{\widetilde{V}Y_{12}}{\widetilde{M}}\left(x[1]+\hat{E}_I\right)\cos\left(\theta_{12}-x[2]-\hat{\delta}_0\right)\\
&\quad-\frac{Y_{11}\cos\theta_{11}}{\widetilde{M}}\left(x[1]+\hat{E}_I\right)^2-\frac{\widetilde{D}(x)}{\widetilde{M}}x[3]+\frac{P_{in}}{\widetilde{M}}\\
\widetilde{D}(x) &= \widetilde{V}^2\left\{\frac{T''_{d0}(X'_d-X''_d)}{(X'_d+X_e)^2}\sin^2\left(x[2]+\hat{\delta}_0\right)\right.\\
&\quad\left.+\frac{T''_{q0}(X_q-X''_q)}{(X_q+X_e)^2}\cos^2\left(x[2]+\hat{\delta}_0\right)\right\}\\
g_1(x) &= \frac{1}{kT'_{d0}},\quad k=1+\left(X_d-X'_d\right)Y_{11}\sin\theta_{11}.
\end{aligned}$$

Assume that the constrained input is subject to

$$u_{min}+\hat{E}_{fd}\le E_{fd}\le u_{max}+\hat{E}_{fd}.$$

Parameters are $\widetilde{M}=0.016095[pu]$, $T'_{d0}=5.09907[sec]$, $\widetilde{V}=1.0[pu]$, $P_{in}=1.2[pu]$, $X_d=0.875[pu]$, $X'_d=0.422[pu]$, $Y_{11}=1.04276[pu]$, $Y_{12}=1.03084[pu]$, $\theta_{11}=-1.56495[pu]$, $\theta_{12}=1.56189[pu]$, $X_e=1.15[pu]$, $X''_d=0.238[pu]$, $X_q=0.6[pu]$, $X''_q=0.3[pu]$, $T''_{d0}=0.0299[pu]$, $T''_{q0}=0.02616[pu]$.
Steady state values are $\hat{E}_I=1.52243[pu]$, $\hat{\delta}_0=48.57°$, $\hat{\dot{\delta}}_0=0.0[deg/sec]$, $\hat{E}_{fd}=1.52243[pu]$. Set $\mathbf{X}=[x^T,x[4]]^T=[x[1],x[2],x[3],x[4]]^T$, $n=3$, $\hat{X}_0=\hat{\delta}_0=48.57°$, $C(x)=x[2]$, $L=1$, $\mathbf{Q}=\mathrm{diag}(1,1,1,1)$, $\mathbf{R}=1$, $d_0=1$ and $x[4](0)=1$. Experiments are carried out for the new control(AACC) and the ordinary linear optimal control(LOC)[1][2].

**1) AACC**($d_i$:GA):
The parameters are suboptimally selected along the algorithm of section 3. $u_{max}=-u_{min}=0.5$. $\Omega=\{M,N,d_i,a_{ij},b_{ij},\hat{X}_i\}$. $\widetilde{G}=100$, $\widetilde{q}=100$, $\widetilde{L}=8$, $P_c=0.8$, $P_m=0.03$, $\mathbf{D}$=[-1,1]×[-1,1.5]×[-5,5]×[0,1.5]. It results that $M=1$, $N=1.03$, $d_1=0.20$, $a=53.8°$ and $\hat{X}_1=90°$.

**2) AACC**($d_i$:fix):
The parameters are suboptimally selected by using a similar way of the **AACC**($d_i$:GA) when the weight is fixed at $d_i=1(i=1,\cdots,M)$. $\Omega=\{M,N,a_{ij},b_{ij},\hat{X}_i\}$. It results that $M=1$, $N=1.14$, $a=49.4°$ and $\hat{X}_1=80°$.

Table 1 shows performances by the AACC and the LOC. The cost function of Table 1 is $\widetilde{J}=\frac{1}{2}\int_0^{20}\left(\mathbf{X}^T\mathbf{Q}\mathbf{X}+u^T\mathbf{R}u\right)dt$. These results indicate that the AACC is better than the LOC.

## 5. Conclusions

We have studied an augmented automatic choosing control using the weighted automatic choosing functions for nonlinear systems with constrained inputs. This approach have been applied to a field excitation control problem of power system. Computer simulations have shown that this controller using the GA can improve performance remarkably well.

## References


[1] Y. N. Yu, K. Vongsuriya and L. N. Wedman, "Application of an Optimal Control Theory to a Power System", *IEEE Trans. Power Apparatus and Systems*, 89-1, pp.55–62, 1970.

[2] A. P. Sage and C. C. White III, "Optimum Systems Control (2nd edition)", *Prentice-Hall, Inc.*, 1977.

[3] M. Vidyasagar, "Nonlinear Systems Analysis", *Prentice-Hall, Inc.*, 1978.

[4] A.Isidori, "Nonlinear Control Systems : An Introduction (2nd edition)", *Springer-Verlag*, 1989.

[5] Y.Takagi and T.Shigemasa, "An Application of State-Space Linearization to a Power System Stabilizer", *Proc. 29th IEEE CDC*, TA8-1, pp.1553–1558, 1990.

[6] H.Takata, "An Automatic Choosing Control for Nonlinear Systems", *Proc. 35th IEEE CDC*, FA10-5, pp.3453–3458, 1996.

[7] D. E. Goldberg, "Genetic Algorithms in Search, Optimization, and Machine Learnings", *Addison-Wesley Pub. Co. Inc.*, 1989.